CHAPTER 1.

ENGINEERING ANALYSIS

Introduction:

To heighten the understanding of behavior of a structure or a machine component, the analyst has three standard tools.

- Analytical Methods
- Numerical Methods
- Experimental Techniques

Analytical method provides quick close form solutions, but they treat only simple geometries and capture only the idealized structural theory.

Relative to analytical methods, numerical methods require very few restrictive assumptions and can treat complex geometries. They are far more cost effective than experimental techniques. The current interest in the engineering community for development and application of computational tools based on numerical methods is thereby justified. The most versatile numerical method in the hands of engineers is the finite element method. The goal of analysis is to verify a design prior to manufacture. While there are several methods of engineering analysis, the most comprehensive is Finite Element Analysis.

Using experimental techniques, representative or full scale models can be tested. Experimentation is costly, however, both in terms of the test facilities, the model instrumentation and the actual test. While the virtues of experimental solution of static, elastic, two-dimensional problems are now largely overshadowed by analytical and numerical methods, problems involving three-dimensional geometry, multiple-component assemblies, dynamic loading and inelastic material behavior are usually more amenable to experimental analysis. In some cases, experimental methods are inevitable and are also useful in validating solutions from numerical and analytical methods.

Figure 1.1 gives a brief classification of methods of structural analysis.

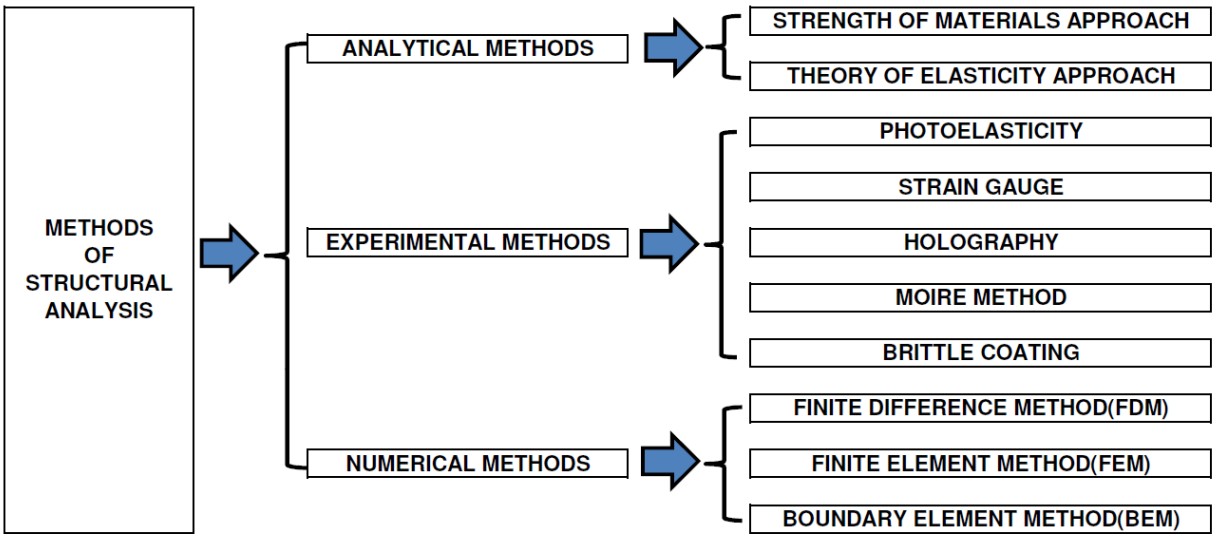


Fig. 1.1: Methods of structural analysis

Experimental stress analysis:

When designing mechanical components, it is essential to investigate and provide crucial information for mechanical failure due to stress generation. Experimental and numerical methods are well known for stress determination and design evaluation purposes. With the proper knowledge and tools, a designer identifies areas of high stresses (i.e. potential failure points), as well as the areas with low stresses (i.e. potential for material removal, weight reduction and cost saving). A Mechanical engineer should be exposed to the most commonly used tools of stress analysis. A summary of attributes of the three most commonly used stress analysis tools is presented in Table 1.1

Table 1.1 Summary of features of the three most commonly used stress analysis tools.

Method	Advantage	Disadvantage
Strain Gauge	Relatively easy to measure Remote data collection	Point field technique
Photoelasticity	Full field interpretation	Must have visual access. Limited temperature range
Numerical methods	Handles complex geometries and boundary conditions. Suitable for parametric studies	Difficult in building Exact model

PART A

Chapter 2.0, Introduction to basics of vibration

Vibration is a term that describes oscillation in a mechanical system. The frequency and amplitude describes it.

Vibration of a physical structure often is thought of in terms of model consisting of a mass and a spring. The vibration of such a model as a system may be "FREE" or "FORCED". In "Free Vibration", there is no energy added to the system but rather the vibration is the result of an initial disturbance. "Forced Vibration", in contrast to free vibration, continuous under "Steady State" conditions because energy is supplied to the system continuously to compensate for that dissipated in the system.

Vibrating systems are comprised of means for storing kinetic energy (spring), means for storing kinetic energy (mass or initial) and means by which the energy is lost (damper). The vibration of a system involves the alternately transfer of energy between its potential and kinetic forms. In a damped system some energy is dissipated at each cycle of vibration and must be replaced from an external source if steady vibration is to be maintained. Although a single physical structure may store both potential energy and kinetic energy, and may dissipate energy, here only lumped parameter systems comprised of ideal spring, masses and dampers where in each element only one function has is considered

Translation Motion:

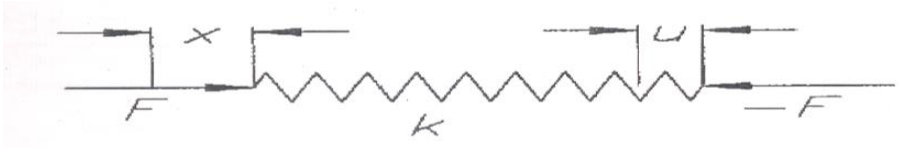


Fig 2.1 Linear Spring with Stiffness K

The change in the length of the spring is proportional to force acting along the length.
$F = k(x - u)$

Mass: A mass is a rigid body as shown in fig. 2.2 whose acceleration x mass, according to Newton's second law is proportional to the resultant F of all forces acting on the mass.

i.e. $F = m\ddot{x}$

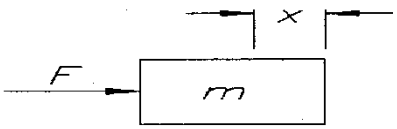


Fig 2.2

Damper: In the viscous damper shown in fig. 2.3, the applied force is proportional to the relative velocity of its end points.

$$\text{i. e. } F = c\left(\dot{x} - \dot{u}\right)$$

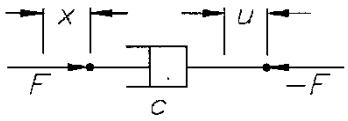


Fig. 2.3 Viscous Damper

Degree of freedom (DOF)

The number of independent parameters required to define the distance of all the masses from their reference positions is called the number of degree of freedom N. for example, if there are n masses in a system constrained to move only in translation in the x and y directions, the system has 2N degree of freedom.

Single degree of freedom system

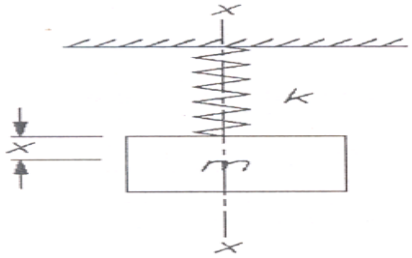


Fig 2.4 Single degree spring mass system

Fig. 2.4 shows the single degree spring mass system. It consists of mass 'm' attached by means of a spring k to in immovable support. The mass is constrained to translation motion in the direction of x – axis so that its change of position from an initial reference is described fully by the value of single quantity 'x'. This is called single degree of freedom.

Terminology:

Cycle: One complete movement of any repeated motion.
Period: Time taken for one complete cycle.
Frequency: Number of cycles of motion occurring in unit time.
Degree of freedom: Number of independent coordinates required to specify the configuration.
Free vibration: A vibrating system not influenced by any external force.
Forced vibration: A vibrating system influenced by any external force.
Damped vibration: A vibrating system influenced by any external resistance to its motion.
Undamped vibration: A vibration system not influenced by any external resistance to its motion.
Spring constant (modulus): Force required to produce unit deformation of spring.
Damping coefficient: Resistance developed per unit velocity.
Damping factor: Ratio of damping coefficient to critical damping.
Logarithmic decrement: Natural logarithmic of two successive amplitudes.
Resonance: The condition, for which forced frequency is equal to natural frequency.

Chapter-3

Experiment – 1, Analysis of undamped & damped free vibration in single degree longitudinal vibration system

Aim: To determine the natural frequency, logarithmic decrement, damping ratio, damping coefficient and damped natural frequency of a longitudinal vibration setup with single degree of freedom.

Theory:

Free vibration

If the mass 'm' is displaced from its equilibrium position and then allowed to vibrate free without the aid of forces, it is said to have free vibration.

The symbolic representation of a spring mass damper system is show in Fig. 3.1

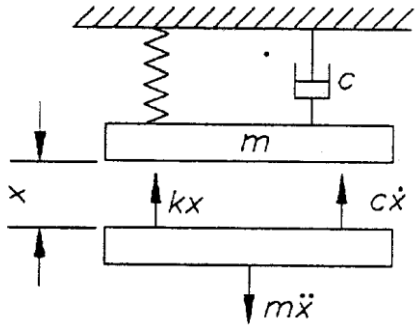


Fig.3.1 Spring Mass Damper System

The usual representations are:

K = spring constant (N / m)

C = spring coefficient (Ns/ m)

m = mass (kg)

W= weight (N)

x = displacement of mass from its men position at time "t" (mm)

= static deflection of spring with load W (mm)

From Newton's law $m\ddot{x} = \sum resisting$ forces

or $m\ddot{x} = -\left(c\dot{x} + kx\right)$

i.e $m\ddot{x} + c\dot{x} + kx = 0$ -------3.1

Expression (3.1) is the governing equation of above system, which is a second order linear differential equation.

The solution for such an expression is of the form,

$X = e^{st}$ -------3.2

Substituting in 9.1 $ms^2 + cs + k = 0$

Or $s^2 + (c/m)s + k/m = 0$ ------3.3

The expression (3.3) is a quadratic equation, which has the roots,

$$S_1 = (C/2m) + \sqrt{(C/2m)^2 - (K/m)}$$
$$S_2 = (C/2m) - \sqrt{(C/2m)^2 - (K/m)} \quad \text{------3.4}$$

Above mentioned set of roots (3.4), may be real, distinct of complex conjugates, depending upon the values of (k / m) and $(C / 2m)^2$.

$\omega_n = \sqrt{k/m}$. Let a = (C/2m).we have

$$S_1 = (a) + \sqrt{(a)^2 - (\omega_n)^2}$$
$$S_2 = (a) - \sqrt{(a)^2 - (\omega_n)^2} \quad \text{-------3.5}$$

(i) When $a^2 > \omega_n^2$ the roots are real and separate, and the solution yields to

$x = c_1 e^{(-a+\sqrt{a^2-\omega_n^2})t} + c_2 e^{(-a-\sqrt{a^2-\omega_n^2})t}$ ------3.6

This represents the over damped case, where the mass moves to its mean position gradually at infinite time, without any oscillation.

(ii) When, $a^2 = \omega_n^2$, the roots are real and separate, and the solution yield to

$x = c_1 e^{-\frac{c}{2m}} + c_1 e^{-\frac{c}{2m}t} = (c_1 + c_2)e^{-\omega n^t}$ ------3.7

This represents critical damping situation, where the mass moves back rapidly to its original position without any oscillations.

$x = c_1 e^{(-a+i\sqrt{\omega_n^2-a^2})^2} + c_2 e^{(-a-i\sqrt{\omega_n^2-a^2})}$

$x = c_1 e^{(-a+i\sqrt{\omega_n^2-a^2})^2} + c_2 e^{(-a-i\sqrt{\omega_n^2-a^2})}$ ------3.8

Where i = $\sqrt{-1}$

For critical damping $\left(\frac{c}{2m}\right)^2 = \frac{k}{m}$

Or C_c = critical damping coefficient = 2m* $\sqrt{k/m} = 2m\omega_n$ ------3.9

Since = damping natural frequency, == $\sqrt{\omega_n^2 - a^2}$

Expression (3.8) can be rewritten as,

$x = e^{-at}(c_1 e^{i\omega_d} + c_2 e^{i\omega_d t})$ ------3.10

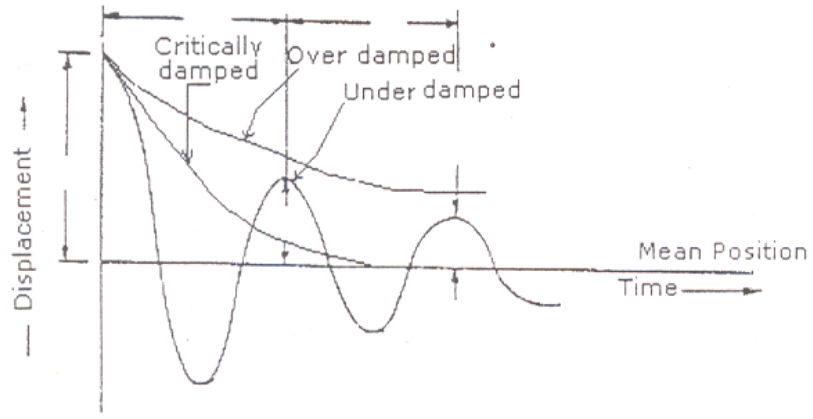
Let A = (C1 + C_2); B = i (C_1 + C_2); C = = $\sqrt{A^2 + B^2}$ & θ = tan $^{-1}$(B/A)

(Note: 'C' is not damping co efficient)

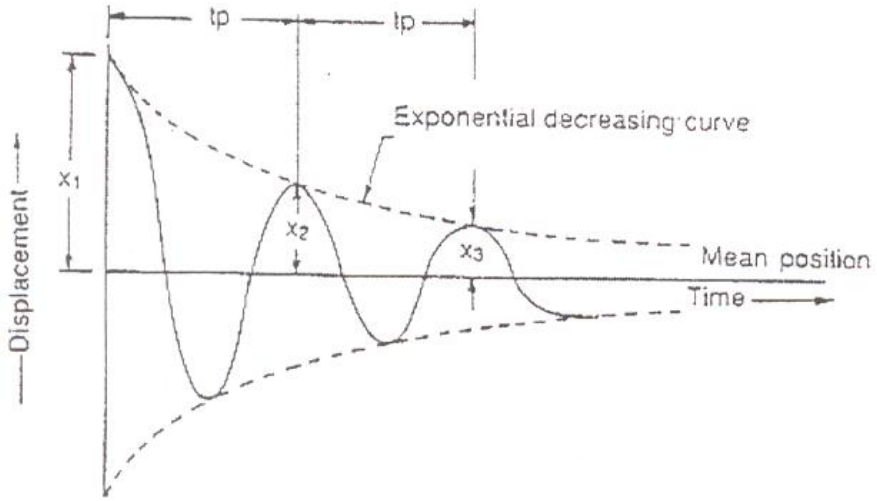
Expression (3.10) can be reduced to

X = C. e $^{-at}$ cos (ω_dt-θ) -----3.11

Expression (3.11) represent simple harmonic motion with diminishing amplitude as shown in fig.3.2 (a and b)



(a)



(b)

Fig: 3.2 Response of free undamped vibration system

The time period

$$t_p = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\omega_n^2 - a^2}} \qquad \text{------3.12}$$

And the frequency of damped vibration,

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi}\sqrt{\omega_n^2 - a^2} \qquad \text{------3.13}$$

It can be shown that, (from 3.11) $\frac{x_1}{x_2} = e^{atp}$

Thus logarithmic decrement,

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = at_p = \left(\frac{C}{C_C}\right)\frac{2\pi}{\sqrt{1-\left(\frac{C}{C_C}\right)^2}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \qquad \text{------3.14}$$

Damping ratio $\zeta = \frac{C}{C_c} = \frac{C}{2m\omega_n}$

In general,

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = ... = \frac{x_n}{x_{n+1}} = e^{at_p} = constant.....$$

$$.....\frac{x_n}{x_{n+1}} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = ... = \frac{x_n}{x_{n+1}} = e^{at_p}e^{at_p}e^{at_p}....e^{at_p}$$

$$or.\frac{x_n}{x_{n+1}} = e^{nat_p} \qquad \text{------3.15}$$

$$\text{or } \delta = \frac{1}{n}\log_e\left(\frac{x_1}{x_{n+1}}\right) \qquad \text{------3.16}$$

Expression (3.16) is useful to determine the value of the damping coefficient c, if the amplitudes of first cycle and (n+1) cycle are known, then

$$\frac{1}{n}\log_e\left(\frac{x_1}{x_{n+1}}\right) = (at_p) = \delta$$

SETUP FOR DAMPED FREE VIBRATION SYSTEM:

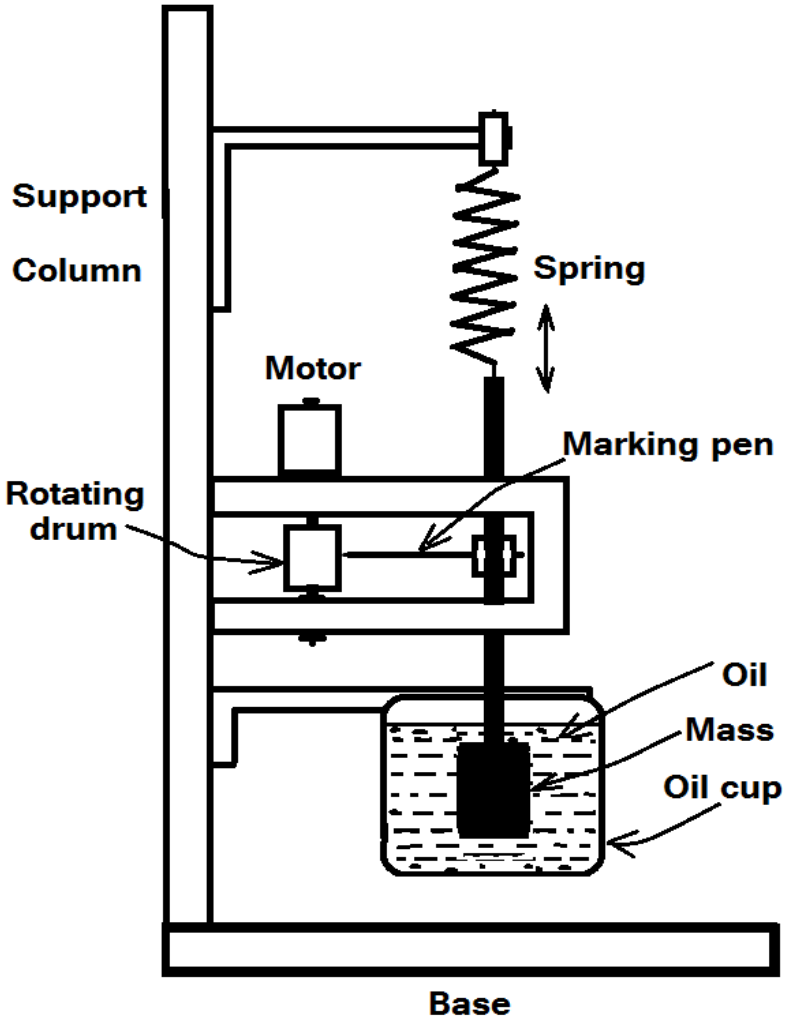


Fig 3.3 Experimental setup for free damped vibration system

Note: The setup has coulomb damping and not viscous damping.

