FIR Filter design using Wavelet Coefficients

Pushpavathi.K.P
Department of Telecommunication Engineering
BMS.College of Engineering
Bangalore, India
Email:pushpakp.tce@bmsce.ac.in

B.Kanmani
Department of Telecommunication Engineering
BMS.College of Engineering
Bangalore, India
Email:bkanmani.tce@bmsce.ac.in

***Abstract* — The recent developments in the field of digital signal processing makes use of the wavelet mathematical function. The multi-resolution ability of the wavelets makes it outperform Fourier transformations in terms of data compression and quality. This paper shows that wavelet coefficients can be considered as digital Finite Impulse Response (FIR) filter coefficients in the FIR filter design. Other most popular FIR filter design methods are the windowing method, Frequency sampling method and optimal filter design method. The paper also presents the comparison of the popularly used rectangular window method filter design with that of the wavelets.**

Keywords—wavelets, FIR filters, window method

I. Introduction

Digital filters find various applications in signal processing. Digital filters can be broadly classified as the IIR (Infinite Impulse Response) or the FIR (Finite Impulse Response), based on the duration of the impulse response of the filter. FIR filters are always stable and can be designed to be linear phase, and hence have a preference over the IIR filters, where one needs to consider and ensure stability. Design of FIR filters is well established and few methods of design are usually included in the under graduate course on 'Digital Signal Processing'. Prominent design techniques for the FIR digital filters are: (i) design using the Window method, (ii) design through the frequency sampling method, and (iii) Optimization techniques. In the design of FIR filters using the Window method, one represents the desired frequency response using the Fourier series representation (which results in an impulse response of infinite duration), and use suitable Window functions to truncate and arrive at the desired FIR filter. With increase in the length of the filter, the error between the desired and the designed filter can be reduced. On the other hand, design using the frequency sampling method involves taking fixed frequency samples from the desired frequency response, and computing its inverse discrete Fourier transform to obtain the desired FIR filter coefficients. Here, there is an exact representation of the designed and the desired filter at the instants of sampling, and ripples in between. In this method also the error decreases with increase in the length of the filter. The third method of FIR filter design is based on applying optimization techniques to arrive at the desired filter.

The wavelet transforms are another well-developed mathematical functions widely used in signal processing. We know that the Fourier Transforms are used to obtain the frequency domain representation of a signal, and any signal can be completely represented either by its time-domain representation or its frequency domain representation (using the Fourier Transforms). In other words, when we compute the Fourier transform of a signal, we move to the frequency domain from the time domain. This mapping from the time to the frequency domain is unique (for all signals that obey the Dirichlet's condition). In computing the wavelet transform of a given signal, there is a need to recognize that there is no change in the domain. In this aspect, the wavelet transform can be compared to the Hilbert transform, where the original signal and the Hilbert transformed signals are both in the time domain. Wavelets have found application in varying domains, and are specially used in de-noising and data compression. Wavelets are also used in the solution of partial differential equations. However, the use of wavelets in filter design is not observed. In this work, we attempt to use the wavelet coefficients as FIR filters. We compare the performance of filter design using the rectangular window function with that of Daubechies wavelet coefficients as FIR filters.

The rest of the paper is as follows: in Section II we have a brief introduction about wavelets. We have considered the example of Haar wavelet and Daubechies 2 wavelet. Section III explains that wavelet functions can be obtained from the wavelet coefficients. In section IV, we have shown that the wavelet coefficients exhibit the filter characteristics. We have a brief explanation of the window method of FIR filter design in section V followed by the simulation study of both the window method and wavelet method of FIR filter design.

II. Wavelet Transform

A wave is an infinite length, oscillating continuous function in time or space that is periodic while, a wavelet is a waveform of limited duration with an average of zero value [1]. Wavelets are mathematical functions that break up the signal into different frequency components. Each component is then analyzed with different resolution matched to its scale. Thus frequency information at instants of time can be obtained. Therefore, Wavelet transform (WT) behaves as a tool that provides the time-frequency representation of the signal [1]. The wavelet transforms are broadly classified into i) continuous wavelet transform and ii) discrete wavelet transform. In the case of long signals, the time complexity of the continuous wavelet transform (CWT) is overcome by the discrete wavelet transform (DWT).

