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Frequency Sampling method of FIR Filter design: A comparative study

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***Abstract*— In this paper, attempt is made to study the frequency sampling method of FIR (Finite Impulse Response) filter design in comparison with rectangular window method. The design methods are studied for the cases of the low pass filter, high pass filter and for an arbitrary response filter. The paper demonstrates that the method proposed here is the simplest way of designing any arbitrary response filter.**

Keywords- Digital filters, FIR and IIR filters, frequency sampling

I. INTRODUCTION

Digital signal processing has developed over the years in the field of computer technology and micro industry. It is used in the area of signal processing and wireless communication [1]. FIR filters also called the all- zero filters, find their applications where linear phase response is required. Certain signal processing systems include the narrow band linear phase filters. They are generally designed using the window method or Optimal Filter design method that uses direct convolution structure to implement them [2]. The frequency sampling method of FIR filter design is used where the frequency sampling structures are used. This is because the frequency sampling structure has its coefficients that are the values of the filter's frequency response sampled at N equally spaced points. Such a structure can efficiently realize the filter with very less number of non-zero samples [3].

II. METHODS FOR DESIGN OF FIR FILTERS

The well-known linear phase FIR filter design methods are the windowing, frequency sampling and optimal filter design methods [4-6]. Brief comparison of the FIR design methods are presented in this section. One of the better ways to obtain the FIR filter is the windowing method. Here the Fourier coefficients h[n] are modified by the finite duration weighting sequence w[n]. In the frequency sampling method, the desired frequency response is approximated by sampling it at N points spaced equally and evaluated for the sampled frequency response. Optimal filter designing can be done using different methods. The basic idea in these methods is to obtain the filter coefficients repeatedly till a particular error is reduced to minimum.

In this paper we compare the method of rectangular window method with the Frequency sampling method.

A. *Window method*

The window method of FIR filter design is based on the Fourier Series representation of the desired filter frequency response $H_{desired}\ \omega$, as the frequency response of digital filters are periodic with period 2π . Hence, the impulse response of the filter, that matches the desired frequency response $H_{desired}\ \omega$, can be obtained directly by the expression for Fourier coefficients, given by equation (1).

$$\mathrm{h}[n]=\frac{1}{2\pi}\int_{<2\pi>} H_{desired}\ \omega\ e^{i\omega n}d\omega \qquad (1)$$

$\forall$ n varying from $-\infty$ to ∞

where, the frequency response is related to the sequence h[n] through equation (2) [7],

$$H_{desired}\ \omega\ =\sum_{n=-\infty}^{\infty}\mathrm{h}[n]e^{-i\omega n} \qquad (2)$$

From equation (1), we observe that the filter coefficients, representing the filter impulse response, have infinite duration, and hence represent the IIR (Infinite Impulse Response) filter.

A. *Rectangular window design*

The impulse response of infinite duration is truncated to a finite length, through the use of a rectangular window, to obtain the FIR (Finite Impulse Response) filter coefficients, as shown below:

$$\mathrm{h}\ [n]=\frac{1}{2\pi}\int_{<2\pi>} H_{desired}\ \omega\ e^{i\omega n}d\omega \qquad (3)$$

$\forall$ n varying from -N/2 to N/2

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This truncation results in error, and also the overshoot due to the Gibb's phenomenon [7-9]. The error due to truncation reduces with increasing N, while the overshoot is not dependent on N. Through introducing a suitable time shift in the impulse response, we then have the causal FIR filter of length (N+1). For the designed filter, frequency response is given by equation (4)

$$H_{designed}(\omega) = \sum_{n=0}^{N} h[n]e^{-j\omega n} \tag{4}$$

The mean square error in the designed filter is then given by equation (5)

$$mse = \frac{1}{2\pi}\int_0^{2\pi} \left|H_{desired}(\omega) - H_{designed}(\omega)\right|^2 d\omega \tag{5}$$

a) Low Pass Filter:

We now attempt to find the filter coefficients for the ideal low pass digital filter with cut-off ω_c for $H_{desired}(\omega)$ defined by

$$H_{desired}(\omega) = \begin{cases} 1 & if\ |\omega| \le \omega_c \\ 0 & if\ \omega_c < |\omega| < \pi \end{cases} \tag{6}$$