Procedure:

- Find stiffness of the spring (slope of load Vs deflection curve – undamped system).
- Fix the pen to the vibrating platform and graph sheet of the drum recorder.
- Adjust the drum recorder in vertical position perfectly perpendicular to the axis of the spring so that, the pen tip makes contact with the drum.
- Add known mass to the platform and apply the initial force by pressing the platform downward manually.
- Start the drum recorder and release the platform.
- Form the graphs obtained measure the amplitude of the two successive cycles and hence calculate the logarithmic decrement, damping factor and other parameters

Observations

Radius of recording drum r = mm

Speed of recording drum N = rpm

Mass m = kg

Undamped free vibration:

Sl. No.	Weight W in N	Deflection mm	Stiffness K N/m	Undamped natural Frequency (Hz)
1				
2				
3				
4				
5				

Specimen calculation:

Undamped free vibration:

i) Stiffness of spring, K =

Slope of load deflection curve = N/m

ii) Natural frequency with a damped mass m, $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ *Hz*

Damped free vibration

Sl. No.	Time for n oscillations t Sec (n=)	x_i	From graph (x_i/x_{i+1})	Average	Logarithmic decrement δ	Damping ratio ξ	Undamped frequency	
							F_n Hz	ω_n rad/s
1		x_1 = x_2 = x_3 = x_4=	x_1/x_2 = x_2/x_3 = x_3/x_4 =					

Average of successive amplitude

Logarithmic decrement (from graph) $\delta = \ln\left(\frac{x_n}{x_{n+1}}\right)$ =

Damping ratio (ξ) $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ i.e $\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$ =

Undamped natural frequency = $\omega_n = \sqrt{k/m}$ = rad. / sec

Critical damping coefficient C_c = $2m\omega_n$ = Ns/m

Actual damping coefficient $C = \xi C_c$ = Ns/m

Damped natural frequency $\omega_d = \omega_n\sqrt{1-\xi^2}$ = rad/sec

$f_d = f_n\sqrt{1-\xi^2}$ = Hz

Note:

Angular velocity of drum = $2\pi N/60$ rad/sec

Radius of drum r = mm

Surface velocity V = mm/sec

Distance between consecutive peaks t_p = mm

Damped time period T_d =

F_d = $1/T_d$

Conclusions

Chapter 4.0
Experiment-2 ANALYSIS OF UNDAMPED & DAMPED FREE VIBRATION IN SINGLE DEGREE TORSIONAL VIBRATION SETUP

Aim: Determination of natural frequency, logarithmic decrement, damping ratio and damping coefficient of a torsional vibration setup with single degree of freedom.

Theory:

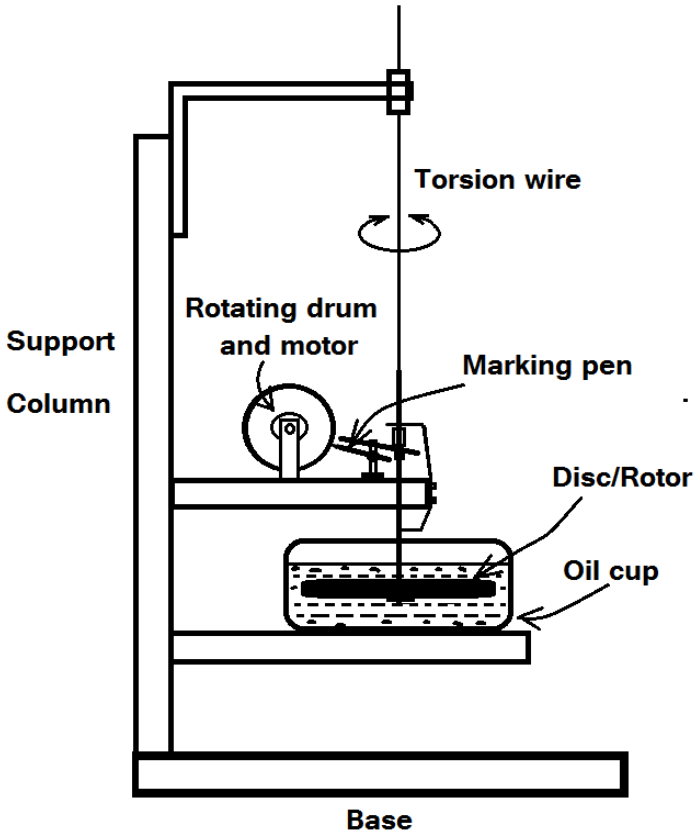


Fig 4.1 Free Torsional Vibration Setup

The usual representations are:

K_t = spring constant (N-m/rad) $\quad K_t = \frac{M_t}{\theta} = \frac{G\,J}{l}$

C = damping coefficient N-m-s

m = mass of rotor (kg)

$I = mk^2$

Where k is the radius of gyration

θ = displacement of rotor from it mean position at time 't'(radian)

From Newton's law

$$I\ddot{\theta} = \sum resisting\ torque$$

or $$I\ddot{\theta} = -\left(c\dot{\theta} + k\theta\right)$$

$$I\ddot{\theta} + \left(c\dot{\theta} + k\theta\right) = 0 \qquad \text{------4.1}$$

Expression (4.1) is the governing equation of above system which is a second order linear differential equation. As discussed in the previous experiment, the system may be over damped, critically damped, or under damped. For under damped system

$$\theta = Ce^{-at}\cos(\varpi_d t - \phi) \qquad \text{------4.2}$$

Expression (4.2) represents simple harmonic motion with diminishing amplitude as shown in fig. 4.2.

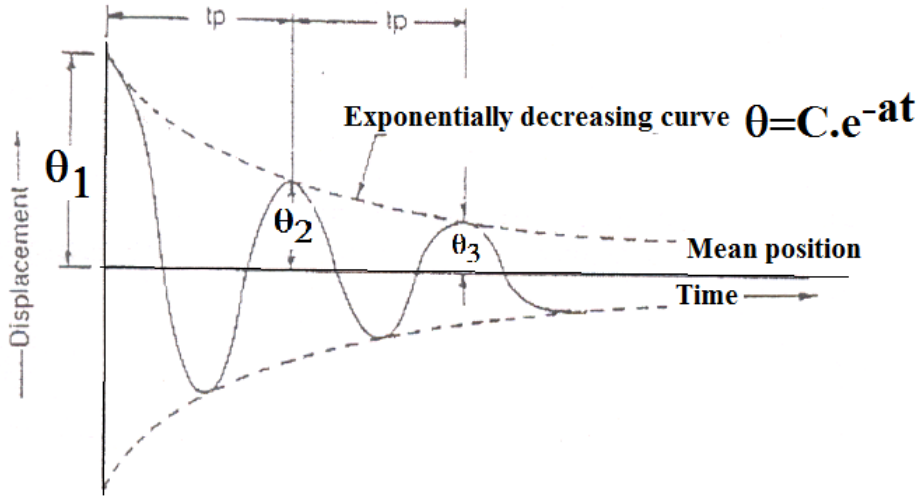


Fig: 4.2 Response of free undamped torsional system

The time period

$$t_p = \frac{2\pi}{\sqrt{\omega_n{}^2 - a^2}} = \frac{2\pi}{\omega_d} \qquad \text{------4.3}$$

And the frequency of damped vibration

$$f_d = \frac{\omega_d}{2\pi} - \frac{1}{2\pi}\sqrt{\omega_n{}^2 - a^2} \qquad \text{------4.4}$$

It can be shown that, (from 10.2)

$\frac{\theta_1}{\theta_2} = e^{at}$ Thus logarithmic decrement,

$$\delta = \log\left(\frac{\theta_1}{\theta_2}\right) = \text{at} = \left(\frac{c}{c_c}\right)\frac{2\pi}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}} = \frac{2\pi}{\sqrt{1-\zeta^2}}$$

Where damping ratio $\zeta = \left(\frac{c}{c_c}\right) = \frac{C}{2I\omega_n}$

Or $\delta = \frac{1}{n}.\log_e\left(\frac{\theta_n}{\theta_{n+1}}\right)$

4.3 Procedure:

- Find stiffness of the torsion wire.
- Attach one end of the torsion wire to the torsion head and other end to the rotor. The apparatus is leveled.
- Adjust the length of the wire such that the rotor is at the proper level.
- A cup containing the oil is placed below the rotor with the rotor dipping in oil.
- The torsion head is rotated slowly until the pointer shows zero degree. The flywheel is rotated from zero position to different position and released. Note down the amplitude of oscillation after each cycle and also time taken ‘t’ for 'n 'cycles.
- Draw the graphs of amplitude Vs cycles. Also obtain the ratio of the amplitude of the two successive cycles and hence calculate the logarithmic decrement, damping factor, damping coefficient, and natural frequency of the system.

Observations:

Diameter of the wire d = m
Length of the wire I = m
Modulus of rigidity G of the wire = Mpa
Mass of the rotor m = Kg.
Radius of gyration k = r/√2 (where r is the radius of rotor) = m

Tabulation:

Sl. No.	Time for n oscillations t Sec (n=)	From graph (x_i/x_{i+1})		Logarithmic decrement δ	Damping ratio ξ	Undamped frequency	
			Average			F_n Hz	ω_n rad/s
1		$\theta_1/\theta_2 =$ $\theta_2/\theta_3 =$ $\theta_3/\theta_4 =$					
2		$\theta_1/\theta_2 =$ $\theta_2/\theta_3 =$ $\theta_3/\theta_4 =$					
3		$\theta_1/\theta_2 =$ $\theta_2/\theta_3 =$ $\theta_3/\theta_4 =$					

Critical damping C_c N-sec/mm	Damping coefficient C N-sec/mm	Damped Natural frequency		From graph	
		F_n Hz	ω_n rad/s	F_n Hz	ω_n rad/s

Specimen calculation:

Stiffness of spring, $K_t = \frac{GJ}{I} = \frac{\pi G d^4}{32l}$ = N-m/rad

Un-damped Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{K_t}{I}} =$ Hz

$\omega_n = 2\pi f_n$ = rad/sec

Average of $\frac{\theta_n}{\theta_{n+1}}$ =

Logarithmic decrement $\delta = l_n\left(\frac{\theta_n}{\theta_{n+1}}\right) =$

Damping ratio $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$, $\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} =$

Critical damping coefficient $C_c = 2I\omega_n$ N-m –sec

Actual damping coefficient $C = \xi C_c$ N-m –sec

Damped natural frequency $\omega_d = \omega_n\sqrt{1-\xi^2}$

$f_d = f_n\sqrt{1-\xi^2} =$

Note:

Angular velocity of drum $= 2\pi N / 60$ rad/sec

Radius of drum r = mm

Surface velocity V = mm/sec

Distance between consecutive peaks t_p = mm

Damped time period T_d =

$F_d = 1/T_d$

Chapter 5

Experiment-3 Determination of Frequencies and mode shapes of cantilever beam

Introduction

Free vibration takes place when a system oscillates under the action of forces integral in the system itself due to initial deflection, and under the absence of externally applied forces. The system will vibrate at one or more of its natural frequencies, which are properties of the system dynamics, established by its stiffness and mass distribution. In case of continuous system the system properties are functions of spatial coordinates. The system possesses infinite number of degrees of freedom and infinite number of natural frequencies.

In actual practice there exists some damping (e.g., the internal molecular friction, viscous damping, aero dynamical damping, etc.) inherent in the system which causes the gradual dissipation of vibration energy, and it results in decay of amplitude of the free vibration. Damping has very little influence on natural frequency of the system, and hence, the observations for natural frequencies are generally made on the basis of no damping. Damping is of great significance in restraining the amplitude of oscillation at resonance.

The comparative displacement alignment of the vibrating system for a particular natural frequency is known as the Eigen function in continuous system. The mode shape of the lowest natural frequency (i.e. the fundamental natural frequency) is termed as the fundamental (or the first) mode frequency. The displacements at some points may be zero which are called the nodal points. Generally *n*th mode has (*n*-1) nodes excluding the end points. The mode shape varies for different boundary conditions of a beam.

Modal analysis has become a major technique to determine dynamic characteristics of engineering structures and its components. It is a process by which the natural frequencies, mode shapes and damping factor of structures can be determined with a relative ease. The modal analysis process has two types of method to analysis the structures. First is theoretical modal analysis and second is experimental modal analysis.

In the theoretical modal analysis method (fig.5.1) the spatial properties of the modal (mass, stiffness and damping) are given and using them modal and response modal are obtained. In theoretical modal analysis one cannot forecast accurate boundary conditions, actual rigidity and damping for complex engineering structures and component. So the calculated results often have certain error with actual result. The Numerical modal analysis method using the Finite element modeling software's like NASTRAN, ANSYS enables engineers to get a better understanding of dynamic properties of structures

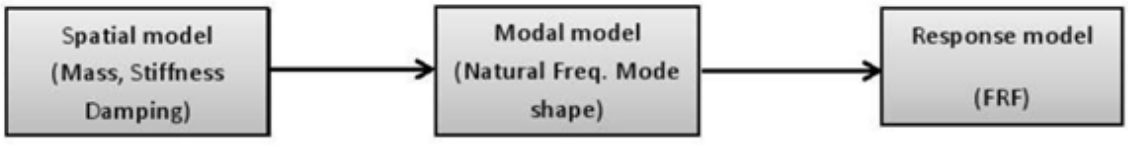


Fig:5.1Theoretical route to vibration analysis

The experimental modal analysis path from response modal (fig 5.2) and not so often ending with spatial modal. Experimental modal analysis used to derive the modal of a linear time-invariant vibratory system. Modal analysis using vibro-meter is non- destructive testing, based on vibration response of the structures. For excitation impact hammer is widely used in modal analysis. It is well known that for structures falling under resonant conditions small force can result in large deformation, and possibly, damage can be induced in the structure. The interaction between the inertial and elastic properties of the materials causes resonant vibration in the structures. Modal is frequently used to find mode of vibration of machine component in the structure.

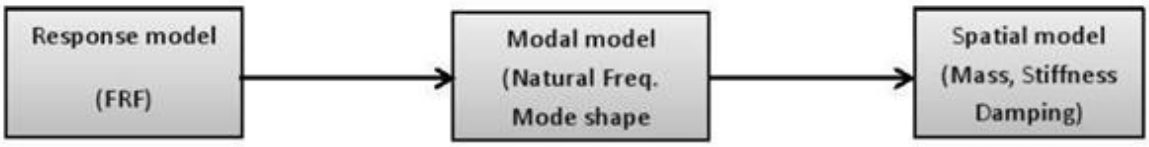


Fig5.2 Experimental route to vibration analysis

THEORTICAL MODAL ANALYSIS OF BEAM: Cantilever beam: fixed - free

Consider an Euler-Bernoulli uniform cantilever beam undergoing transverse vibration condition Fig.5.3 shows a cantilever beam having rectangular cross section, which is subjected to bending vibration by giving a small initial displacement at the free end; and Fig. 18 depicts a cantilever beam under the free vibration.