The computation of the DWT can be done using convolution based procedure where input sequence is decomposed into low pass and high-pass sub-bands with each of them consisting of half the number of samples in the original sequence [1].

Every wavelet is represented by the scaling function, $\varphi(t)$ and wavelet function, $\eta(t)$. The discrete wavelet transform (DWT) can be scaled and translated in discrete steps. Here the time-scale space is sampled at discrete intervals [2].

A. Example 1: Haar wavelet

The simplest wavelet is the Haar wavelet. It is also called Daubechies 1 (db1) wavelet. A discrete signal is decomposed into two sub signals of half its length by the Haar transform. One sub signal is the running average also called trend while the other is the running difference called fluctuation [3]. As the Haar wavelet is not continuous, they are not differentiable.

Considering Haar wavelet, equation (1) shows the Haar scaling coefficients ak = [1 1] and Haar wavelet coefficient, bk= [1 -1] [4]. Let the Haar scaling function be given as in equation (2) and Haar wavelet function be given as in equation (3). Haar scaling function and Haar wavelet function is shown in figure 1.

$$\text{Haar wavelet : } \begin{cases} \text{scaling coefficients , ak=[1 1]} \\ \text{wavelet coefficients, bk=[1 -1]} \end{cases} \tag{1}$$

$$\varphi(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & otherwise \end{cases} \tag{2}$$

$$\eta(t) = \begin{cases} 1 & 0 < t < 0.5 \\ -1 & 0.5 < t < 1 \end{cases} \tag{3}$$

Now consider that the Haar scaling function φ(t). The function is time scaled as φ(2t). This is then time shifted to obtain φ(2t-1). By adding them we obtain equation (4) as shown in figure 2.

$$\varphi(2t) + \varphi(2t-1) = \varphi(t) \tag{4}$$

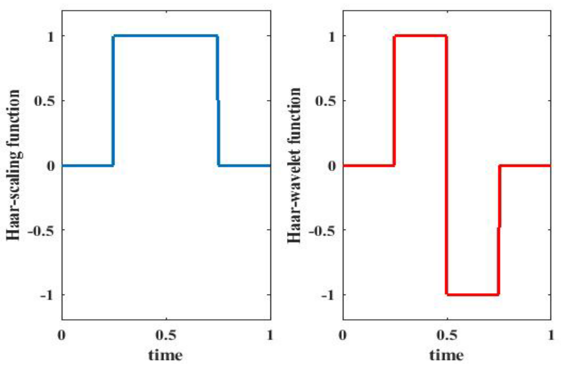
With the suitable modifications as in equation (5)

$$a_0\varphi(2t) + a_1\varphi(2t-1) = \varphi(t) \tag{5}$$

we obtain φ(t) [4], given by equation (6).

$$\varphi(t) = \sum_{k=0}^{2N-1} a_k \varphi(2t-k) \tag{6}$$

Now consider the Haar wavelet function, η(t). Taking the difference of time scaled, φ(2t) and time shifted, φ(2t-1) signal, we have equation (7), as shown in figure 3.