Through the use of equation (3), we then have the filter coefficients [7-10], given by:

$$h[n] = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega$$

$$= \frac{\omega_c}{\pi} sync(\omega_c n) \tag{7}$$

$\forall$ n varying from -N/2 to N/2

Hence, through equation (7) we have the filter coefficients of the FIR low pass filter with (N+1) coefficients.

b) High Pass Filter

We now apply the method to obtain the filter coefficients for the ideal high pass digital filter with cut-off ω_c for $H_{desired}(\omega)$ defined by

$$H_{desired}(\omega) = \begin{cases} 0 & |\omega| \le \omega_c \\ 1 & \omega_c < |\omega| < \pi \end{cases} \tag{8}$$

Through the use of equation (3), we then have the filter coefficients, for the high pass filter given by:

$$h[n] = \frac{1}{2\pi}\int_{\pi-\omega_c}^{\pi+\omega_c} e^{j\omega n} d\omega$$

$$= (-1)^n \frac{\omega_c}{\pi} sync(n\omega_c) \tag{9}$$

$\forall$ n varying from -N/2 to N/2

Comparing the results for the filter coefficients of the low pass filter equation (7), with that of the high pass filter equation (9), we observe that the low pass filter can be converted to a high pass filter purely by multiplying the filter coefficients with $(-1)^n$.

c) Arbitrary filter

The same method can be used to design the FIR filter for any given frequency response.

One among the major constraints in design of the FIR filters is the need to compute the integral as given in equation (1). For simple frequency response, this method is computationally feasible and can be easily implemented. However, when the desired filter frequency response is arbitrary, a closed form of expression for the filter coefficients is not possible, and this becomes a major constraint for the FIR filter design using the rectangular window method.

B. Frequency sampling method

The frequency sampling method of FIR filter is based on obtaining N uniformly spaced samples from the magnitude of the given frequency response $H_{desired}(\omega)$. Let the uniform spaced samples of frequency response be represented by $H[k]$ $k = 0,1,...,N-1$. For N being odd, we know that to ensure linear phase characteristic, the phase response has slope $\alpha = (N-1)/2$ [7]. The magnitude and the phase together specify the frequency samples H[k], which can be identified as the Discrete Fourier Transform (DFT) of the filter impulse response. Hence, the frequency samples from the desired frequency samples are given by equation (10).

$$H[k] = |H(\omega)|\Big|_{\omega=\frac{2\pi}{N}k} e^{-j\frac{2\pi(N-1)}{N}k} \qquad k = 0,..,N-1 \tag{10}$$

Hence, through the inverse DFT of H[k], we obtain the filter coefficients of the desired FIR filter, using the equation (11).

$$h[n] = \frac{1}{N}\sum_{k=0}^{N-1} H[k]e^{j2\pi nk/N} \qquad n = 0,1,...,N-1 \tag{11}$$

The greater the number of frequency samples, the smaller the error of approximation [7]. The main advantage of this method is that any computation complexity is identical for any frequency response, and the method does not involve any integration as in the case of the design of rectangular window method. This leads to the FIR filter of length N. The advantage of this method over the windowing method is that it can be used for any given magnitude response.

In the next Section, we implement the FIR filter design for LPF, HPF and an arbitrary response, and compare the filter response for both the rectangular and the frequency sample method, and compare their performance in terms of mean square error and the overshoot.

III. Simulation Study

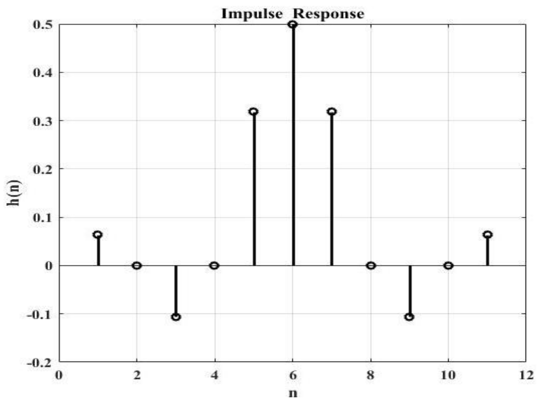
We now implement the FIR design using the rectangular window method and the frequency sampling method and compare the performance for three cases: (i) Low pass filter with cut-off $\omega_c = \pi/2$, (ii) high pass filter with cut-off $\omega_c = \pi/2$, and (iii) filter with arbitrary magnitude response. The frequency response of the desired response is compared with that of the designed filter, for various values of filter order N, for two parameters: the mean square error and the percentage overshoot.