The boundary conditions for a cantilever beam (Fig. 5.3) are given by;

$$at \quad x = 0, Y(x) = 0, \frac{dY(x)}{dx} = 0$$

$$at \quad x = l, \frac{d^2Y(x)}{dx^2} = 0, \frac{d^3Y(x)}{dx^3} = 0$$

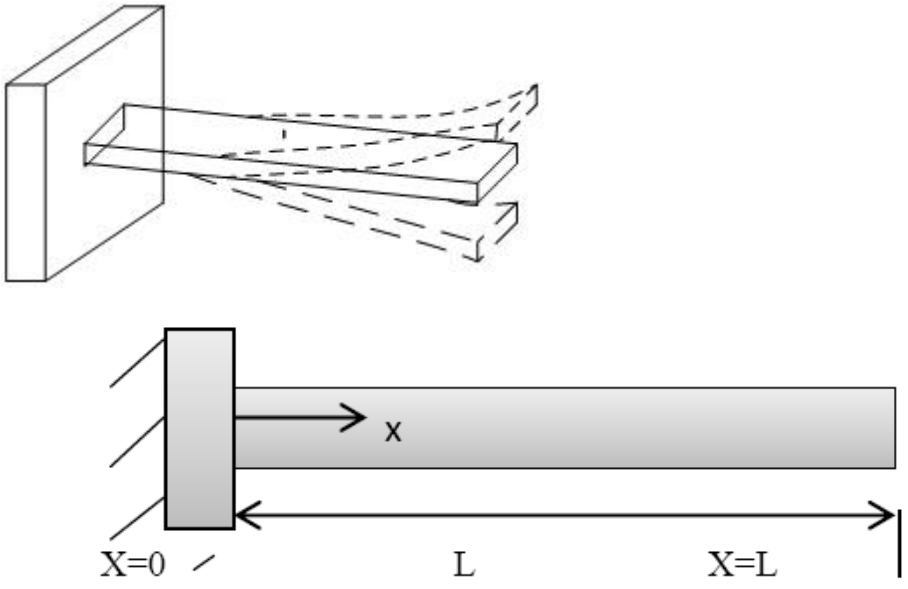


Fig: 5.3 Cantilever Beam

Mathematical analysis

For a cantilever beam exposed to free vibration, and the system is considered as continuous system considering the beam mass as distributed along with the stiffness of the shaft, the equation of motion can be written as given by the following equations

$$EI\ \frac{d^4w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{------ 5.1}$$

$$C^2 \frac{d^4w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{------ 5.2}$$

Where $C = \sqrt{\frac{EI}{\rho A}}$ ------ 5.3

Where, *E* is the modulus of rigidity of beam material, *I* is the moment of inertia of the cross section of the beam, *w*(*x*) is displacement in *z* direction at distance *x* from fixed end, *ω* is the circular natural frequency, *m* is the mass per unit length, *m = ρA(x)*, *ρ* is the density of the material, *x* is the distance measured from the fixed end.

W(x,t) = w(x) T(t) ------ 5.4

$$\frac{c^2}{w}\frac{d^4w}{dx^4}\ T = -\frac{1}{T}\frac{d^2T}{dT^2} = a \qquad \text{------ 5.5}$$

Where $a=\omega^2$ can be shown as a constant. The equation 5.5 can be written as 2 equation.

$$\frac{d^4w(x)}{dx^4} - \beta^4 w(x) = 0 \qquad \text{------ 5.6}$$

Where

$$\beta^4 = \frac{\rho A \omega^2}{EI} \qquad \text{------ 5.7}$$

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0 \qquad \text{------ 5.8}$$

From the equation 5.7 the natural frequency of beam ω can be written as

$$\omega = (\beta L)^2 \sqrt{\frac{El}{\rho A L^4}} \qquad \text{-------5.9}$$

The solution equation 5.8 is

$$T(t) = A\cos\omega t + B\sin\omega t \qquad \text{------- 5.10}$$

Where A and B are constant that can be determined be boundary conditions.

Assuming the solution for equation 5.6 as
$W(x) = Ce^{sx}$ -------5.11

Using 5.6 and 5.11 one can be obtain the general solution

$$W(x) = C_1(\cos\beta x + \cosh\beta x) + C_2(\cos\beta x - \cosh\beta x) + C_3(\sin\beta x + \sinh\beta x) + C_4(\sin\beta x - \sinh\beta x)$$
---------5.12

Where, the constant C_1,C_2,C_3,C_4 can be determined from the boundary conditions. For a **Cantilever Beam** the transverse deflection and its slope must be zero at the fixed end and at free end the bending moment and shear force must be zero.

Thus the boundary condition become

W (0) =0 ------- 5.13

$\frac{dw}{dx}(0) = 0$ -------- 5.14

$\frac{d^2w}{dx^2}(L) = 0$ -------- 5.15

$\frac{d^3w}{dx^3}(L) = 0$ -------- 5.16

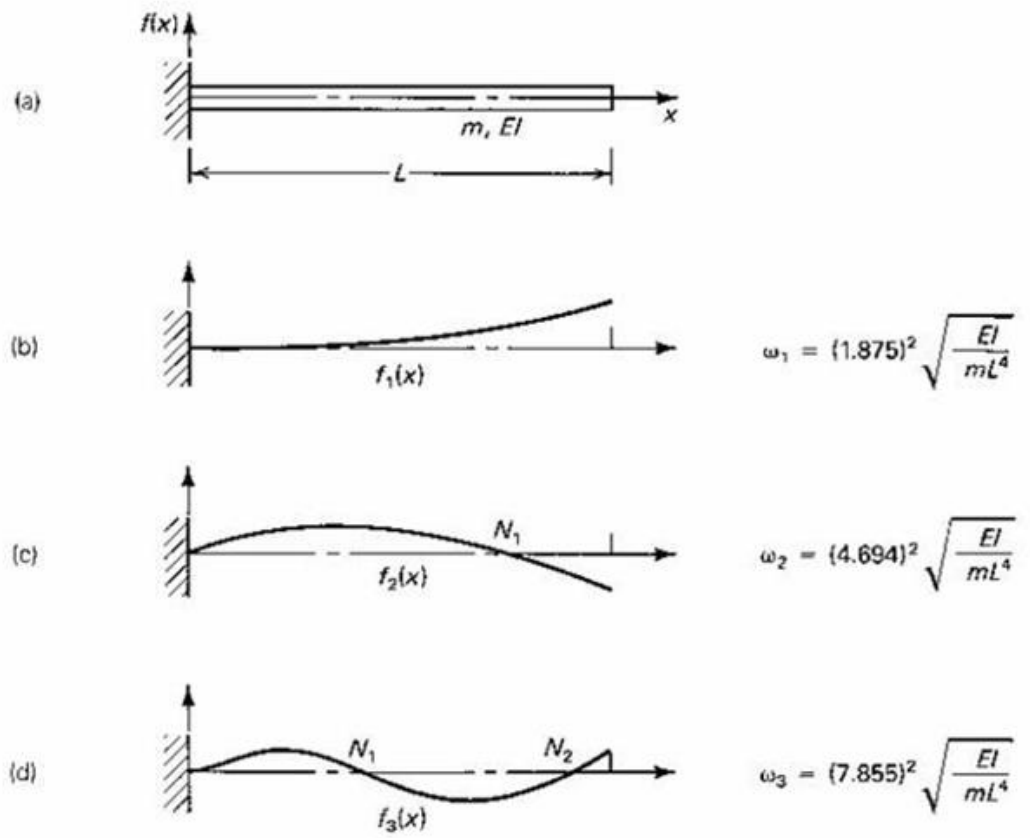
Substituting the equations from 5.13 to 5.16 in 5.12 to obtain the solution

$\cos\beta L + \cosh\beta L + 1 = 0$ ---- 5.17

Equation 5.17 is the frequency equation. This transcendental equation can be solved to obtain the value of $(\beta L)^2$ and for the cantilever beam the values are given in the below table 5.1.

Table 5.1

Mode	**$(\beta L)^2$**
Mode 1	1.875104
Mode 2	4.694091
Mode 3	7.854757



Quality Factor and Bandwidth

The Quality factor is defined as the ratio of natural frequency to the frequency bandwidth. The estimation of Q factor is the indirect method of finding the damping factor. Higher the Q factor implies less in energy loss. The Q Factor is used to represent how structure is. The magnitude of peak is dependent on the damping ratio. From the quality factor damping ratio can be determined.

Quality factor is also defined as the resonant frequency divided by the half power bandwidth around the peak. Figure 5.5 shows the bandwidth and resonant frequency.

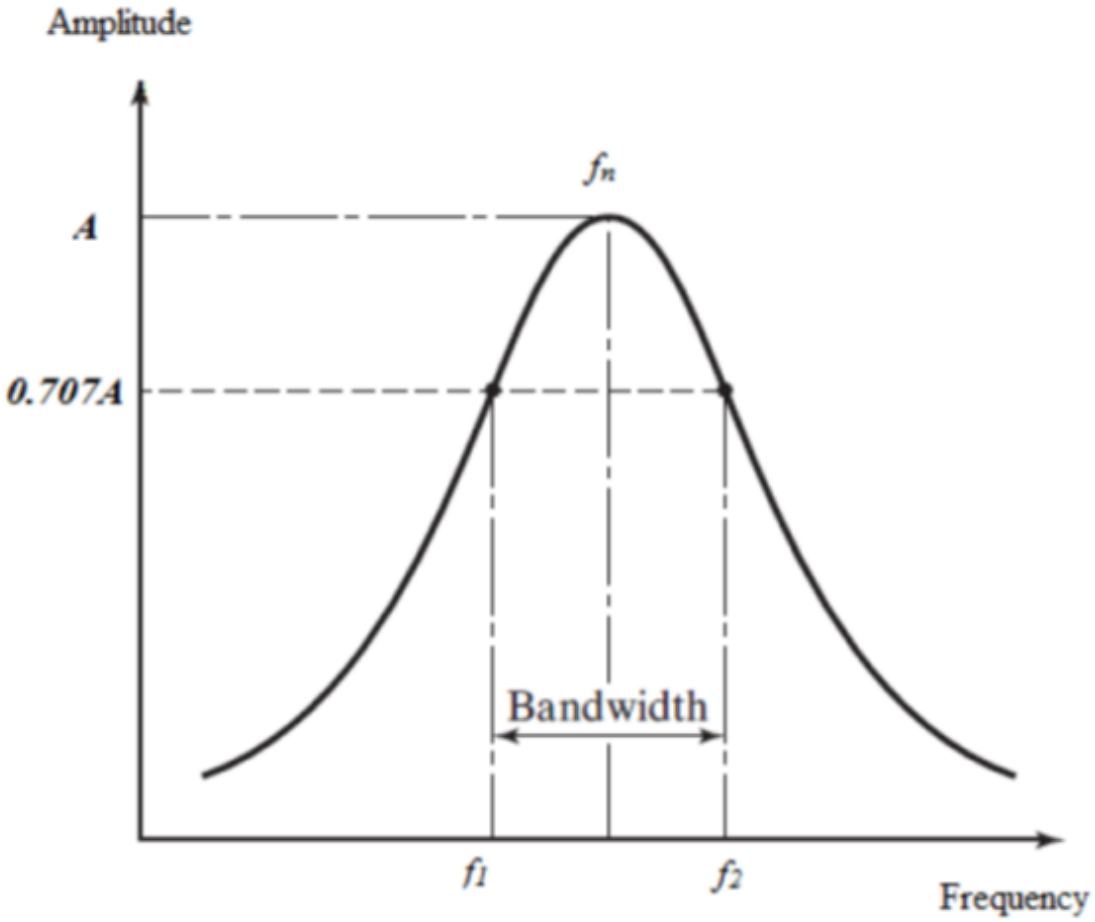


Figure 5.5 : bandwidth and resonant frequency

$$Q = \frac{f_n}{f_1 - f_2}$$

Were, Q is the quality factor,
f_n is the natural frequency (resonant frequency),
f_2- f_1 is the bandwidth.
The quality factor for the plates of various configurations is estimated by frequency response functions. In experimental Q factor is calculated by obtaining the frequency response function from FFT. The results Q factor are presented in the chapter

Experimental Setup

Fig.5.6 shows an experimental setup of the cantilever beam. It includes a beam specimen of particular dimensions with a fixed end and at the free end an accelerometer is clamped to measure the free vibration response. The fixed end of the beam is gripped with the help of clamp. For getting defined free vibration cantilever beam data, it is very important to confirm that clamp is tightened properly; otherwise it may not give fixed end conditions in the free vibration data.

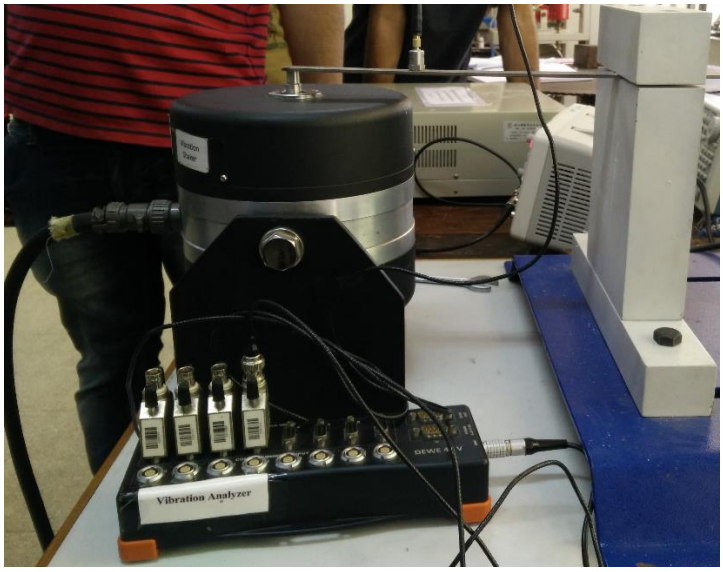
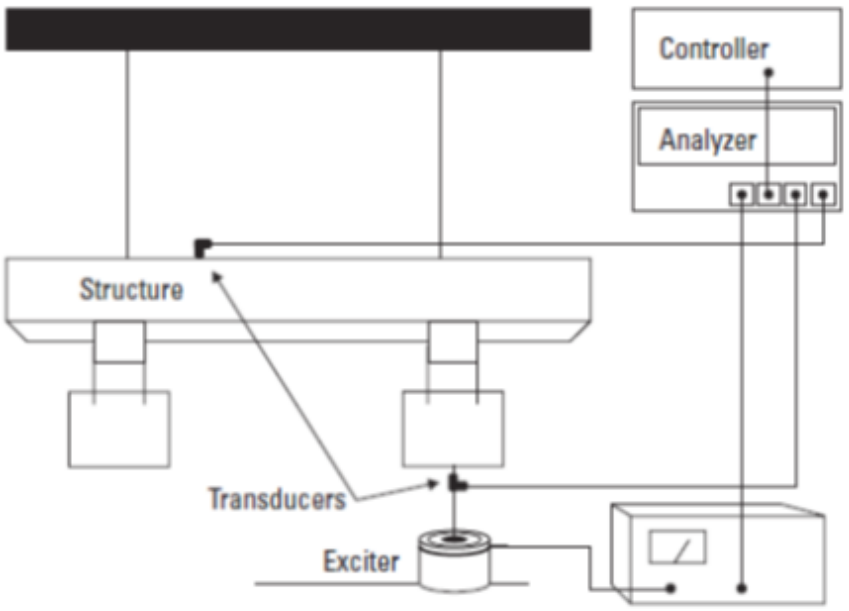


Fig 5.6 Cantilever Beam Setup

To obtain the natural frequency of the cantilever beam the following apparatus are used accelerometer (0.5 Hz-10000 Hz), signal conditioner (9V, 2mA), vibration shaker with power amplifier setup, function generator, Data acquisition system, Atalon application software. The complete experiment setup is shown in figure 5.7.

Figure 5.7
General test
configuration



Description of apparatus

a) Accelerometer

An accelerometer is used as sensor in the experimentation, in which it senses the amplitude of vibration of the structure and sends the signals to the analyzer so that information can be used for the study. Accelerometer measures in units of gravity which is equal to 9.81 m/s². The figure 5.8 shows the accelerometer.

Accelerometer measures the acceleration of the vibrating structure. The description of accelerometer is shown in table 5.2.

Accelerometer is connected to input socket of DC power supply unit (5110 type). The output signal from the power supply unit is connected to USB port of PC, which leads to AD card. The outputs data are acquired by a Lab view based program which captures and display the time domain signal of vibration.

Table 5.2: Specification of accelerometer

Specification	Unit	Type 8702B50
Model		Single axis linear
Sensitivity	Mv/g	±100
Frequency range	Hz	0.5-10000
Mass	Gram	8.7
Range	G	±50

Figure **5.8** : Accelerometer

Figure **5.9**: Shaker

b) Vibration shaker

The vibration shaker is a devise used to excite the structure at the defined frequency range. The excitation of the structure is possible in two ways with use of shaker one is constant amplitude and other is the constant excitation force. The shakers works on the principal of electrodynamics or hydraulic, in this work used an electro dynamic shaker. The figure 5.9 shows the one such shaker works on the principal of electrodynamics.

c) Function generator

Function generator is electronic device used to generate the electrical signals that operates shakers, these signals may be in the sinusoidal, rectangular, and triangular. Generators also generates the sweep and continuous wave frequency signals which are useful in getting the FRF from the experiments. With use of AFG 12 standard waveforms can be generated Figure 5.10 shows the Function generator.

Figure **5.10**: Arbitrary function generator

d) Oscilloscope

Oscilloscope is also an electronic equipment used to check the signals produced by function generators, it is very important to know the signals type and some information such as amplitude range and period of oscillation before sending signals to the shakers. Some of the delegated instrument should not be operated in the certain range so the oscilloscope is useful to check the condition of the signals generated. The screen of the instrument shows the 2D plot of the signals generated the simples sinusoidal signals can be seen in figure 5.11.