a b

Fig.1.a) Haar scaling function and b) Haar wavelet function

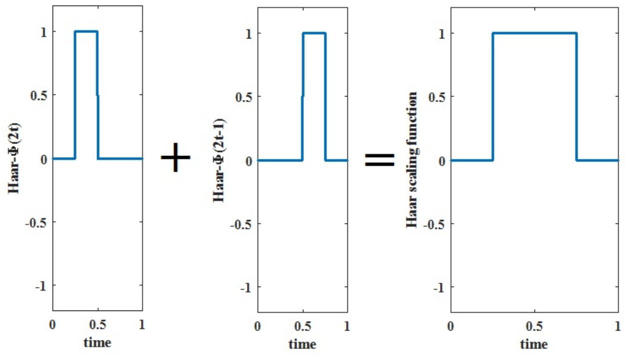


Fig 2. Time scaled and time shifted Haar scaling function

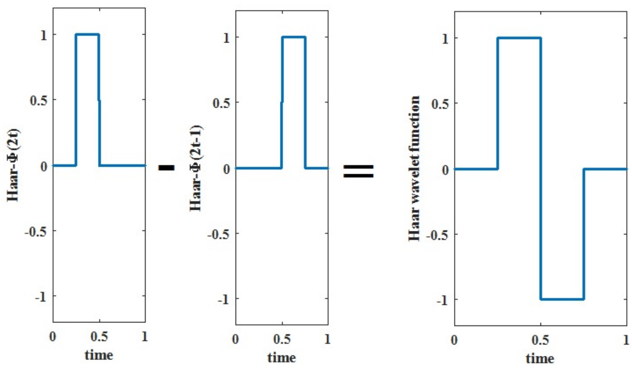


Fig.3. Time scaled and time shifted Haar wavelet function

$$\varphi(2t) - \varphi(2t-1) = \eta(t) \tag{7}$$

This holds true with suitable modification as in equation (8).

$$b_0\varphi(2t) + b_1\varphi(2t-1) = \eta(t) \tag{8}$$

This is equivalent to equation (9), where we obtain η(t)[4].

$$\eta(t) = \sum_{k=0}^{2N-1} b_k \varphi(2t-k) \tag{9}$$

The Haar scaling function and Haar wavelet function with suitable coefficients ak and bk is shown in figure 1.

In general, for any wavelets, scaling function and wavelet function is given by equation (6) and (9) respectively, where ak and bk are the filter coefficients, showing the relationship between wavelet and filter.

B. Example 2: Daubechies wavelets

The Daubechies wavelets are the family of orthogonal wavelets that defines the discrete wavelet transform and are denoted as dbA or DN, where A represents the zero moments and N represents the length or the number of taps or the number of coefficients. Most commonly used Daubechies orthogonal wavelets are db1 (D2) to db10 (D20) respectively. D2 or db1 is also called the Haar wavelet. Their wavelet function coefficients are derived based on the scaling function coefficients of the respective Daubechies wavelet. That is, the order of scaling function coefficients

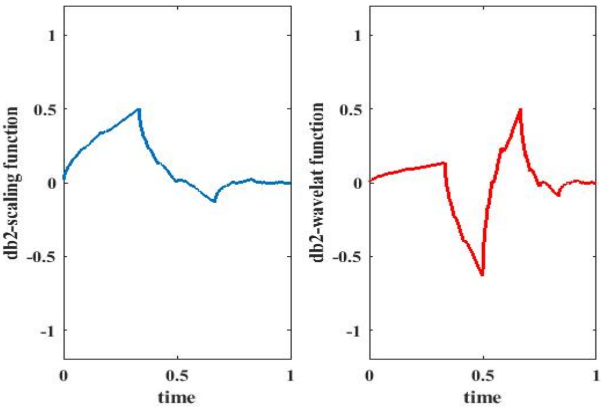


Fig.4. Daubechies 2 scaling function and the wavelet function

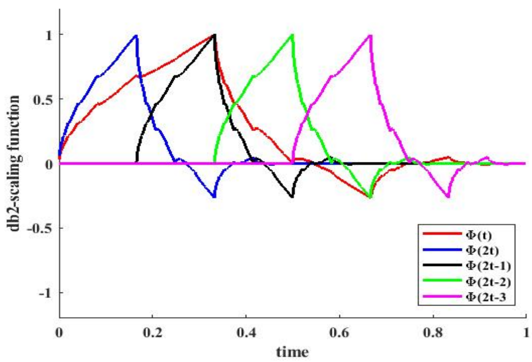


Fig.5. Daubechies 2 scaling function

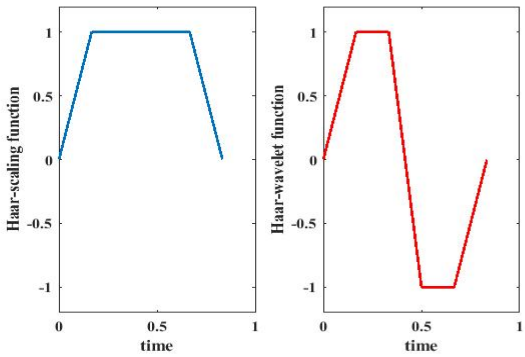
are reversed and then the sign of every second one is reversed. Figure 4 shows the Daubechies 2 wavelet. Any signal can be constructed by summing the orthogonal wavelet basis function, weighted by the wavelet transform coefficients [5] [2] as shown in figure 5.