A. *Low pass filter*

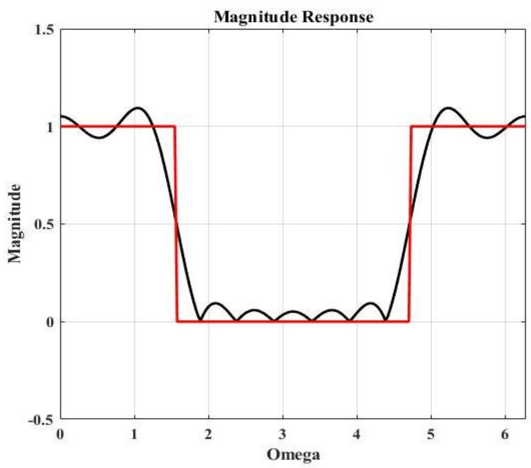
The cut-off frequency of the low pass filter being $\omega_c = \pi/2$, results in the filter coefficients of the desired filter using, equation (7), the rectangular window as:

$$h[n] = \frac{1}{2} sync\left(\frac{n\pi}{2}\right) \qquad n = -N/2,....., N/2 \qquad (12)$$

The above method applied for N=11, is shown in Figure 1.



(a)



(b)

Figure 1. (a)The impulse response of the FIR filter using the rectangular window method, (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).

We now attempt to obtain the filter design using the frequency sampling method, wherein the Frequency samples H[k], can be obtained by using equation (10).Considering

$$H_{desired}(\omega) = \begin{cases} 1 & if\ |\omega| \le \omega_c \\ 0 & if\ \omega_c < |\omega| < \pi \end{cases}$$

as that of the ideal Low pass filter with similar cutoff of $\omega_c = \pi/2$, it can be observed from Figure 2, that few frequency samples are zero, for this specified cut-off.. N equal samples in an interval of $[0,2\pi)$ considered are obtained by sampling the desired frequency response at equally spaced frequencies as shown in the Figure 2, for N=11.

Using equation (11), we obtain the filter coefficients h[n], for the frequency response shown in Figure 2, and the results are shown in Figure 3. Figure 4, includes the pole-zero plot for the filter designed using the rectangular window method and using the frequency sampling method, for N=11.

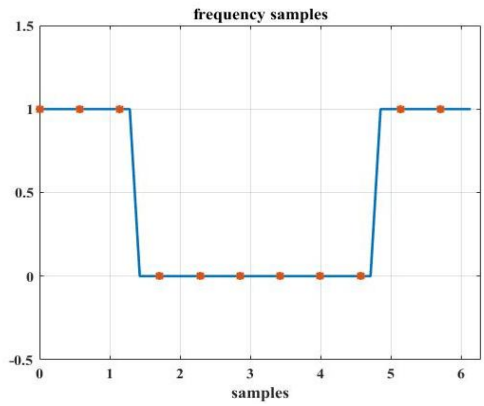
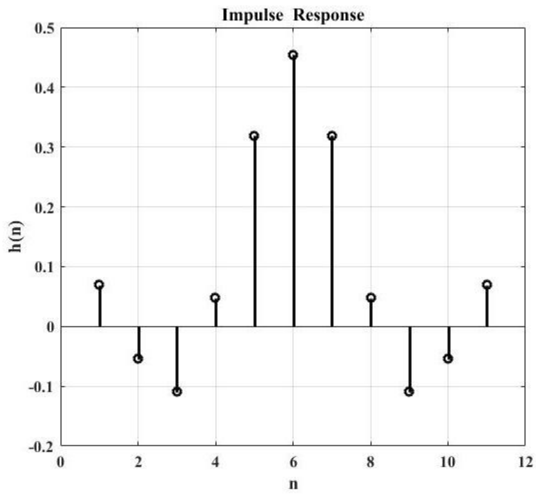