The oscilloscope used in our experiment is to check the signals generated by the arbitrary function generator. Once the waveform generated by AFG is checked and this waveform is transferred to shaker with use of amplifier.

Figure **5.11** Oscilloscope

e) Amplifier

The generated sinusoidal signals has the lower amplitude these signals needs to be amplified to excite the shaker. For the purpose of amplification used an equipment called Amplifier. The instrument has the circuits to amplify the input signals by using external power.

An amplified signal is supposed to excite the structure, necessary gain required to excite is to set manually. In amplifier the vibration amplitude can be controlled by varying the gain. Amplifier is may be used to control the excitation during testing depending on the capability of the equipment.

For example of applicability of amplifier in radio stations, the signals are used to amplify up to 300GHz.Amplifiers shown in figure 5.12 can be used in almost all electronic application were signal is to be amplified.

Figure**5.12**: Amplifier.

f) Vibration analyzer

This is the devise which collects the data from the accelerometer and sends to the application software. The analyzer used in experiment is to facilitate the vibration study.

The analyzer used for this experiment has 8 channel, each channel required for the each equipment during experiment. The main function of the vibration analyzer is to record and store and also to plot the results as requires. The figure 5.13 shows the vibration analyzer.

Figure**5.13**: Vibration analyzer

g) Atalon application software

Atalon application software is used to obtain the frequency response from structures. The software used for this purpose is DEWE Soft, input signals for this software is obtained analyzer through accelerometer. The experimental data can be stored and FFT can be stored and FFT can be obtained.

5.3.2 Experimental procedure

The experiment on vibration characteristic measurements involves many aspects like position of shaker to excite, position of accelerometer, and fixity of the plate etc... Decision is taken for these instalments is based on the FE analysis. The brief procedure for the experimentation is listed below.

- Measure the dimension of plate structure and note the material. All edges of plate clamped firmly for CCCC condition, fixing of plate can be done as per the requirement.
- Apply wax to the accelerometer, and fix it firmly to the given structure.
- Check the accelerometer is connected correctly to the power supply unit and the Atalon application software is started in the PC.
- Operate the shaker in sine wave sweep mode.

Open the Atalon application software. Enter sampling rate/samples to be collected for measuring vibration. Typically 1000 samples per sec for 5 seconds is adequate.

- Acquire samples by strongly tapping the structure or by exciting structure.
- Export this data to an FFT analyzer and transform into a frequency domain.
- Plot the frequency domain and find the frequency peaks which correspond to natural frequencies.
- Compare those frequency with theoretical or FEM analysis results.

5.4 Comparison of Solution

The results obtained from the Finite element analysis and experimental analysis compared in this section. The Q factor and natural frequencies obtained from both experimental and the finite element analysis is compared here. These parameters is obtained for three plate with CCCC, CCCF, and CFCF boundary conditions. The procedure for finding the Q factor is mentioned in earlier sections.

The experimentally obtained FFT are used for finding these parameters. The below tables and graph shows the comparison of results. The below table 3.2 gives the comparison of modal frequencies of both FEA and experimental, and figure 3.10 to figure 3.12 shows the bar chart of results.

5.5 Experimental Procedure

1. Choose a beam of a particular material (steel, aluminum or copper), dimensions (L, w, d) and transducer (i.e., measuring device, e.g. strain gauge, accelerometer, laser vibrato meter.
2. Clamp one end of the beam as the cantilever beam support.
3. Place an accelerometer (with magnetic base) at the free end of the cantilever beam, to observe the free vibration response (acceleration).
4. Induce initial deflection to the cantilever beam and allow to oscillate on its own. To get the higher frequency it is recommended to give initial displacement at an arbitrary position apart from the free end of the beam (e.g. at the mid span).
5. This could be done by bending the beam from its fixed equilibrium position by application of a small static force at the free end of the beam and suddenly releasing it, so that the beam oscillates on its own without any external force applied during the oscillation.
6. The free oscillation could also be started by giving a small initial tap at the free end of the beam.
7. Record the data obtained from the chosen transducer in the form of graph (variation of the vibration response with time).
8. Repeat the procedure for 5 to 10 times to check the repeatability of the experimentation.
9. Repeat the whole experiment for different material, dimensions, and measuring devices.
10. Record the whole set of data in a data base.

Calculation of experimental natural frequency

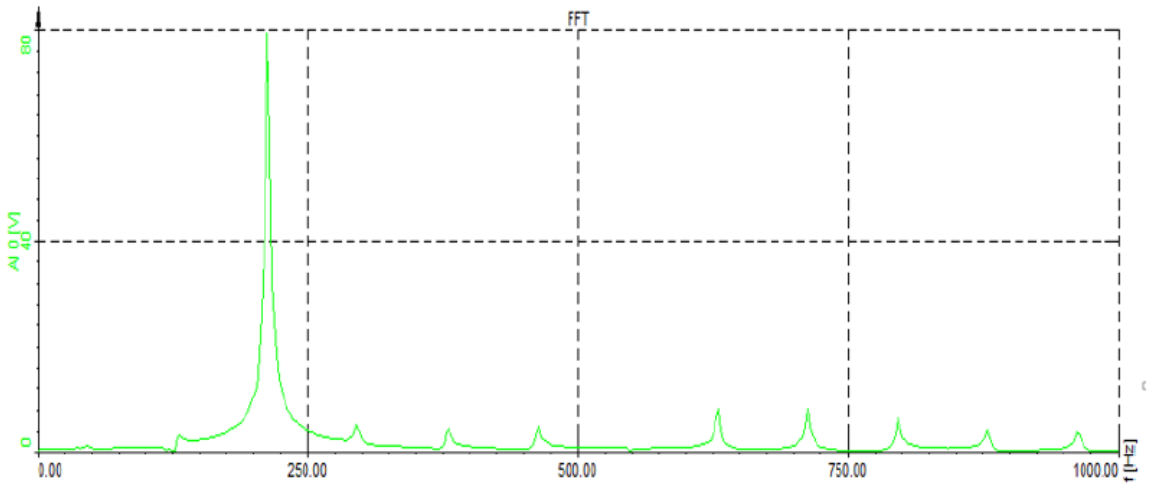


Fig 5.14

To observe the natural frequencies of the cantilever beam subjected to small initial disturbance experimentally the experiment should be conducted with the specified cantilever beam specimen. Record the data of time history (Displacement-Time), and FFT plot. The natural frequencies of the system can be obtained directly by observing the FFT plot. The location of peak values relates to the natural frequencies of the system. Fig.5.14 above shows a typical FFT plot.

Observation and Tabulation
Material :
Density (ρ): kg/m^3
Young's Modulus (E) kg/m^2
Poisson's ratio
Length L m
Breadth b m
Height h m

Sl.No	Mode	Theoretical Frequency	Experimental Frequency	Mode shape
1.	1.			
2.	2.			
3.	3.			

Results

Good agreement between the theoretically calculated natural frequency and the experimental one is found. The correction for the mass of the sensor will improve the correlation better. The present theoretical calculation is based on the assumption that one end of the cantilever beam is properly fixed. However, in actual practice it may not be always the case because of flexibility in support.

CHAPTER 6. INTRODUCTION TO PHOTOELASTICTY

Photo-elasticity is an experimental technique for stress and strain analysis that is particularly useful for members having complicated geometry, complicated loading conditions, or both. For such cases, analytical methods (that is, strictly mathematical methods) may be cumbersome or impossible, and analysis by an experimental approach maybe more appropriate. While the virtues of experimental solution of static, elastic, two-dimensional problems are now largely overshadowed by analytical methods, problems involving three-dimensional geometry, multiple-component assemblies, dynamic loading and inelastic material behavior are usually more amenable to experimental analysis.

The name photo-elasticity reflects the nature of this experimental method: *photo* implies the use of light rays and optical techniques, while *elasticity* depicts the study of stresses and deformations in elastic bodies. Through the photo-elastic-coating technique, its domain has extended to inelastic bodies, too. Photo-elastic analysis is widely used for problems in which stress or strain information is required for extended regions of the structure. It provides quantitative evidence of highly stressed areas and peak stresses at surface and interior points of the structure and often equally important, it discerns areas of low stress level where structural material is utilized inefficiently.

Nature of Light:

Electromagnetic wave theory: Maxwell (1831 – 1879):

Light is an electromagnetic disturbance propagating in space, represented by two vector fields an electric field and a magnetic field mutually perpendicular to each other.

Polarised light:

In ordinary light beam the light waves describe a random vibratory motion in a plane transfers to the direction of propagation. Usually the tip of the light moves randomly and is called as Natural or Un-Polarised light.

When the waves in a beam of light are constrained to vibrate in a systematic manner in planes normal to the direction of propagation, the beam is said to be polarized.

Types of Polarised light:

a) Plane / Linearly Polarised Light:

When the waves in a beam of light vibrate in parallel planes so that the orientation of light vector is constant, the beam of light is said to be plane polarized, as shown in the Fig. 6.1

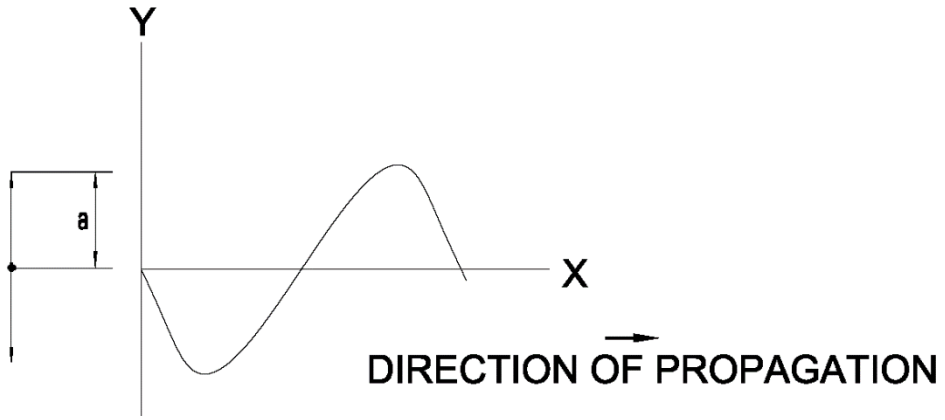


Fig. 6.1: Plane polarized light

b) Elliptically polarized:

If the vibration is such that both the amplitude and orientation of the light vector changes uniformly, so that tip of vector traces out an ellipse, it is said to be elliptically, as shown in the Fig. 6.2

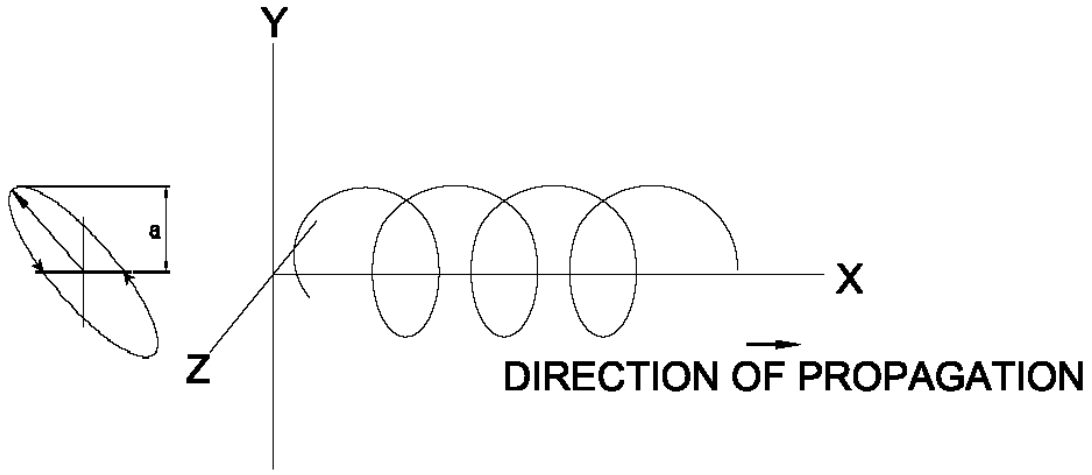


Fig. 6.2: Elliptically polarized light

c) Circularly polarized Light:

If the vibration is such that the amplitude remains constant while the orientation of the light vector changes uniformly so that the tip of the vector traces out a circle, the light is said to be circularly polarized. Direction of Propagation of Light remains same as shown in the Fig. 6.3

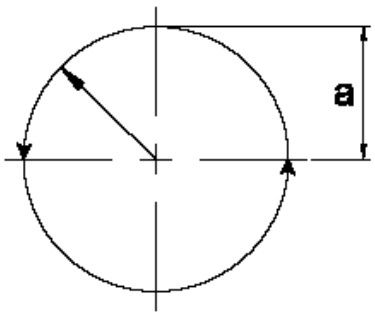


Fig. 6.3: Circularly polarized light

Superposition of waves:

Two waves having the same wavelength and amplitude but a different phase are shown in Fig. 6.4. The two waves can be expressed by

$E_1 = A \cos [2\pi/\lambda (z + \Phi_1 - ct)]$, $E_2 = A \cos [2\pi/\lambda (z + \Phi_2 - ct)]$ ------6.1

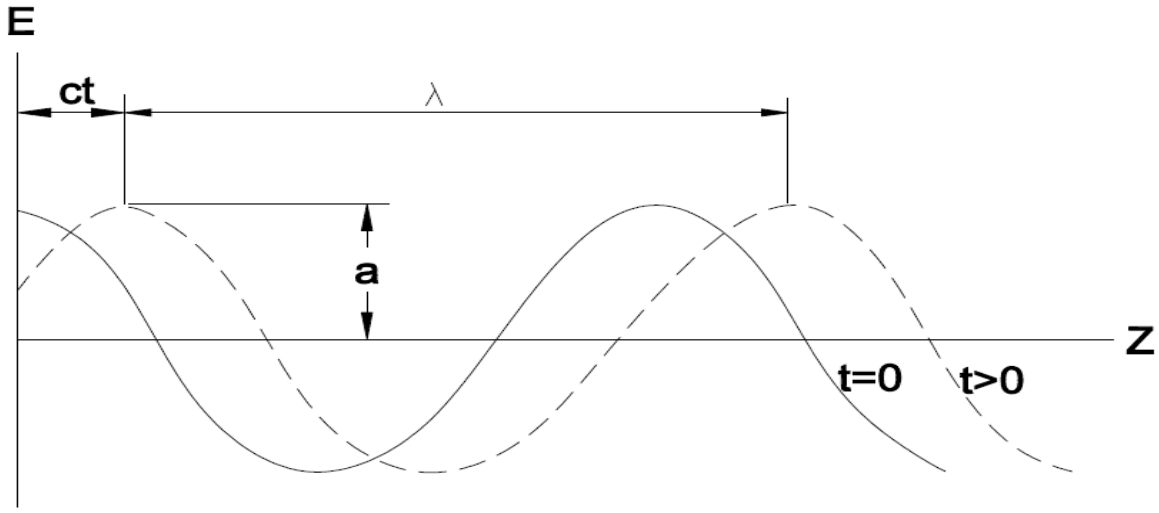


Fig. 6.4 : Magnitude of the light vector as a function of position along the axis of propagation for two waves with different initial phases

Where, Φ_1 = initial phase of wave E_1, Φ_2 = initial phase of wave E_2.

$\Phi = \Phi_2 - \Phi_1$ = the linear phase difference between waves.

The intensity of the light wave resulting from superposition of two waves of equal amplitude is a function of the linear phase difference Φ between the waves. The intensity of the resultant wave assumes its maximum value when $\Phi = n\lambda$, n = 0,1,2,3,…. or when the linear phase difference is an integral number of wavelengths. i.e Constructive interference

The intensity of the resultant wave assumes its minimum value when $\Phi = [(2n+1)/2]\lambda$, n=0,1,2,3,……, or when the linear phase difference is an odd number of half wavelengths. i.e. destructive interference. Interference effects have important application in photoelasticity, moiré and holography.

Two other important forms of polarized light arise as a result of the superposition of two linearly polarized light waves having the same frequency but mutually perpendicular planes of vibration

A special case of elliptically polarized light occurs when the amplitudes of the waves E_x and E_y are equal and $\Phi = [(2n+1)/4]\lambda$, n=0,1,2,3

A second special case of elliptically polarized light i.e circularly polarized light occurs when the linear phase difference Φ between the two waves E_x and E_y is an integral number of half wavelengths ($\Phi = n\lambda/2$, n = 0, 1, 2, ……). Light exhibiting this behavior is known as plane or linearly polarized light.

Optical instrument: The polariscope

The polariscope is an optical instrument that utilizes the properties of polarized light in its operation. For experimental stress-analysis work, two types of frequently employed, the plane polariscope and the circular polariscope. The names follow from the type of polarized light used in their operation. In practice, plane-polarized light is produced with an optical element known as a plane or linear polarizer. Production of circular polarized light or the more general elliptically polarized light requires the use of a linear polarizer together with an optical element known as a wave plate. A brief discussion of linear polarizers, wave plates, and their series combination follows.

Linear or plane polarizers: When a light wave strikes a plane polarizer, this optical element resolves the wave into two mutually perpendicular components. The component parallel to the axis of polarization is transmitted while the component perpendicular to the axis of polarization is either absorbed, as in the case of Polaroid, or suffers total internal reflection, as in the case of a calcite crystal such as Nicol prism.