III. WAVELET FUNCTIONS FROM THE COEFFICIENTS

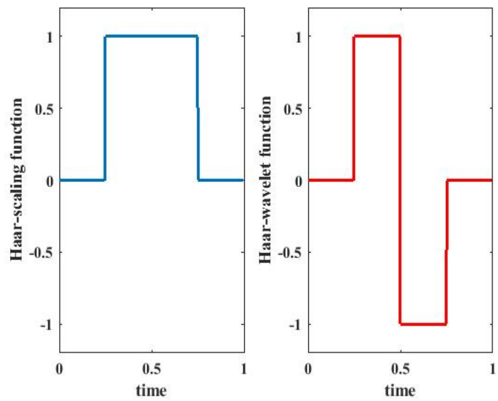
The basis functions, namely the scaling function and the wavelet function can be obtained from their respective coefficients, ak and bk. Let the scaling coefficients be ak and wavelet coefficients be bk. The coefficients are up sampled and are convolved with the original scaling coefficients. The resulting coefficients are again up sampled and convolved with the original scaling coefficients. The process is repeated till the desired function is obtained. The steps involved are given by the following:

Scaling function:
Step A: phi =ak
Step B: up_phi = up sample (phi)
Step C: phi = convolve (up_phi, ak)
Step D: repeat step B and step C until the scaling function is well represented.

Similarly, the wavelet function
Step A: eta =bk
Step B: up_eta = upsample (eta)
Step C: eta = convolve (up_eta, ak)
Step D: repeat step B and step C until the wavelet function is well represented.



a)

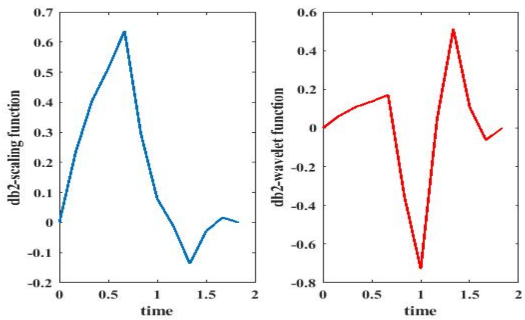


b)

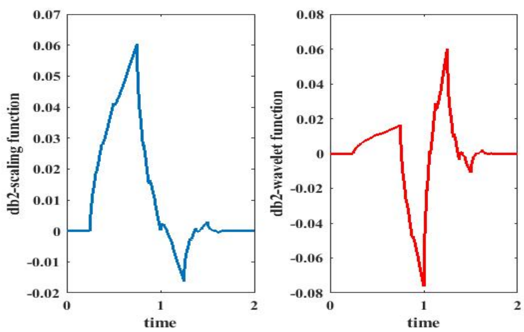
Fig.6. Haar basis function for a) N=1 and b) N=8 where N is the number of iterations

We observe that after 8 to 10 iterations we have a good representation of the functions. Haar basis function from coefficients is as shown in figure 6.

Similarly let us consider the Daubechies wavelets.



a)



b)

Fig.7. Daubechies 2 basis function for a) N=1 and b) N=8 where N is the number of iterations

Let Daubechies 2 wavelet have coefficients given as,

ak = [0.4830   0.8365   0.2241   -0.1294]

bk = [-0.1294   -0.2241   0.8365   -0.4830]

They represent the scaling and wavelet coefficients respectively. The resultant functions obtained from coefficients are as given in equation (10) and is as shown in figure 7.

$$\varphi(t) = 0.4830\varphi(2t) + 0.8365\varphi(2t-1) + 0.2241\varphi(2t-2) - 0.1294\varphi(2t-3) \tag{10}$$

$$\eta(t) = -0.1294\varphi(2t) - 0.2241\varphi(2t-1) + 0.8365\varphi(2t-2) - 0.4860\varphi(2t-3)$$

IV. Wavelet coefficients in frequency domain

Wavelets can be viewed to have a band-pass like spectrum [2]. This uses the fact that the time compression of the wavelet by a factor of 2 will stretch its frequency spectrum by a factor of 2 and shift all the frequency components up by the factor of 2. This wavelet spectrum can be obtained by scaling the mother wavelet in the time domain. Scaling function may be considered as a signal with low-pass spectrum while the wavelet function can be viewed to resemble the high pass. The figure 8 and figure 9 shows that the scaling coefficients and the wavelet coefficients exhibit the low-pass filter and the high-pass filter response respectively. This explains that the wavelet coefficients ak and bk can be considered the filter coefficients.