Figure 2. The desired frequency response for Low pass filter cut-off frequency $\omega_c = \pi/2$, together with the N equally spaced frequency samples for N=11



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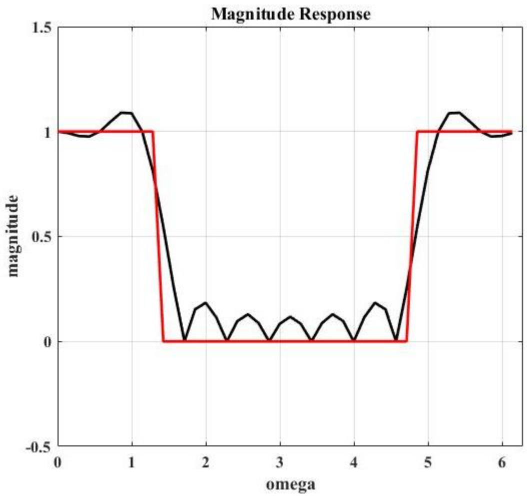
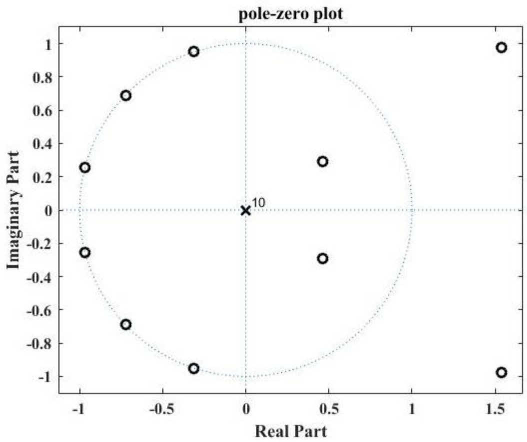


(b)

Figure 3(a). The impulse response of the FIR filter using the frequency sampling method for N=11. (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).



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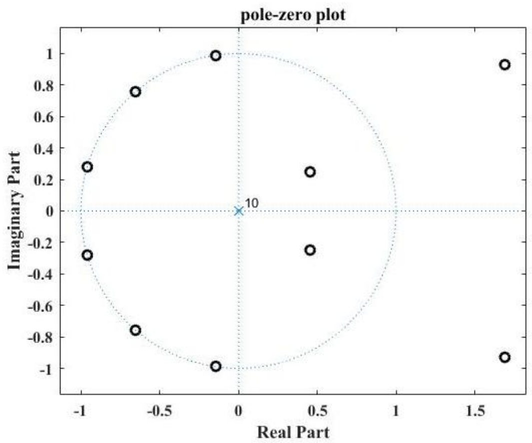


(b)

Figure 4. **(a)**The pole-zero plot for N=11, for the low pass FIR using the rectangular window method of design, (b) The frequency sample method of design.

TABLE I COMPARISON OF MEAN-SQUARE-ERROR (M.S.E) AND THE OVERSHOOT FOR DIFFERENT VALUES OF FILTER ORDER FOR THE LOW PASS FILTER

Filter Order N	**Rectangular Window Method of filter design**		**Frequency sampling method of filter design**	
	M.S.E	***overshoot***	***M.S.E***	***overshoot***
11	0.1052	1.0941	0.1393	1.0895
23	0.0528	1.0892	0.0666	1.1150
35	0.0351	1.0892	0.0438	1.1224
47	0.0262	1.0849	0.0326	1.1261

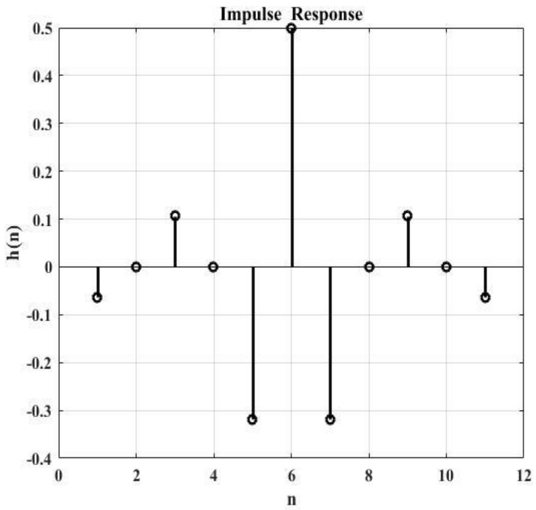
The design is repeated for different values of N, and the results are presented in Table I. It can be observed the mean square error decreases with increasing N, while there is little change in the overshoot. It can also be observed that the design using rectangular window method is marginally better than the filter designed using the frequency sampling method. However, as mentioned earlier, the need to compute the integral, (equation 1), is avoided in this method without significant deterioration in performance.