Most modern polariscopes containing linear polarizers employ Polaroid H sheet, a transparent material wit stained and oriented molecules. In the manufacture of H-type Polaroid films, a thin sheet of polyvinyl alcohol is heated, stretched, and immediately bonded to a supporting sheet of cellulose acetate butyrate. The polyvinyl face of the assembly is then stained by a liquid rich in iodine. The amount of iodine diffused into the sheet determines its quality, and Polaroid Corporation produces three grades, denoted according to their transmittance of light as HN-22, HN-32, and HN-38. Since the quality of a polarizer is judged by its transmission ratio, HN-22 (with a transmission ratio of the order of 10^5 at wavelengths normally employed in photo elasticity) is recommended for photo-elastic purposes.

WAVE PLATES

A wave plate is defined as an optical element which has ability to resolve a light vector into two orthogonal components and to transmit the components with different velocities. Such a material has been referred to as doubly refracting or birefringent. The doubly refracting plate has two principal axes labeled 1 and 2. The transmission of light along axis 1 proceeds at velocity c1 and along axis 2 at velocity c2. Since c1 is greater than c2, axis 1 is often called the fast axis and axis 2 the slow axis.

6.1 Arrangement of the Optical Elements in a Polariscope

a) Plane polariscope: The plane polariscope is the simplest optical system used in photoelasticity; it consists of two linear polarizers and a light source arranged as illustrated in Fig. 6.5

The linear polarizer nearest the light source is called the polarizer, while the second linear polarizer is known as the analyzer. In the plane polariscope the two axes of polarization are always crossed; hence no light is transmitted through the analyzer, and this optical system produces a dark field. In operation a photoelastic model is inserted between the two crossed elements and viewed through the analyzer, and this optical system produces a dark field. In operation photoelastic model is inserted between the two crossed elements and viewed through the analyser.

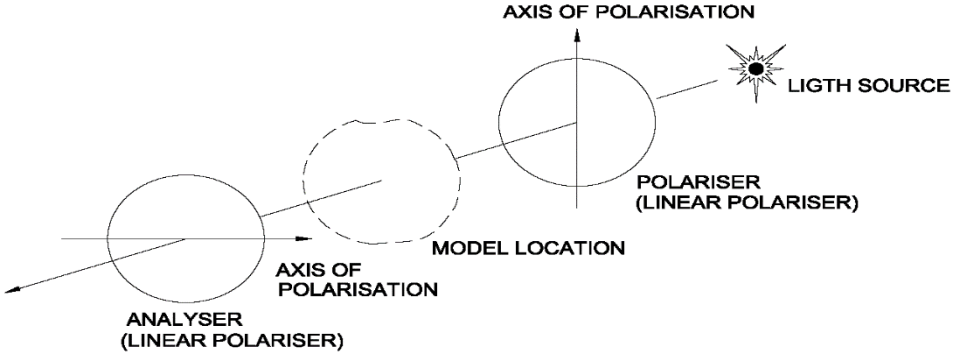


Fig. 6.5 Arrangement of the optical elements in a plane polariscope

a) Circular polariscope: As the name implies, the circular polariscope employs circularly polarized light; consequently, the photoelastic apparatus contains four optical elements and a light source, which is illustrated in Fig.6.6

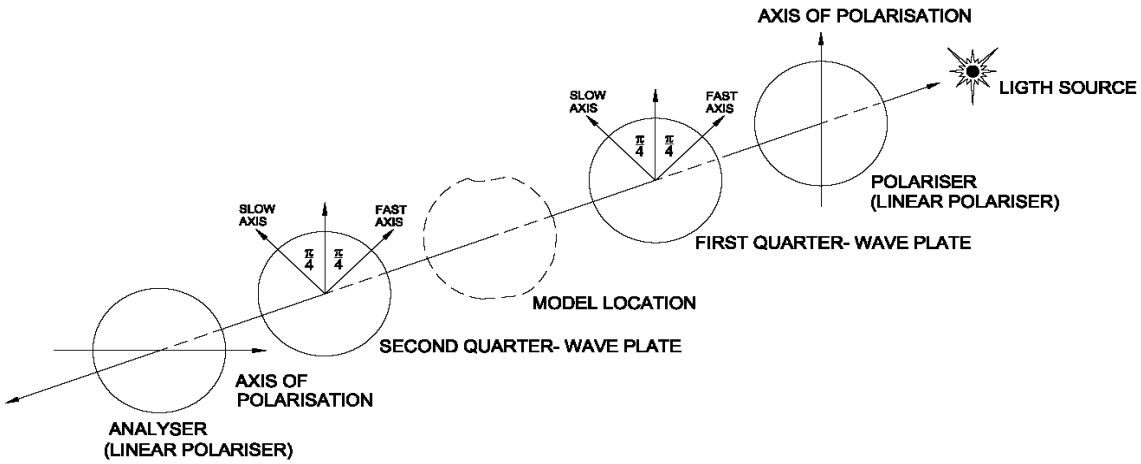


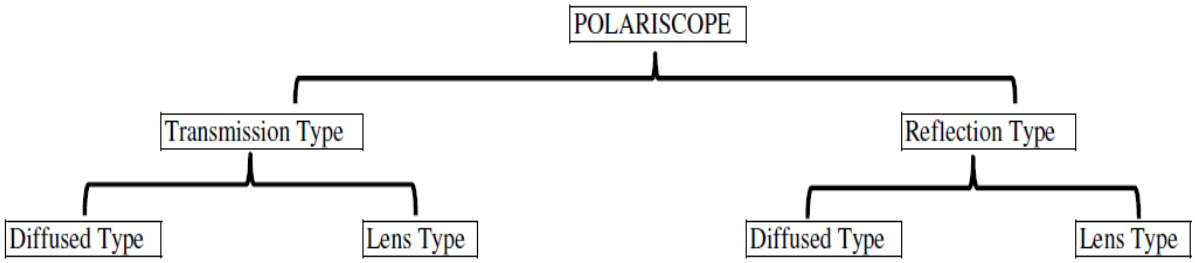
Fig. 6.6 Arrangement of the optical elements in a circular polariscope

The first element following the light source is called the polarizer. It converts the ordinary light into plane-polarized light. The second element is a quarter-wave plate set at an angle $\beta = \pi/4$ to the plane of polarization. This first quarter-wave plate converts the plane-polarized light into circularly polarized light. The second quarter-wave plate is set with its fast axis parallel to slow axis of the first quarter-wave plate. The purpose of this element is to convert the circularly polarized light into plane-polarized light, which is again vibrating in the vertical plane. The last element is the analyzer, with its axis of polarization in the horizontal plane, and its purpose is to extinguish the light. This series of optical elements constitutes the standard arrangements of the optical elements in the polariscope are possible, depending upon whether the polarizers and quarter-wave plates are crossed or parallel. These four optical arrangements are described in table 6.1. Arrangements A and B are normally recommended for light and dark field use of the polariscope since a portion of the error introduced by imperfect quarter-wave plates, i,e., both quarter-wave plates differ from $\pi/2$ by an amount, is cancelled out. Since quarter-wave plates are often of poor quality, this fact is important to recall in aligning the polariscope.

ARRANGEMENT	QUARTERWAVE PALTE	POLARISER & ANALYSER	FIELD
A	CROSSED	CROSSED	DARK FIELD
B	CROSSED	PARALLEL	BRIGHT FIELD
C	PARALLEL	CROSSED	BRIGHT FIELD
D	PARALLEL	PARALLEL	DARK FIELD

Table 6.1 Four arrangements of optical elements in a circular polariscope

CLASSIFICATION OF POLARISCOPES



STRESS OPTIC LAW

When a Photo elastic material is loaded, the resulting stresses produce proportional optical effects which appear as isochromatic fringes when viewed with a polariscope.

Starting with the unloaded model and applying load in increments, fringes will appear first at most highly stressed points. As the load is increased, new fringes appear and the earlier fringes are pushed towards the areas of lower stress.

The Fringe orders observed are proportional to difference between principal stresses. Their relationship is expressed by Stress Optic Law.

i.e. $\sigma_1 - \sigma_2 = Nf/t$ ------6.2

Where, σ_1 and σ_2 = Algebraically Maximum and Minimum Principal Stresses, in N/mm^2

N= Fringe Order

f= Material Fringe Value, in N/Fringe/mm.

t= Thickness of the model, in mm.

Since, for a given material and a light source, value of 'f' and 't' are constant, the $\sigma_1 - \sigma_2$ Value is a function of 'N' only. Therefore at any point of interest where $\sigma_1 - \sigma_2$ is to be determined, the fringe order 'N' is to be measured very accurately even in fractions.

Normally Polariscope is used as a dark field instrument, which means that with no stress in the model, all light is extinguished and the model appears uniformly black. As load is gradually applied to the model, the most highly stressed region begins to take colour –first gray then white; and yellow is seen with further load when the violet is extinguished. With the further load, the blue is extinguished to produce orange; and then green, to give red. The next colour to vanish with increasing load is yellow, leaving a purple colour; and this is followed by the extension of orange, producing a deep blue fringe.

The purple fringe, which is easily distinguished from the red and blue on either side and is very sensitive to small change in stress level, is referred to as the tint-of-passage. Because of its distinctiveness and resolution, the purple tint-of-passage is selected to mark the increment in relative retardation equal to a fringe order of unity (N-1). Subsequently, recurrence of the tint-of-passage with greater relative retardation signifies the presence of higher integral fringe orders (N=2, N=3, etc,)

color	**Approximate fringe order**
Black	0.0
Yellow	0.6
Red	0.9
Purple (tint-of-pasdsage)	1.0
Blue-green	1.2
Yellow	1.5
Red	1.75
Red/green transition	2.0
Green	2.2
Yellow	2.5
Red	2.8
Red/green transition	3.0
green	3.2

Table 6.2 Fringe colors for full-field interpretation

EFFECT OF STRESSED MODEL IN TRANSMISSION POLARISCOPE

PLANE POLARISCOPE

Effects of a stressed model in a plane polariscope: It has been established that the principal-stress difference $\sigma_1 - \sigma_2$ can be determined in two-dimensional model if N is measured at each point in the model.

Consider first the case of a plane-stressed model inserted into the field of a plane polariscope with its normal coincident with the axis of the polariscope, as illustrated in Fig. 6.7. Note that the principal-stress direction at the point under consideration in the model makes an angle α with the axis of polarization of the polarizer.

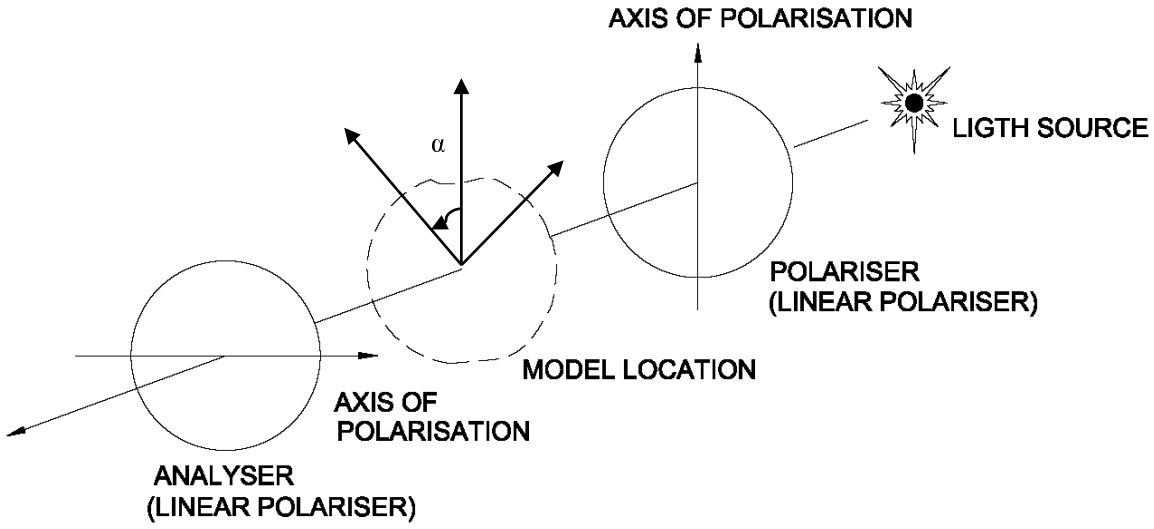


Fig. 6.7 A stressed photoelastic model in a plane polariscope

The amplitude of the light wave emerging from the analyzer of a plane polariscope is given by

$$I = k \sin^2 2\alpha \sin^2 \delta/2 \qquad \text{------6.3}$$

Where, $\delta = \delta_2 - \delta_1 = 2\pi h/\lambda\ (n2 - n_1) = 2\pi h/\lambda\ (\sigma 2 - \sigma_1)$

Examination of equation (6.3) indicates that extinction (I = 0) occurs either when $\sin^2 2\alpha = 0$ or when $\sin^2 (\delta/2) = 0$. Thus, one of the conditions for extinction is related the principal-stress directions and the other related to the principal-stress difference.

a) **Effect of principal-Stress Directions**

When $2\alpha = n\pi$, where n = 0, 1, 2,…… $\sin^2 2\alpha = 0$ and extinction occurs. In other words one of the principal-stress directions coincides with the axis of the polarizer ($\alpha = 0, \pi/2$, or any exact multiple of $\pi/2$), the intensity of the light is zero. The fringe pattern produced by the $\sin^2 2\alpha$ term in equation(2.3) is known as an **isoclinic fringe pattern**. Isoclinic fringe pattern are used to determine the principal-stress direction at all points of a photoelastic model.

b) **Effect of Principal – Stress Difference**

When $\delta/2 = n\pi$, where n = 0, 1, 2, …., $\sin^2(\delta/2) = 0$ and extinction occurs. In other words, when the principal-stress difference is either zero (n=0) or sufficient to produce an integral number of wavelengths of retardation (n = 1, 2, 3, ……), the intensity of light emerging from the analyzer is zero. The fringe pattern produced by the sin2 ($\delta/2$) term in equation (6.3) is known as an **isochromatic fringe pattern**.

Examination of equation (6.3) indicates that the order of extinction n depends on both the principal-stress difference $\sigma_1 - \sigma_2$ and the wavelength λ of the light being used. Thus, for a given principal-stress difference, the order of extinction n can be an integer only for light of a single wavelength (monochromatic light). When a model is viewed in monochromatic light, the isochromatic fringe pattern appears as a series of dark bands since the intensity of light is zero when n = 0, 1, 2, 3,….. When a model is viewed in white light (all wavelength of the visible spectrum present), the isochromatic fringe pattern appears as a series of colored bands. The intensity of light is zero, and a black fringe appears only when the principal-stress difference required to produce a given order of extinction is different for each of the wavelengths. Thus, not all the wavelengths can be extinguished simultaneously to produce a condition of zero intensity. The various colored bands form in regions where the principal-stress difference is sufficient to produce extinction of a particular wavelength of the white light. For example, when the principal-stress difference is sufficient to produce extinction of the green wavelengths, the complementary color, red, appears as the isochromatic fringe. At the higher levels of principal-stress difference, where several wavelengths of light can be extinguished simultaneously, e.g., second order red and third order violet, the isochromatic fringes become pale and very difficult to identify; therefore, they are seldom used for stress analysis work.

c) Frequency Response of a Polariscope

The circular frequency ω for light in the visible spectrum is approximately 10^{15} rad/s. As a result, neither the eye nor any type of existing high-speed photographic equipment can detect the periodic extinction associated with the ωt term of the expression for the light wave.

CIRCULAR POLARISCOPE

When a stressed photoelastic model is placed in the field of a circular polariscope with its normal coincident with the z axis of the polariscope, the optical effects differ somewhat from those obtained in a plane polariscope. The use of a circular polariscope eliminates the isoclinic fringe pattern while it maintains the isochromatic fringe pattern, and as a result the circular polariscope is more widely used than the plane polariscope.

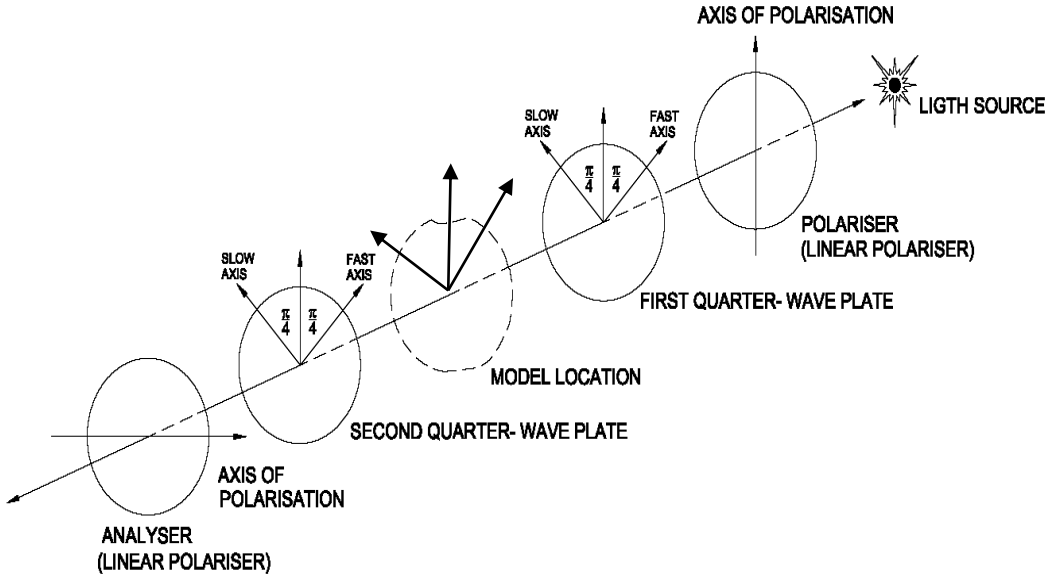


Fig. 6.8 A stressed photoelastic model in a circular polariscope

The intensity of light emerging from the analyzer of a circular polariscope is give by,

$$I = k \sin^2 \delta /2 \qquad (6.4)$$

Inspection of equation (6.4) indicates that the intensity of the light beam emerging from the circular polariscope is a function only of the principal-stress difference since the angle Φ does not appear in the amplitude of the wave. This indicates that isoclinics have been eliminated from the fringe pattern observed with the circular polariscope.