a)

b)

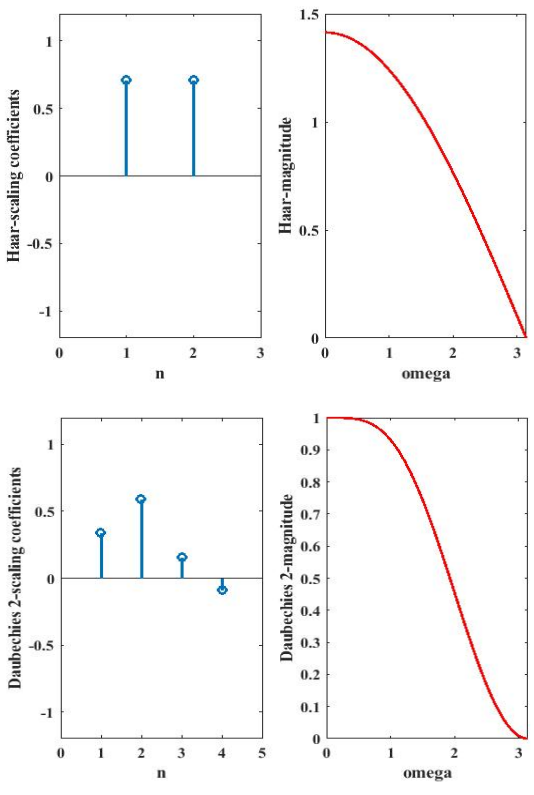


Fig.8 Magnitude response for a) Haar scaling function b) Daubechies 2 scaling function

a)

b)

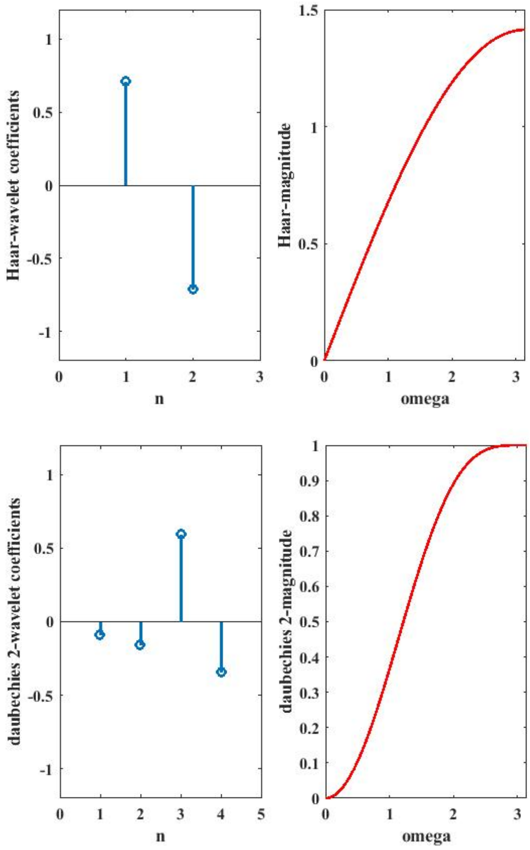


Fig.9. Magnitude response for a) Haar wavelet function b) Daubechies 2 wavelet function

V. Window Method

Most commonly used method of designing the linear phase FIR filter is the window method. In this method, the Filter coefficients h[n] are obtained by the finite duration weighing sequence w[n]. The impulse response of the digital FIR filter with the desired frequency response $H_{desired}(\omega)$ can be obtained directly by the equation (11) and (12) [6].

$$\mathrm{h}[n] = \frac{1}{2\pi} \int_{<2\pi>} H_{desired}(\omega) e^{i\omega n} d\omega \tag{11}$$

$\forall$ n varying from -∞ to ∞

And the frequency response is given by

$$H_{desired}(\omega) = \sum_{n=-\infty}^{\infty} \mathrm{h}[n] e^{-i\omega n} \tag{12}$$

Equation 11 represents the Infinite Impulse Response (IIR) filter as the filter impulse response has infinite duration. The infinite impulse response is truncated to a finite length using the window. Of the commonly used windows like hamming window, Kaiser Window, we use the rectangular window for our study.