B. High pass filter

The cut-off frequency of the high pass filter being $\omega_c = \pi / 2$, results in the filter coefficients of the desired filter using, equation (9), the rectangular window as:

$$h[n] = (-1)^n \frac{1}{2} sync\left(\frac{n\pi}{2}\right) \qquad n = -N/2, \ldots\ldots, N/2 \tag{13}$$

The above method applied for N=11, is shown in Figure 5.



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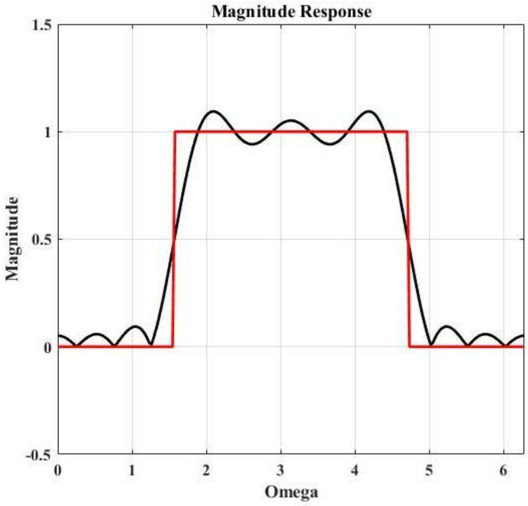


(b)

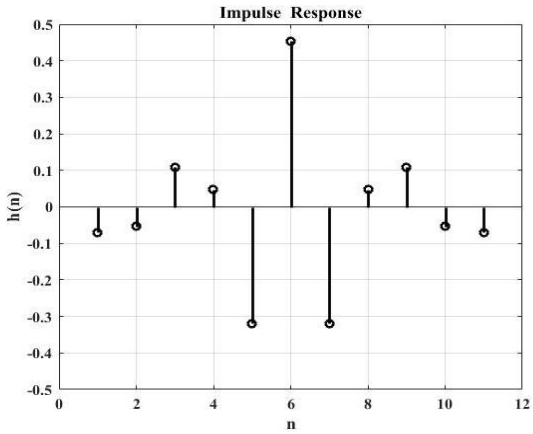
Figure 5. (a) The impulse response of the FIR filter using the rectangular window method for N=11, (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).

We now attempt to obtain the filter design using the frequency sampling method, wherein the Frequency samples H[k], can be obtained by using equation (10).Considering

$$H_{desired}(\omega) = \begin{cases} 0 & |\omega| \le \omega_c \\ 1 & \omega_c < |\omega| < \pi \end{cases}$$

as that of the ideal high pass filter with similar cutoff of $\omega_c = \pi / 2$, N equal samples in an interval of [0,2π) are obtained by sampling the desired frequency response at equally spaced frequencies for N=11. The high pass filter coefficients are obtained by multiplying the alternate coefficients of the low pass filter by (-1).

Using equation (11), we obtain the filter coefficients h[n], for the frequency response of the high pass filter, and the results are shown in Figure 6. Figure 7, includes the pole-zero plot for the filter designed using the rectangular window method and using the frequency sampling method, for N=11.