PHOTO ELASTIC MATERIAL

COMMONLY USED MATERIALS

Today most elastic-stress analyses are conducted by employing one of the following materials:

1. Colombia resin CR-39
2. **Homalite 10**
3. **Polycarbonate**
4. Epoxy resin
5. Urethane rubber

In our country in all the R and D organizations where photoelastic stress analysis is carried out, specimens made out of Araldite are employed for two-dimensional analyses. The sheets are cast cold using Cy 230 resin and Hy 951 hardener manufactured by Hindustan CIBA-Geigy Ltd.

PROPERTIES OF PHOTO ELASTIC MATERIAL

One of the most important factors in a photoelastic analysis is the selection of the proper material for the photoelastic model. Unfortunately, a perfectly ideal photoelastic material does not exist, and the investigator must select from the list of available materials the one which most closely fits his needs. The quality of photoelastic plastic used each year is not sufficient to entice a chemical company into the development and subsequent production of a polymeric material especially designed for photoelastic applications. As a consequence, the analyst must select a model material which is commercially available for some purpose other than photoelasticity.

The following list gives properties which an ideal photoelastic material should exhibit. These criteria are discussed individually below.

1. The material must be transparent to the light employed in the polariscope.
2. The material should be quite sensitive to either stress or strain, as indicated by a low material fringe value in terms of either stress fσ or strain fε.
3. The material should exhibit linear characteristics with respect to
4. Stress-strain properties.
5. Stress-fringe-order properties
6. Strain-fringe-order properties
7. The material should have both mechanical and optical isotropy and homogeneity.
8. The material should not exhibit time dependent properties such as creep.
9. The material should exhibit a high modulus of elasticity and a high proportional limit.
10. The material sensitivity, that is f_σ or f_ε, should not change markedly with small variations in temperature.
11. The material should not exhibit time-edge effects.
12. The material should be capable of being machined by conventional means.
13. The material should be free of residual stresses.
14. The material should not be prohibitively expensive.

COMPENSATION TECHNIQUES (i.e. Fractional fringe order determination)

The various methods employed for measurement of fractional fringes are:

1) Babinet - Soleil Compensation
2) Tardy's Compensation.

1) BABINET – SOLEIL COMPENSATION

It consists of two Quartz wedges cut similarly with respect to their optical axes. The thickness of the combination can be varied with the help of micrometer screw. The compensator is kept before or after the model and is oriented along the principal stress axis at the point of interest in the mode. The required compensation is achieved by rotating the micrometer dial until a dark fringe passes through the point of interest. The retardation given by the compensator is added or subtracted according to the fringes observed with reference to lower or higher order fringe.

2) TARDY'S COMPENSATION

The Tardy's Method of Compensation is a relatively fast, simple and accurate technique for measuring fractional fringe orders. The basis for Tardy's method is as follows.

"When the Polariser and the analyser of the Polariscope are aligned with the directions of the principal stresses, and the Quarter Wave Plates are at 45^0 to the Polariser axis, independent rotation of the analyser by θ will move a Fringe to a position where the fractional order is θ/ 180".

PROCEDURE FOR TARDY'S COMPENSATION

a) Set the Apparatus in Circular Polariscope Mode and examine the area where stress measurements are to be made. Observe the integral order fringes on the model and mentally label each fringe with its order.

b) Select the specific points at which the stress is to be measured, and use a pencil to mark each point on the model with a small cross (+). Number the test points.

c) Now convent the apparatus from Circular to Plane Polariscope mode for the observation of isoclinics.

d) Now, rotate all the Polaroid's Unit back and forth over a small angle, noticing the black isoclinics which move with the rotation Carefully rotate the assembly to bring the darkest area of an Isoclinic directly over the first test point. The Polariser/ Analyser axes now coincide with directions of the principal stress at the point.

e) Convert the Apparatus back into Circular Polariscope mode, restoring the instrument to a crossed circular polariscope, and eliminating the isoclinics.

f) Rotate the Analyser alone slowly and observe that Isochromatic Fringes move with it. Continue the rotation until a fringe is brought to the test point.

Carefully adjust the Analyser to centre the fringe over ht test point and read the fraction, 'r' on the compensator scale directly, if the scale is graduated in fractional numbers or read the angle, if the scale is graduated in degree from '0' to '180'.

g) When the analyser is rotated clockwise, if the lower order fringe moves to the test point, the total fringe order at that point is:

$N = n + \text{fraction} = n + r$ (N is positive): When the scale is graduated in fractional numbers.

or

$N = n + (\alpha / 180)$: When the scale is graduated in degrees from 0 to 180.

If the higher order Fringe moves to the test point for clockwise analyser rotation, the total fringe order is:

$N = -(n+1-r)$ (N is Negative)

or

$N = -\{n+1-(\alpha/180)\}$.

h) Now the difference in the principal stresses at the test point can be calculated from the stress optic law.

CALIBRATION OF PHOTO ELASTIC MATERIAL

By employing the plane polariscope and white light source, the directions of the principal stresses at a desired point in a model under a plane state of stress can be determined. The use of circular polariscope and one of the compensation methods will enable one to determine accurately the relative retardation at the point of interest. The

relative retardation can be expressed either in the form of fringe order or in terms of wavelengths. If *N* is the number of wavelengths of relative retardation then the value of (σ1- σ2) at the point.

$\sigma_1 - \sigma_2 = Nf/t$ or $\sigma_1 - \sigma_2 = FN$

Where *f* is the material fringe constant and *F* the model fringe constant. The model fringe constant was defined as the value of $(\sigma_1 - \sigma_2)$ necessary to cause a relative retardation of 1λ in the model of given thickness d. The material fringe constant f is defined as the value of $(\sigma_1 - \sigma_2)$ to cause a relative retardation of 1λ in a model of unit thickness. The method to determine either *f* or F for the given model material is called the calibration method. The three commonly used methods are 1.Use of a tension specimen, 2.Use of a circular disc and 3.Use of a rectangular beam

Chapter 7

Experiment 4. Calibration of Photo-elastic Material using circular disc under Diametric Compression

Aim: To calibrate the given photoelastic material using circular disc under compression.

Apparatus And Material: Circular Polariscope with accessories, photoelastic model in the form of circular disc (model).

Theory: Based on Theory of Elasticity, the stress distribution along the horizontal diameter in a circular disk under compression is given by

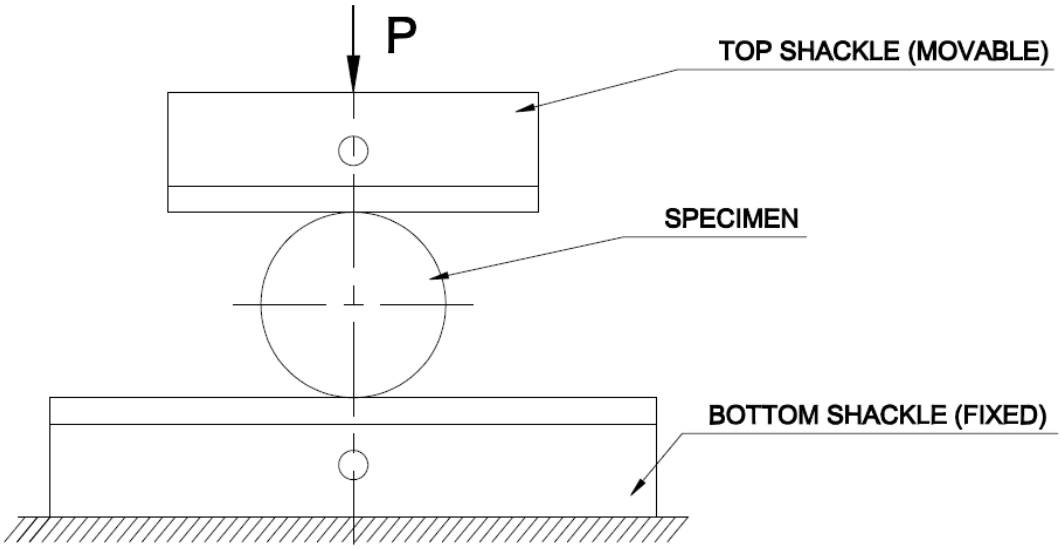


Fig7.1 Circular disc under diametric compression

$$\sigma_1 = \frac{2P}{\pi tD}\left[\frac{D^2 - 4X^2}{D^2 + 4X^2}\right]^2$$

$$\sigma_2 = \frac{-2P}{\pi tD}\left[\frac{4D^4}{\left(D^2 + 4X^2\right)^2} - 1\right]$$

At the centre i.e at x = 0

$\sigma_1 = 2P/\pi tD$ and $\sigma_2 = -6P/\pi tD$

$(\sigma_1-\sigma_2) = 8P/\pi tD$ -----7.1

From stress optics law for 2D stress system $(\sigma_1-\sigma_2) = Nf_\sigma / t$ ------7.2

Equating (7.1) & (7.2), we will get $Nf_\sigma / t = 8P/\pi tD$

or $f_\sigma = \frac{8}{\pi D}\left(\frac{P}{N}\right) = \frac{8}{\pi D}\left(\frac{\Delta P}{\Delta N}\right)$ = N/mm/fringe ------7.3

By knowing the loads required for producing different numbers of fringes a graph of P Vs N is plotted and the slope of this linear graph gives (ΔP/ ΔN) which is used to estimate f_σ (i.e. the fringe constant of the material). This is referred to as calibration.

Procedure:

1. Make initial set up for the experiment, set the digital display to zero.
2. Place the model between loading screw and the bottom surface of the frame.
3. Apply load by rotating the screw handle.
4. Observe the specimen through analyzer.
5. Determine the loads required for getting integral fringe orders (0, 1, 2, 3) at the center of circular disk and tabulate.

   Or

   Using Tardy's compensation technique, obtain fringe orders for different loads and tabulate (Note: The graphical stress directions are aligned along x and y axis at the center of circular disc).
6. Draw a graph between load Vs fringe pattern (linear graph passing through origin).
7. Determine slope of the line (ΔP/ΔN).
8. Calculate the material fringe constant by using the formula … (7.3)

Observation:

Diameter of the specimen: D = mm
Thickness of the specimen: t = mm

Tabular Column:

Method1 (By using graph):

Sl No	**Fringe Order (N)**	**Load Applied (P)**		**Slope Of Line (ΔP/ΔN)**	**Material Fringe Constant (f_σ)**
		Kg	**N**	**N/fringe**	**N/mm/fringe**
1	0 (Black)				
2					
3					
4					
5					
6					

Specimen Calculation:

Load (P) = kg = x 9.81 = **N**

Slope (from graph) = **(ΔP/ΔN) =** **N/fringe**

Material Fringe Constant:

$$f_\sigma = \frac{8}{\pi D}\left(\frac{P}{N}\right) = \frac{8}{\pi D}\left(\frac{\Delta P}{\Delta N}\right) =$$ N/mm/fringe

Method 2 (Using Line of Best Fit):

Tabular Column:

Sl No	**X**	**Y**	**XY**	X^2
	N (Fringe Order)	**P (Load in N)**	**NP**	N^2
1				
2				
3				
4				
5				
6				
SUM				

Specimen Calculation:

$a_0 = [\sum Y \sum X^2 - \sum X \sum XY] / [N\sum X^2 - (\sum X)^2]$

$a_0 =$

$a_1 = [N \sum XY - \sum X \sum Y] / [N\sum X^2 - (\sum X)^2]$

$a_1 =$

$f\sigma = 8/\pi D\ (P/N) = 8^* a1/\pi D$

$f\sigma =$ N/mm/fringe.

Chapter 8.
Experiment 5. Calibration of Photoelastic Material under bending load

Aim: To calibrate the given photoelastic material using constant bending moment specimen.

Apparatus And Material: Circular Polariscope with accessories, photoelastic model in the form of a four point bending specimen.

Theory: A rectangular specimen loaded as shown in fig. 8.1(two/four point loading) is subjected to pure bending i.e. constant bending moment. The BMD and SFD for such a load arrangement as shown in the figure 8.2.

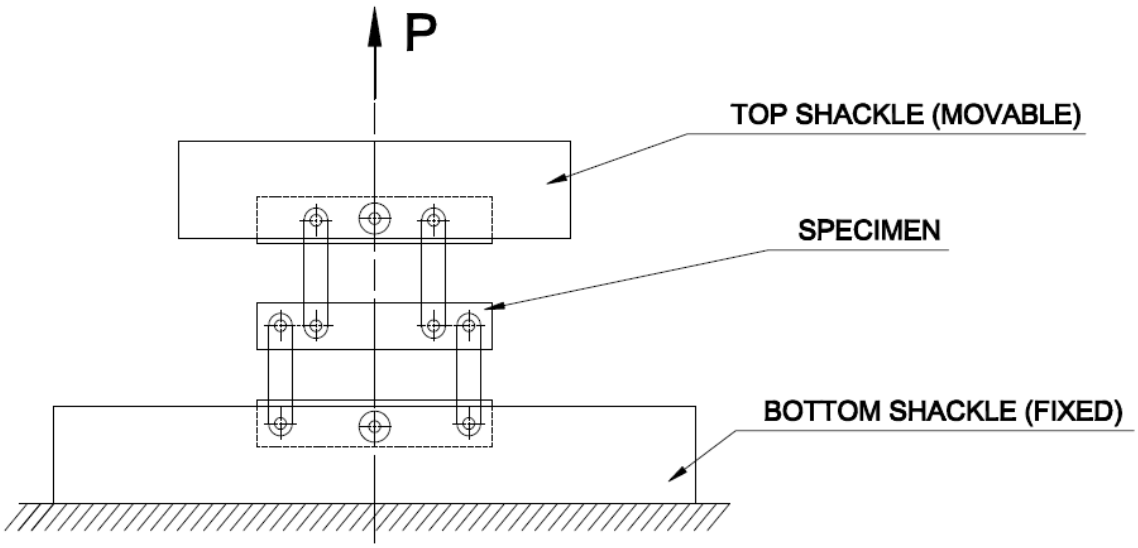


Fig. 8.1 Loading Arrangement for Four Point bending

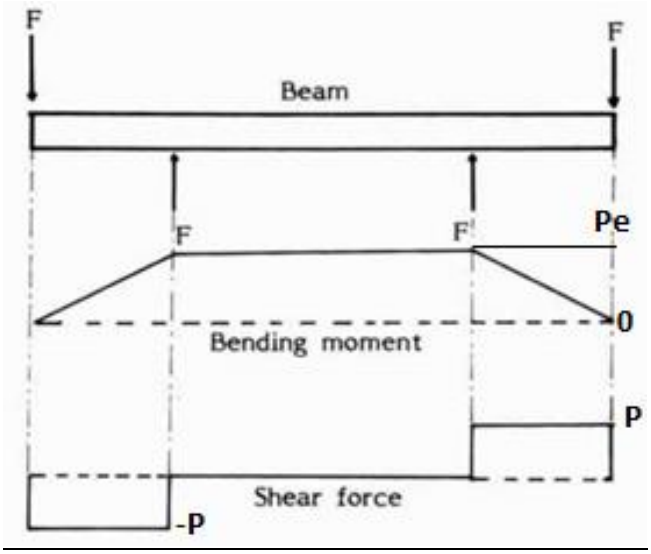


Fig.8.2 Shear force and Bending moment diagrams for point bending.

The bending stress in the specimen can be calculated by the formula,

$\sigma_b = M_b Y/I$

Where, $M_b = (P/2).e$ → (constant bending moment)

Y=distance of the outer fibre from the NA = d/2

I= Moment of inertia about neutral axis = $\frac{td^3}{12}$

$$\sigma_b = \frac{P}{2}\, e\, \frac{d}{2} \Big/ \frac{td^3}{12} = \frac{3Pe}{td^2} \qquad \text{------8.1}$$

From stress optics law,

$(\sigma_2 - \sigma_2) = Nf_\sigma/t$ ------8.2

But $\sigma_2 = 0$ on the free boundary and $\sigma_1 = \sigma_b$

$\sigma_b = Nf_\sigma/t$ ------8.3

equating 8.1 and 8.2

$3Pe/td^2 = Nf_\sigma/t$

$f_\sigma = 3e/d^2\ (P/N) = 3e/d^2\ (\Delta P/\Delta N)$ N/mm/fringe ------8.4

Setup details:

A circular polariscope with all the elements as discussed in the beginning is used. A four point bending specimen (photoelastic material) is used and the arrangement of loading and other details are shown in the Fig. 8.1

Procedure:

1. Make initial set up for the experiment, set the digital display to zero.
2. Fix the specimen to the shackles having arrangements as per fig. 8.1 taking care to see that the joints are made free to move with minimum friction.
3. Apply load by rotating the screw handle.
4. Observe the specimen through analyzer for each fractional load applied.
5. Determine the loads required for getting integral fringe orders (0, 1, 2, 3) at the center of the specimen and tabulate.