In the rectangular window method, we obtain the finite Impulse Response (FIR) filter coefficients from the Infinite duration response, through the use of a rectangular window as given by equation (13) [6]

$$h_{designed}[n] = \frac{1}{2\pi} \int_{<2\pi>} H_{desired}(\omega) e^{i\omega n} d\omega \tag{13}$$

$\forall$ n varying from -N/2 to N/2

The results of this truncation gives rise to errors and overshoot due to Gibbs Phenomenon [7]. Overshoot occurs at the discontinuity, when a Fourier series is used to approximate a function with discontinuities [8]. It describes the quality of the approximation.

VI. SIMULATION STUDY

In this section, we discuss the design of FIR filter using the rectangular window method and the wavelets. We compare the frequency response of the desired response with that of the designed filter for different values of filter order, N. The percentage overshoot and the mean square error are the two parameters considered for our study.

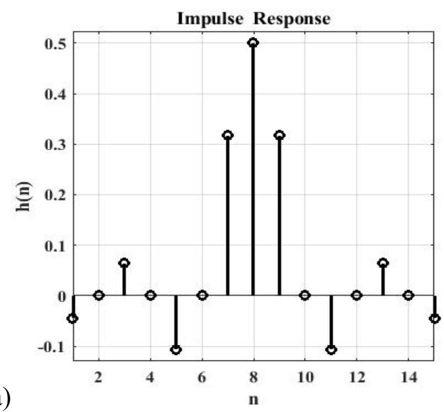
From equation 13, we have the FIR low-pass filter coefficients given by using equation (14) [6]

$$h[n] = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega$$

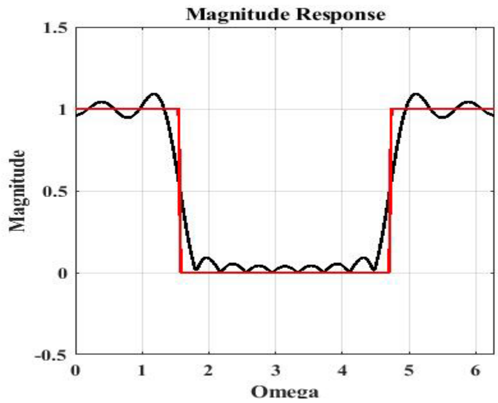
$$= \frac{\omega_c}{\pi} sync(\omega_c n) \tag{14}$$

$\forall$ n varying from -N/2 to N/2

Here the cut-off frequency of the low-pass filter is taken to be $\omega_c=\pi/2$ and the response obtained is shown in figure 10 and figure 11 for N=15 and N=25, respectively, N being the order of the filter.

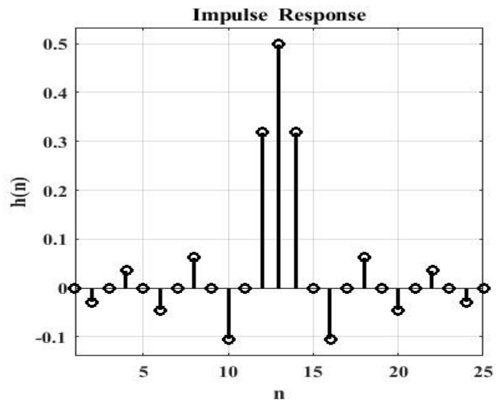


a)

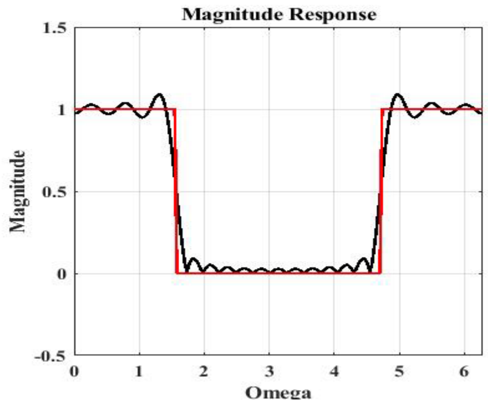


b)

Fig.10. a) the impulse response and b) the plot of desired frequency response (red) along with the plot of frequency response of designed (black) FIR filter using the rectangular window method