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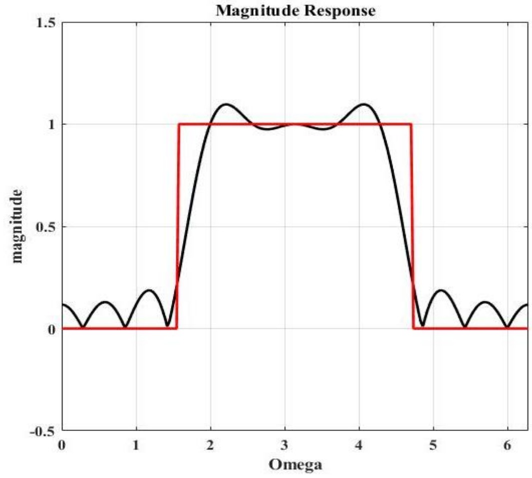
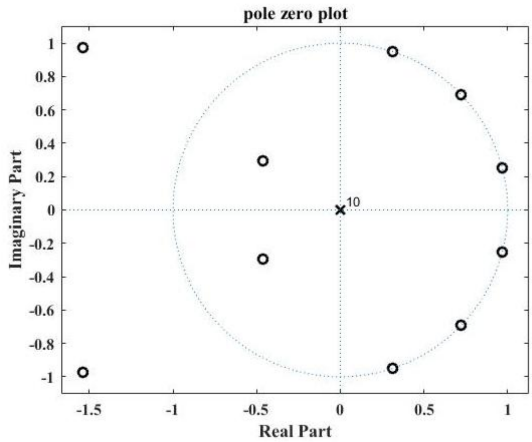


(a)

Figure 6. (a)The impulse response of the FIR filter using the frequency sampling method for N=11. (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).



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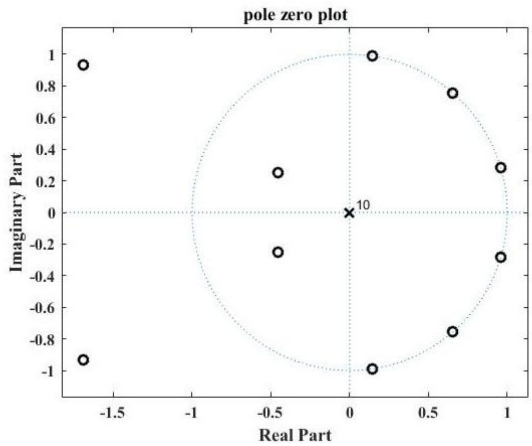


(b)

Figure 7. (a). The pole-zero plot for N=11, for the high pass FIR using the rectangular window method of design (b). The frequency sample method of design.

TABLE II COMPARISON OF MEAN-SQUARE-ERROR (M.S.E) AND THE OVERSHOOT FOR DIFFERENT VALUES OF FILTER ORDER FOR THE HIGH PASS FILTER

Filter Order N	Rectangular Window Method of filter design		Frequency sampling method of filter design	
	M.S.E	***overshoot***	***M.S.E***	***Overshoot***
11	0.1052	1.0940	0.1918	1.0967
23	0.0528	1.0906	0.1020	1.1198
35	0.0351	1.0887	0.0711	1.1251
47	0.0262	1.0895	0.0556	1.1271

The design is repeated for different values of N, and the results are presented in Table II. It can be observed the mean square error decreases with increasing N, while there is little change in the overshoot. It can also be observed that the design using rectangular window method is marginally better than the filter designed using the frequency sampling method. However, as mentioned earlier, the need to compute the integral, (equation 1), is avoided in this method without significant deterioration in performance.

C. Arbitrary response

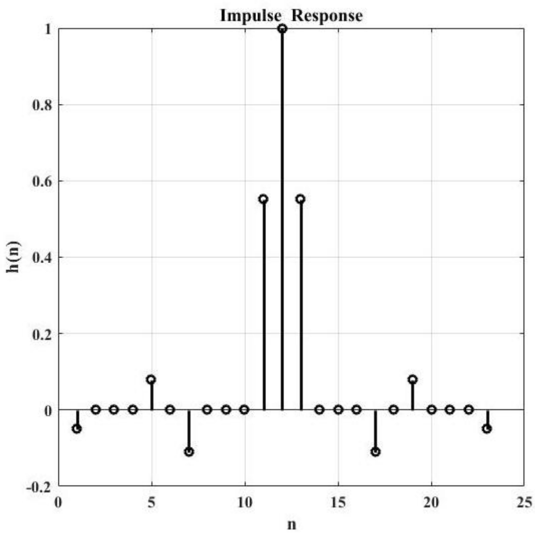
The cut-off frequency of the arbitrary filter being $\omega_1 = \pi/3$ and $\omega_2 = 2\pi/3$, results in the filter coefficients of the desired filter using, equation (14), the rectangular window as:

$$h[n] = \frac{1}{2\pi}\left[\int_{-\omega_2}^{-\omega_1} H_2\ \omega\ e^{i\omega n} d\omega + \int_{-\omega_1}^{\omega_1} H_1\ \omega\ e^{i\omega n} d\omega + \int_{\omega_1}^{\omega_2} H_2\ \omega\ e^{i\omega n} d\omega\right]$$