Or

Using Tardy's compensation technique, obtain fringe orders for different loads and tabulate (Note: The principal stress directions are aligned along X and Y axes in the constant bending moment region)

6. Draw a graph between load Vs fringe order (Linear and passing through origin).
7. Determine slope of the line $\Delta P/\Delta N$, from the graph a line of best fit.
8. Calculate the material fringe constant by using the equation 8.4

Observation:

Depth of the specimen: d = mm

Thickness of the specimen: t = mm

Eccentricity: e = mm

Tabular Column:

Sl No	**Fringe Order (N)**	**Load Applied (P)**		**Slope Of Line (ΔP/ΔN)**	**Material Fringe Constant (fσ)**
		Kg	**N**	**N/fringe**	**N/mm/fringe**
1	0 (Black)	0	0		
2					
3					
4					
5					
6					
7					

Specimen Calculation:

Load (P) = kg

= x 9.81 = N

Slope (from the graph) = ΔP/ΔN = N/fringe

Material fringe constant fσ = N/mm/fringe

Model fringe constant F= fσ/t = = N/mm^2/fringe

Method 2 (Using Line of Best Fit):

Tabular Column:

Sl No	**X**	**Y**	**XY**	X^2
	N (Fringe Order)	**P (Load in N)**	**NP**	N^2
1	0	0	0	0
2				
3				
4				
5				
6				
7				
SUM				

Specimen Calculation:

$a_0 = [\sum Y \sum X^2 - \sum X \sum XY] / [N\sum X^2 - (\sum X)^2]$

$a_0 =$

Slope= $(\Delta P/\Delta N) =$

$a_1 = [N \sum XY - \sum X \sum Y] / [N\sum X^2 - (\sum X)^2]$

$a_1 = (\Delta P/\Delta N) =$

Material fringe constant,
$f_\sigma = 3e/d^2 (P/N) = 3e/d^2 (\Delta P/\Delta N)$

$f_\sigma =$ N/mm/fringe.

Model fringe constant,
$F_\sigma = f_\sigma/t =$ N/mm^2/fringe

Conclusions:

Chapter 9. STRESS CONCENTRATION EXPERIMENTS

Any machine component having sudden changes in the cross section will have non-uniform stress distribution when subjected to external load. The magnitude of stress is maximum at the discontinuity in the component, i.e. the stress is concentrated at a narrow area. This is referred to as stress concentration. Fillets, notches, holes, keyways, splines, etc. causes stress concentration in the component. A factor which takes into account the ratio of maximum stress to nominal stress is called a stress concentration factor. In the design of a component this factor is taken into account. Stress concentration factor depends on shape and size of the discontinuity and also the material of the component.

Chapter 10
Experiment 10. Determination of Stress Concentration Factor of Hollow circular disc

Aim: To determine the stress concentration of a circular disc with a circular hole under diametral compression.

Apparatus And Material: Circular Polariscope with accessories, photoelastic model in the form of circular disc with a hole.

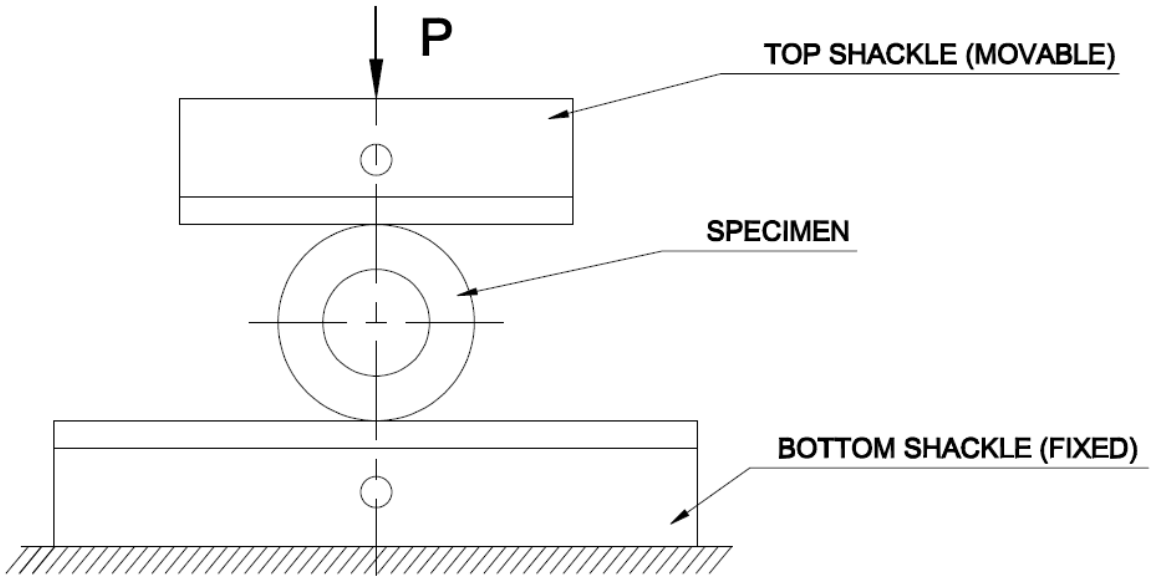


Fig.10.1:Circular plate with circular hole under compression.

Theory:

For circular disc,

σ_{nom} = load /area = P / (D – d).t

By stress optics law $(\sigma_1 - \sigma_2) = Nf_\sigma/t$ and an free boundary $\sigma_2 = 0$

Referring to stress concentration point at the inner boundary along horizontal diameter,
$\sigma_{act} = \sigma_1 = Nf_\sigma/t$

Theoretic stress concentration factor, Kt = maximum actual stress/nominal stress
$Kt = \sigma_{act} / \sigma_{nom}$

Procedure:

1. Make initial set up for the experiment, set the digital display to zero.
2. Place the model between loading screw and the bottom surface of the frame.
3. Apply load by rotating the screw handle.
4. Observe the specimen through analyzer.
5. Determine the loads required for getting integral fringe orders (0, 1, 2, and 3) at the inner boundary along horizontal diameter, and tabulate.

   Or

   Using Tardy's compensation technique, obtain fringe orders for different loads and tabulate (Note: The graphical stress directions are aligned along x and y axis) at the inner boundary along horizontal diameter).
6. Using material fringe constant of a calibration experiment determine σ_{act}
   Also obtain σ_{nom}
7. Calculate Stress concentration factor.

Observation:

Outer diameter of the disc: D = mm
Inner diameter of the disc: d = mm
Thickness of the specimen: t = mm
Material fringe constant from Calibration experiment: f_σ = N/mm/fringe

Tabular Column:

Sl No	**Fringe Order (N)**	**Load Applied (P)**		**σ_{nom}**	**σ_{act}**	**STRESS CONCEN TRATION FACTOR**
		Kg	**N**	**N/mm^2**	**N/mm^2**	
1						
2						
3						
4						

Specimen calculation:

Load P = Kg

= x9.81 = **N**

Nominal stress,

σ_{nom} = P/ (D-d)t = **N/mm^2**

Actual Stress at inner boundary along horizontal diameter,

σ_{act} = N f_σ/t = **N/mm^2**

Stress Concentration factor,$K_\sigma = \sigma_{act}/ \sigma_{nom}$ =

Conclusion:

Chapter 11.
Experiment 11. Determination of Stress Concentration Factor of Crane Hook

Aim: To determine the stress concentration factor in a crane hook.

Apparatus and Material: Circular Polariscope with accessories, photoelastic model in the form of circular disc with a hole.

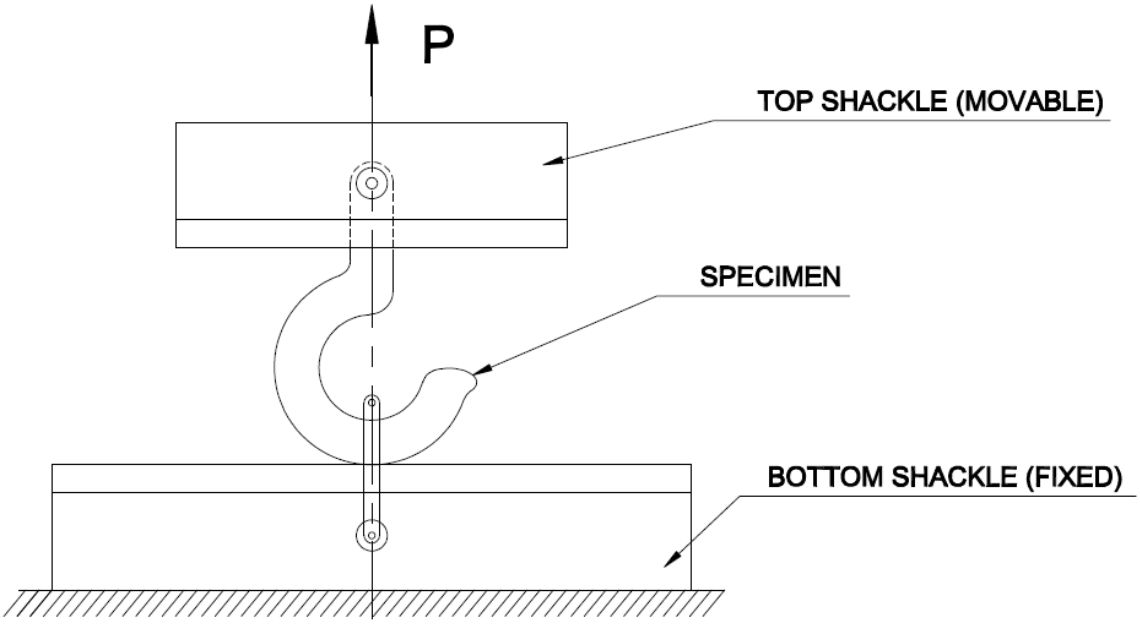


Fig.11.1 Loading Arrangement for crane hook under tension:

Theory:

The stress concentration (K_σ) in a crane hook represents the situation in a curved beam, wherein stress concentration occurs at the concave side. The load applied is tensile in nature. This includes bending moment across the cross section. The stress is maximum at the inner fiber. Both axial stress and bending stress are present in the cross section. But the only axial stress is considered for computing nominal stress.

Procedure:

1. Make initial set up for the experiment, set the digital display to zero.
2. Place the model between loading screw and the bottom surface of the frame.
3. Apply load by rotating the screw handle.
4. Observe the specimen through analyzer.
5. Determine the loads required for getting integral fringe orders (0, 1, 2, and 3) at inner radii (horizontal) and tabulate.

Or

Using Tardy's compensation technique, obtain fringe orders for different loads and tabulate.

6. Using material fringe constant of a calibration experiment determine σ_{act}

   Obtain σ_{nom}

7. Calculate Stress concentration factor.

Observation:

Outer radius of the hook: r_0 = mm

Inner radius of the hook: r_i = mm

Thickness of the hook: t = mm

Fringe constant of the material of hook Calibration experiment:

f_σ = N/mm/fringe

Tabular Column:

Sl. No	**Fringe Order for inner radius (N)**	**Load Applied (P)**		**σ_{nom}**	**σ_{act}**	**STRESS CONCEN TRATION FACTOR**
		Kg	**N/mm^2**	**N/mm^2**	**N/mm^2**	
1						
2						
3						
4						

Specimen Calculation:

Load P = Kg= N

Nominal stress = $\sigma_{nom} = P/(r_o - r_i)t$ = **Mpa**

Actual (max) stress = $\sigma_{act} = Nf\sigma/t$ = **Mpa**

Stress concentration Factor $K_\sigma = \sigma_{act} / \sigma_{nom}$ K_σ =

Conclusion:

Note:

1) If $\sigma_{nom} = P/A + M_b C/I$

   Where, $A= (r_o - r_i)t$, $M_b = Pn (r_o + r_i)/2$, $C= (r_o - r_i)/2$, $I = (r_o - r_i)^3 t/12$

   The stress concentration factor is only due to curvature.

2) For comparision of theoretical and experimental values in curved beam for a given load.

$\sigma_{ri\ theoritical} = F/A + M_b\ C_i/A\ e\ r_i$ $\sigma_{ri\ exp} = Nif\sigma/t$

$\sigma_{ro\ theoritical} = F/A + M_b\ C_o/A\ e\ r_o$ $\sigma_{ro\ exp} = Nof\sigma/t$

Where, $e = r_n - r_c$, $r_n = h/\log (r_o / r_i)$, $r_c = r_o - r_i/2$, $C_i = C_1 - e$,

$Co = C_2 + e$, $h = (r_o - r_i)/2$

Part B

Chapter 12. Governors

Introduction

The function of a governor is to maintain the speed of an engine within specified limits whenever there is a variation of load. In general, the speed of an engine varies in two ways – during each revolution (cyclic variation) and over a number of revolutions. In the former case, it is due to variation in the output torque of the engine during a cycle and can be regulated by mounting a suitable flywheel on the shaft. In the latter case, it is due to variation of load upon the engine and requires a governor to maintain the speed. The operation of a flywheel is continuous whereas that of a governor is more or less intermittent. A flywheel may not be used if there is no undesirable cyclic fluctuation of the energy output, but a governor is essential for all types of engines as it adjusts the supply according to the demand.

Types of Governors

Governors can broadly be classified into two types.

(i) Centrifugal Governor

This is the more common type. Its action depends on the change of speed. It has a pair of masses, known as governor balls, which rotate with a spindle. The spindle is driven by an engine through bevel gears (Fig.12.1). The action of the governor depends upon the centrifugal effects produced by the masses of the two balls. With the increase in the speed, the balls tend to rotate at a greater radius from the axis. Thus causes the sleeve to slide upon the spindle and this movement of the sleeve is communicated to the throttle through a bell crank lever. This closes the throttle valve to the required extent. When the speed decreases, the balls rotate at a small radius and the valve is opened according to the requirement.

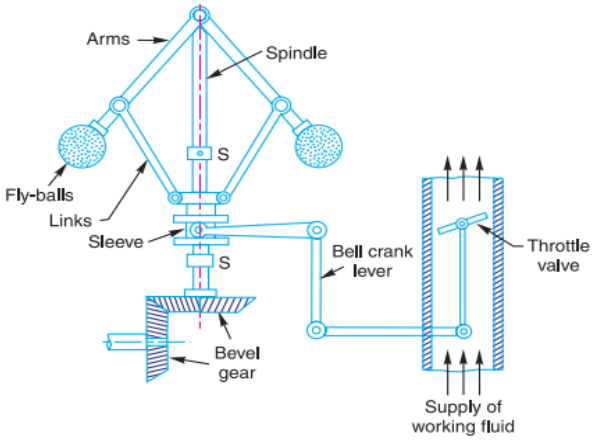


Fig 12.1

(ii) Inertia Governor

In this type, the positions of the balls are affected by the forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls. Using suitable linkages and springs, the change in position of the balls is made to open or close the throttle valve.

Thus, whereas the balls are operated by the actual change of engine speed in the case of centrifugal governors, it is by the rate of change of speed in case of inertia governors. Therefore, the response of inertia governors is faster than that of centrifugal types.

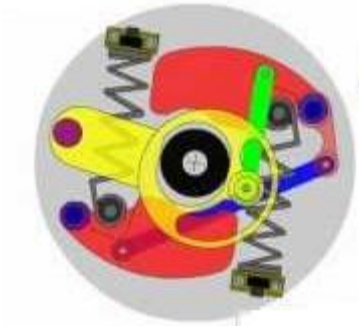


Fig 12.2

Governor Effort: It is the mean force exerted on the sleeve during a given change of speed.
Governor Power: It is defined as the work done at the sleeve for a given change in speed.
Hunting of Governor: It can occur in governor when the fluctuations in engine speed coincides the natural frequency of oscillations of the governor. In that case governor intensifies the speed variation instead of controlling it.
Controlling Force: It is the resultant external force which controls the movement of the ball and acts along the radial line towards the axis.

CHARACTERISTICS OF GOVERNORS
Different governors can be compared on the basis of following characteristics:

Stability: A governor is said to be stable when there is one radius of rotation of the balls for each speed which is within the speed range of the governor.
Sensitiveness: The sensitiveness can be defined under the two situations :
(a) When the governor is considered as a single entity.
(b) When the governor is fitted in the prime mover and it is treated as part of prime mover.

(a) A governor is said to be sensitive when there is larger displacement of the sleeve due to a fractional change in speed. Smaller the change in speed of the governor for a given displacement of the sleeve, the governor will be more sensitive.

(b) The smaller the change in speed from no load to the full load, the more sensitive the governor will be. According to this definition, the sensitiveness of the governor shall be determined by the ratio of speed range to the mean speed. The smaller the ratio more sensitive the governor will be

Where N2 – N1 = Speed range from no load to full load.

Isochronism: A governor is said to be isochronous if equilibrium speed is constant for all the radii of rotation in the working range. Therefore, for an isochronous governor the speed range is zero and this type of governor shall maintain constant speed.

Hunting: Whenever there is change in speed due to the change in load on the engine, the sleeve moves towards the new position but because of inertia if overshoots the desired position. Sleeve then moves back but again overshoots the desired position due to inertia. This results in setting up of oscillations in engine speed. If the frequency of fluctuations in engine speed coincides with the natural frequency of oscillations of the governor, this results in increase of amplitude of oscillations due to resonance. The governor, then, tends to intensity the speed variation instead of controlling it. This phenomenon is known as hunting of the governor. Higher the sensitiveness of the governor, the problem of hunting becomes more acute.

CONTROLLING FORCE AND STABILITY OF SPRING CONTROLLED GOVERNORS

The resultant external force which controls the movement of the ball and acts along the radial line towards the axis is called controlling force. This force acts at the centre of the ball. It is equal and acts opposite to the direction of centrifugal force. For controlling force diagram in which 'F' is plotted against radius 'r', F/r represents slope of the curve. Therefore, for a stable governor slope in controlling force diagram should increase with the increase in speed.