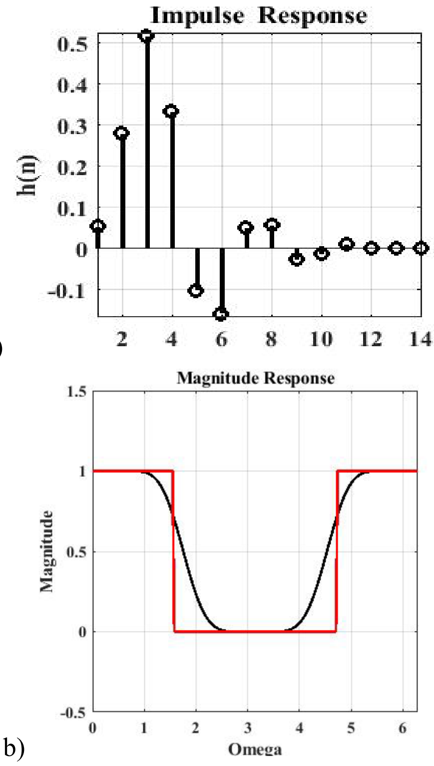
a)



b)

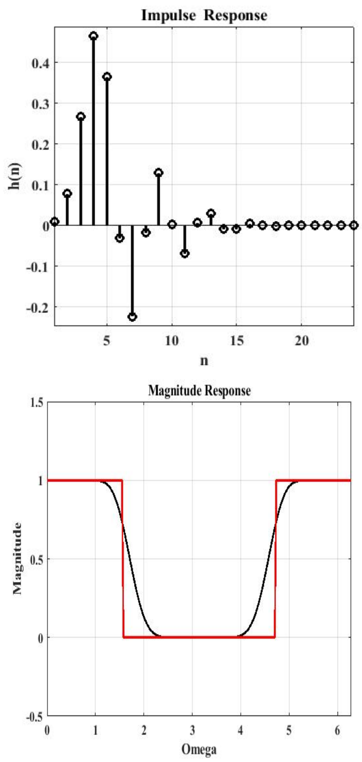
Fig. 11 a) the impulse response and b) the plot of desired frequency response (red) along with the plot of frequency response of designed(black) FIR filter using the rectangular window method

We now attempt to design the filter using wavelet method where the wavelet coefficients are taken as the filter coefficients, from which the frequency response for cut-off of $\omega_c=\pi/2$ is obtained. It is to be taken care that the order of the filter is always even and that the filter coefficients are non-symmetric. Figure 12 and figure 13 shows the results obtained for N=14 and 24 respectively.



a) b)

Fig.12. a) the impulse response and b) the plot of desired frequency response (red) along with the plot of frequency response of designed (black) FIR filter using the wavelet method.



a) b)

Fig.13. a) The impulse response and b) the plot of desired frequency response (red) along with the plot of frequency response of designed (black) FIR filter using the wavelet method.

TABLE 1. COMPARISON OF MEAN-SQUARE-ERROR AND THE OVERSHOOT FOR DIFFERENT VALUES OF FILTER ORDER FOR THE FIR FILTER

Filter order N	**Using Rectangular window coefficients**		**Using Wavelet coefficients**	
	m.s.e	***overshoot***	***m.s.e***	***overshoot***
11	0.1521	0.0941	0.2840	0
15	0.0791	0.0918	0.2405	0
25	0.0528	0.0892	0.1848	0
39	0.0315	0.0889	0.1480	0
43	0.0286	0.0886	0.1411	0

The design being repeated for different values of N, the results are presented in table 1.The table shows that the mean square error (m.s.e) reduces with the increase in the filter order, N. Based on this factor, the rectangular method of design is observed to be marginally better than wavelet method. Observation of the overshoot shows that the overshoot in the case of design using wavelets is always unity while rectangular window method shows the small change in overshot with the increase in N.

VII. CONCLUSION

In this work, it is studied that the scaling and the wavelet coefficients of the wavelets can be considered as the FIR filter coefficients that exhibit the low-pass and the high-pass filter characteristics respectively. Also, the occurrence of the overshoot in the case of rectangular window method and that using wavelets is studied. The results show that the wavelet filters do not exhibit overshoot. From the simulation study, the mean-square error in the rectangular window method is found to be marginally better than the one using wavelets.

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