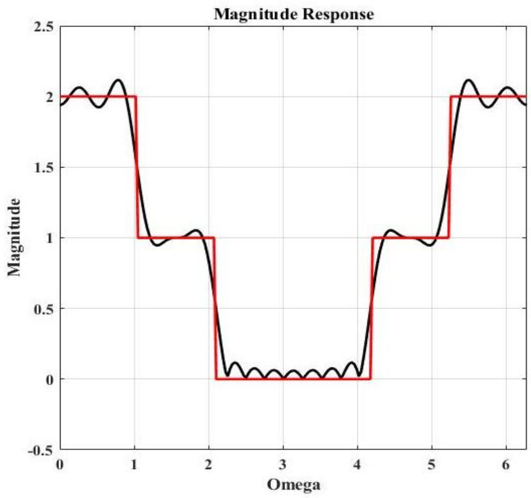
$\forall$ n varying from -N/2 to N/2

(14)

The rectangular window method applied for N=23, is shown in Figure 8.



(a)



(b)

Figure 8. (a). The impulse response of the FIR filter using the rectangular window method for N=23. (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).

We now attempt to obtain the filter design using the frequency sampling method, wherein the Frequency samples H[k], can be obtained by using equation (10).Considering

$$H_{desired}(\omega) = \begin{cases} 2 & if\ |\omega| \le \omega_1 \\ 1 & if\ \omega_1 < |\omega| < \omega_2 \\ 0 & if\ \omega_2 < |\omega| < \pi \end{cases} \qquad (15)$$

as the desired filter with cutoff of as the desired filter with cutoff of $\omega_1 = \pi / 3$ and $\omega_2 = 2\pi / 3$ it can be observed from Figure 9, that few frequency samples are zero, for this specified cut-off. N equal samples in an interval of [0,2π) considered are obtained by sampling the desired frequency response at equally spaced frequencies as shown in the Figure 9, for N=23.

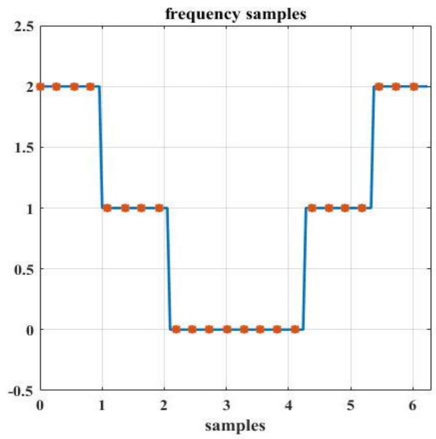
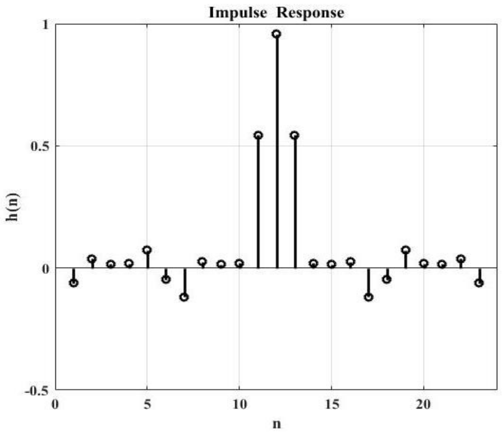


Figure 9: The desired frequency response for cut-off frequency $\omega_1 = \pi / 3$ and $\omega_2 = 2\pi / 3$, together with the N equally spaced frequency samples for N=23

Using equation (11), we obtain the filter coefficients h[n], for the frequency response shown in Figure 9, and the results are shown in Figure 10. Figure 11, includes the pole-zero plot for the filter designed using the rectangular window method and using the frequency sampling method, for N=23.



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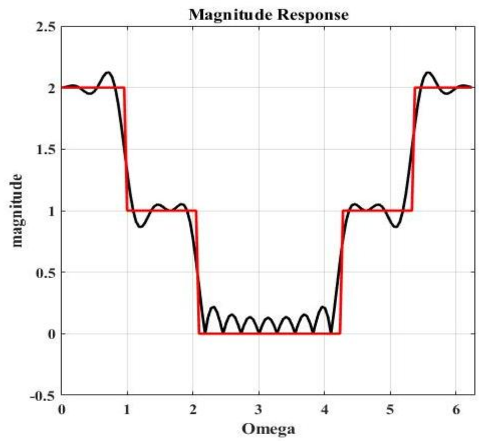
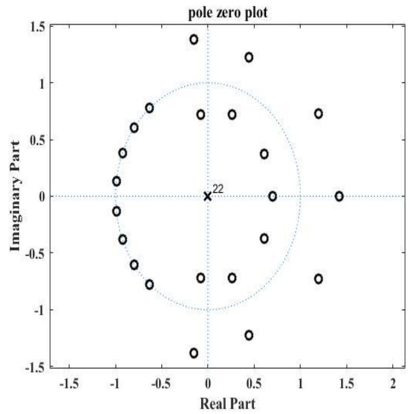
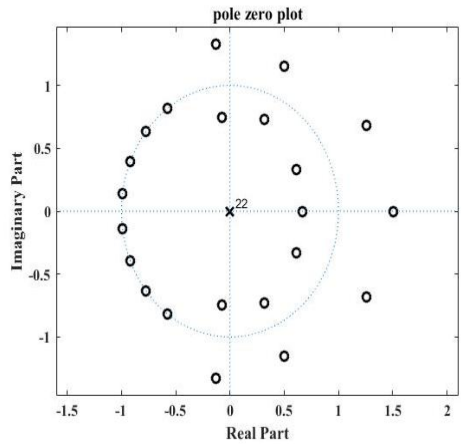


(b)

Figure 10. (a) The impulse response of the FIR filter using the frequency sampling method for N=23. (b) The plot of the desired frequency response (Red color) together with the plot of the frequency response of the designed filter (Black color).



(a)



(b)

Figure 11. (a) The pole-zero plot for N=23, for the given arbitrary response using the rectangular window method of design, (b) The frequency sample method of design.

TABLE III COMPARISON OF MEAN-SQUARE-ERROR (M.S.E) AND THE OVERSHOOT FOR DIFFERENT VALUES OF FILTER ORDER FOR THE ARBITRARY RESPONSE

Filter order N	**Rectangular Window Method of filter design**		**Frequency sampling method of filter design**	
	M.S.E	***Overshoot***	***M.S.E***	***overshoot***
11	0.2154	2.1459	0.2784	2.0836
23	0.1060	2.1167	0.1279	2.1225
35	0.0703	2.1081	0.0833	2.1294
47	0.0524	2.0988	0.0618	2.1319

The design is repeated for different values of N, and the results are presented in Table III. It can be observed the mean square error decreases with increasing N, while there is little change in the overshoot. It can also be observed that the design using rectangular window method is marginally better than the filter designed using the frequency sampling method. However, as mentioned earlier, the need to compute the integral, (equation 1), is avoided in this method without significant deterioration in performance.

IV. CONCLUSION

From the simulation study, the mean square error and overshoot in rectangular window method looks better than the one using frequency sampling method. Instead of performing the integration which is the traditional method of designing FIR filter using windowing method, the frequency sampling method presented here has the advantage of easier computation which is possible through MATLAB. The arbitrary response shown above taken for example requires solving three integrations which is a tedious task and is time consuming. Using frequency sampling method, the filter coefficients are easily computed using MATLAB and hence the design of FIR filters. Also, Frequency sampling technique can be used for any arbitrary magnitude response.

In view of the improved digital processing techniques, the Frequency sampling method of FIR filter design can be considered as the simplest method for FIR filter design. Our results show that there is not much significant deviation in mean-square-error or the overshoot of the frequency sampling method, as compared to that of other methods. We plan to extend the study to non-uniform frequency sampling and also iterative filter design.

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