Figure 12.1 shows the controlling force curves for stable, isochronous and unstable spring controlled governors. The controlling force curve is approximately straight line for spring controlled governors. As controlling force curve represents the variation of controlling force 'F' with radius of rotation 'r', hence, straight line equation can be,

$$F=ar+b$$

Where a and b are constants. In the above equation b may be +ve, or –ve or zero.

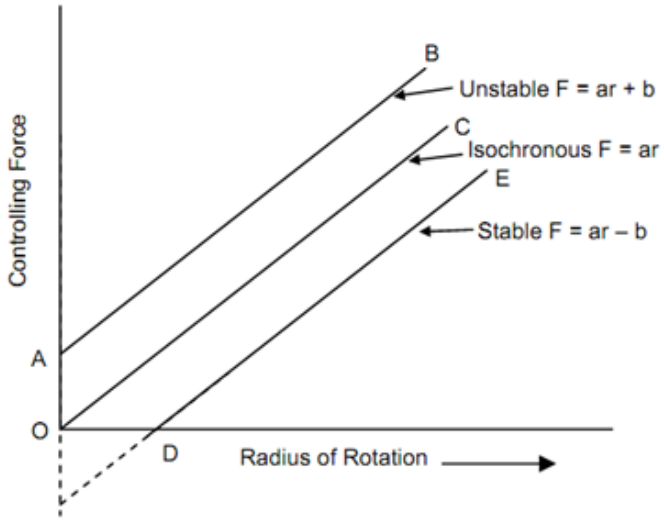


Figure 12.1 : Stability of Spring Controlled Governors

These three cases are as follows:

(a) We know that for a stable governor, the ratio must increase as r increases. Hence the controlling force curve DE for a stable governor must intersect the controlling force axis (i.e. y-axis) below the origin, when produced. Then the equation of the curve will be of the form F=ar-b. Therefore, this equation represents stable governor.

(b) If b in the above equation is zero then the controlling force curve OC will pass through the origin. The ratio F/r will be constant for all radius of rotation and hence the governor will become isochronous. Hence for isochronous, the equation will be F/r =a or constant

(c) If b is positive, then controlling force curve AB will intersect the controlling force axis (i.e. y-axis) above the origin. The equation of the curve will be F=ar+b

Hence this equation cannot represent stable governor but unstable governor.

Unstable F = ar + b

Isochronous F = ar

Stable F = ar – b

Chapter 13
Experiment No. - 8, Porter Governor

Aim: To determine the frictional resistance offered at the sleeve. Effort, power, etc. and in draw the controlling force curve.

Porter Governor:

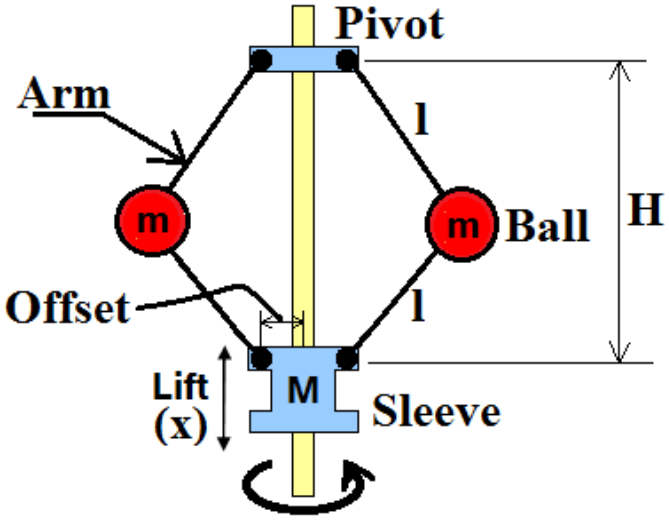


Fig.13.1 Porter Governor

If the sleeve of a Watt governor is loaded with a heavy mass, it becomes a Porter governor as shown in Fig.13.1. Let M = mass of the sleeve, m = mass of each ball, f = force of friction at the sleeve.

The force of friction always acts in a direction opposite to that of the motion. Thus when the sleeve moves up, the force of friction acts in the downward direction and the downward force acting on the sleeve is (Mg + f). Similarly, when the sleeve moves down, the force on the sleeve will be (Mg - f). In general, the net force acting on the sleeve is (Mg ± f) depending upon whether the sleeve moves upwards or downwards.

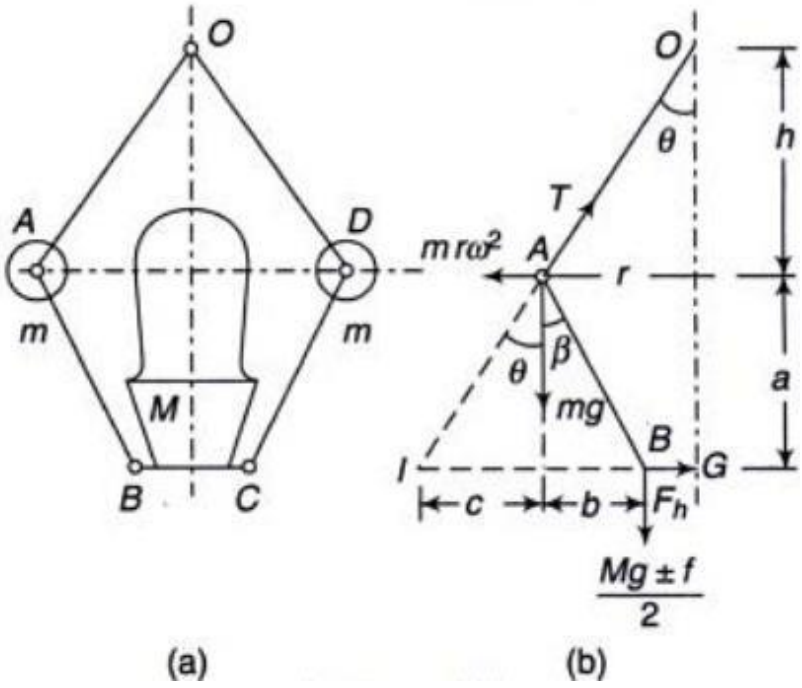


Fig.13.2. Forces acting on the sleeve and on each ball

Let h = height of the governor, r = distance of the center of each ball from axis of rotation.
The instantaneous center of rotation of the link AB is at I for the given configuration of the governor. It is because the motion of its two points A and B relative to the link is known. The point A oscillates about the point O and B moves in a vertical direction parallel to the axis. Lines perpendicular to the direction of those motions locates the point I.

Considering the equilibrium of the left-hand half of the governor and taking moments about I,

$$m r \omega^2 \cdot a = mg\,c + \frac{Mg \pm f}{2}(c+b)$$

or

$$m r \omega^2 = mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+k)\right] \qquad \left(taking \;\; k = \frac{\tan\beta}{\tan\theta}\right)$$

or

$$= \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+k)\right]$$

i.e $$\omega^2 = \frac{1}{mh}\left(\frac{2mg + (Mg \pm f)(1+k)}{2}\right)$$

or

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right)$$

$$N^2 = \frac{895}{h}\left(\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right) \qquad (taking \;\; g = 9.81 m/s^2)$$

This equation would provide two values of N for the same height of the governor. The phenomenon can be explained as below.

First assume that the sleeve has just moved down. Thus it means that the force acting on the sleeve is (Mg - f) downwards. Now, if the speed of the engine increases, the balls would tend to move away from the axis, but now as the friction has to act in the downward direction, the resistance to the motion would be (Mg + f). Thus until the speed rises to such a value as to overcome this resistance to the sleeve movement would be only (Mg - f). Thus, until the speed reduces to such a value as to give a force equal to (MG - f), the sleeve will not move.

Thus, for a given value of h, the governor is insensitive between two values of ω given by the above equation,

* If k = 1, $N^2 = \frac{895}{h}\left(\frac{mg + (Mg \pm f)}{mg}\right)$

* If f = 0, $N^2 = \frac{895}{h}\left(\frac{2m + M(1+k)}{2m}\right)$

* If k = 1, f = 0 $N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right)$

Procedure:

1) Switch on the porter governor
2) Increase its speed such that the sleeve moves upwards for a certain length.
3) Note this distance (sleeve lift) from the sleeve lift scale provided.
4) Wait till the governor stabilises.
5) Then note the speed (in rpm) of the governor using a tachometer.
6) Again increase the speed of the governor slightly more such that the sleeve moves further upwards. Note this sleeve lift.
7) Note the speed of the governor using a tachometer.
8) Continue this procedure for four values of sleeve lift.
9) Now, decrease the speed of the governor such that the sleeve moves downwards. Note the sleeve lift.
10) Note the speed at this instant. Continue this process for four values of sleeve lift.
11) Tabulate the values accordingly.

Observations:

(i) Mass of the rotating discs, m = _____kg
(ii) Mass of the central sleeve assembly, M = _____kg
(iii) Length of the links, l = _____m
(iv) Offset of the link pivots from spindle axis, = _____m
(v) Initial vertical distance between top and bottom pivots, H = _____m

Tabular Column

Sl No	Speed N rpm	Sleeve Lift 'x' m	Distance 'C' m	Distance 'S' m	Angle θ	Radius 'r' m	Height 'h' m	Frictional Force 'f' N	Centrifugal force F, N	Effort 'E' N	Power N-m

Specimen Calculations:

(i) Angular velocity of the spindle $\omega = \frac{2\pi N}{60}$ = _____ rad/sec

(ii) Vertical distance of the ball from the top pivot(with lift x),

$$C = \left(\frac{H}{2}\right) - \left(\frac{x}{2}\right) = _____ \text{ m}$$

(iii) Horizontal distance of the ball from the pivot position(with lift x),

$$S = \sqrt{\left(l^2 - c^2\right)} = _____ \text{ m}$$

(iv) Angle $\theta = \sin^{-1}\left(\frac{c}{l}\right)$ = _____ deg

(v) Radius of rotation r = (0.05 + S) = _____ m
Where, offset of the link pivot from spindle axis is taken as 0.05m

(vi) Height of governor h = r tan θ

(vii) Frictional force $f = \frac{N^2 m g h}{895} - (m + M)g$

(viii) Computational force $F_1 = m\omega^2 r$ = _____ N

(ix) Effort E = 0.01 [(M + m)g + f] = _____ N

(x) Power P = Effort x Sleeve lift = E * x = N-m

Conclusions:

Chapter 14
Experiment No. - 9, Proell Governor

A Porter governor is known as a Proell governor if the two balls (masses) are fixed on the upward extensions of the lower links which are in the form of bent links BAE and CDF. Considering the equilibrium of the link BAE which is under the action in Fig.14.1

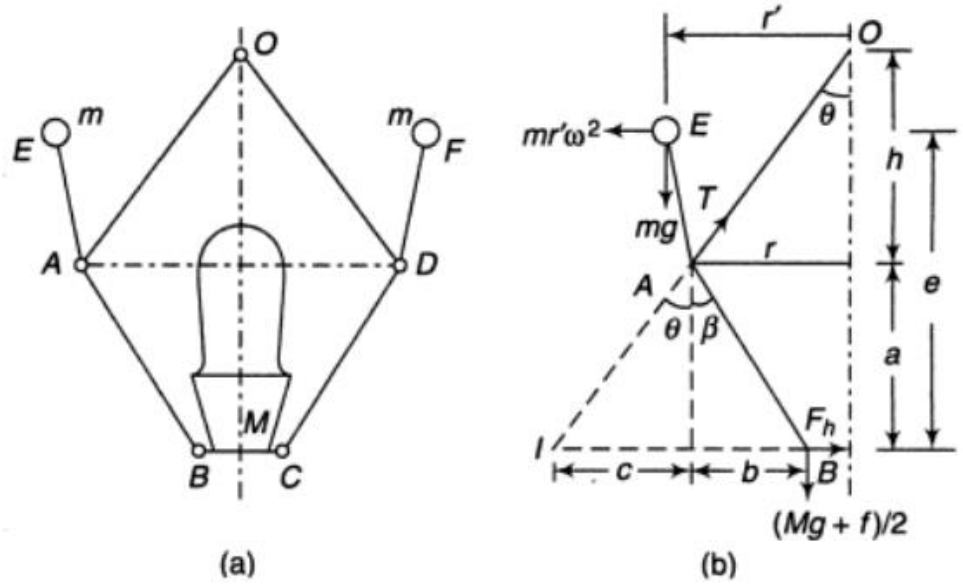


Fig.14.1. Proell Governor

Let the weight of the ball mg, the centrifugal force $mr'\omega^2$, the tension in the link AO, the horizontal reaction of the sleeve, the weight of sleeve and friction, $\frac{1}{2}(Mg \pm f)$.

As before, I is the instantaneous center of the link BAE. Taking moments about I,

$$mr'\omega^2 e = mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)$$

Where b, c, d and r are the dimensions as indicated in the diagram

$$mr'\omega^2 = \frac{1}{e}\left[mg(c + r - r') + \frac{Mg \pm f}{2}(c + b) \right]$$

In the position when AE is vertical, i.e., neglecting its obliquity

$$mr'\omega^2 = \frac{1}{e}\left[mgc + \frac{Mg \pm f}{2}(c + b) \right]$$

$$= \frac{a}{e}\left[mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a} + \frac{b}{a}\right) \right]$$

$$= \frac{a}{e}\left[mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta) \right]$$

$$= \frac{a}{e}\tan\theta\left[mg + \frac{Mg \pm f}{2}(1 + k) \right]$$

$$= \frac{a}{e}\frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1 + k) \right]$$

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{a}{e}\frac{g}{h}\left[\frac{2mg + (Mg \pm f)(1 + k)}{2mg} \right]$$

$$N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{2mg + (Mg \pm f)(1+k)}{2mg}\right]$$ (Taking g = 9.8 m/s^2)

- if k = 1, $N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{mg + (Mg \pm f)(1+k)}{2mg}\right]$
- if f = 0, $N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{2m + M(1+k)}{2m}\right]$
- if k = 1, f = 0 $N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{m+M}{m}\right]$

Observations :

(i) Mass of the rotating discs m = _____kg
(ii) Mass of the central sleeve assembly M = _____kg
(iii) Length of the links l = _____mts
(iv) Length of the extension links e-a = _____mts
(v) Offset of the link pivots from spindle axis = _____mts
(vi) Initial vertical distance between top and bottom pivots H = _____mts

Tabular Column

Sl No	Speed N rpm	Sleeve Lift 'x' mts	Distance 'C' mts	Distance 'S' mts	Angle θ	Radius 'r' mts	Height 'h' mts	Frictional Force 'f' N	Centrifugal force F, N	Effort 'E' N	Power Nm

Specific Calculations

(i) Angular velocity of the spindle $\omega = \frac{2\pi N}{60}$ = _____ rad/sec

(ii) Distance $C = \left(\frac{H}{2}\right) - \left(\frac{x}{2}\right)$ = _____ mts

(iii) Distance $S = \sqrt{(l^2 - c^2)}$ = _____ mts

(iv) Angle $\theta = \sin^{-1}\left(\frac{c}{l}\right)$ = _____ deg

(v) Radius of rotation r = (0.05 + S) = _____ mts

(vi) Height of governor h = r tan θ

(vii) Frictional force $f = \frac{N^2 m g h}{895} - (m+M)g$

(viii) Computational force $F_1 = m\omega^2 r$ = _____ N

(ix) Effort E = 0.01 [(M + m)g + f] = _____ N

(x) Power P = Effort x Sleeve lift = E * x = N-m

Conclusions:

Chapter 15
Experiment No.-10, Hartnell Governor

In this type of governor, the balls are controlled by a spring as shown in Fig.15.1(a). Initially, the spring is fitted in compression so that a force is applied to the sleeve. Two bell-crank levers, each carrying a mass at one end and a roller at the other, are pivoted to a pair of arms which rotate with the spindle. The rollers fit into a groove in the sleeve.

As the speed increases and the balls move away from the spindle axis, the bell-crank levers move on the pivot and lift the sleeve against the spring force. If the speed decreases, the sleeve moves downwards. The movement of the sleeve is communicated to the throttle of the engine. The spring force can be adjusted with the help of a screw cap.

Fig.16.6b shows the forces acting on the bell-crank lever in two positions (assuming that the sleeve moves up so that f is taken positive).

Let F = centrifugal force = $mr\omega^2$
F_s = spring force

Taking moments about the fulcrum A,

$$F_1 a_1 = \frac{1}{2}(Mg + F_{s1} + f)b_1 + mgc_1$$

$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)b_2 + mgc_2$$

In the working range of the governor, θ is usually small and so the obliquity effects of the arms of the bell-crank levers may be neglected. In that case,

$$F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b \qquad \text{(i)}$$

and

$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)b \qquad \text{(ii)}$$

Subtracting (i) from (ii)

$$(F_2 - F_1)a = \frac{1}{2}(F_{s2} - F_{s1})b$$

$$(F_{s2} - F_{s1}) = \frac{2a}{b}(F_2 - F_1)$$

or

Let s = stiffness of the spring
h_1 = movement of the sleeve

$$(F_{s2} - F_{s1}) = h_1 s = \frac{2a}{b}(F_2 - F_1) \text{ or } s = \frac{2}{h_1} \cdot \frac{a}{b} \cdot (F_2 - F_1)$$

But $h_1 = \theta \cdot b = \frac{r_2 - r_1}{a} \cdot b$

$$s = \frac{2}{r_2 - r_1}\left(\frac{a}{b}\right)^2 \cdot (F_2 - F_1) = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_2 - F_1}{r_2 - r_1}\right)$$

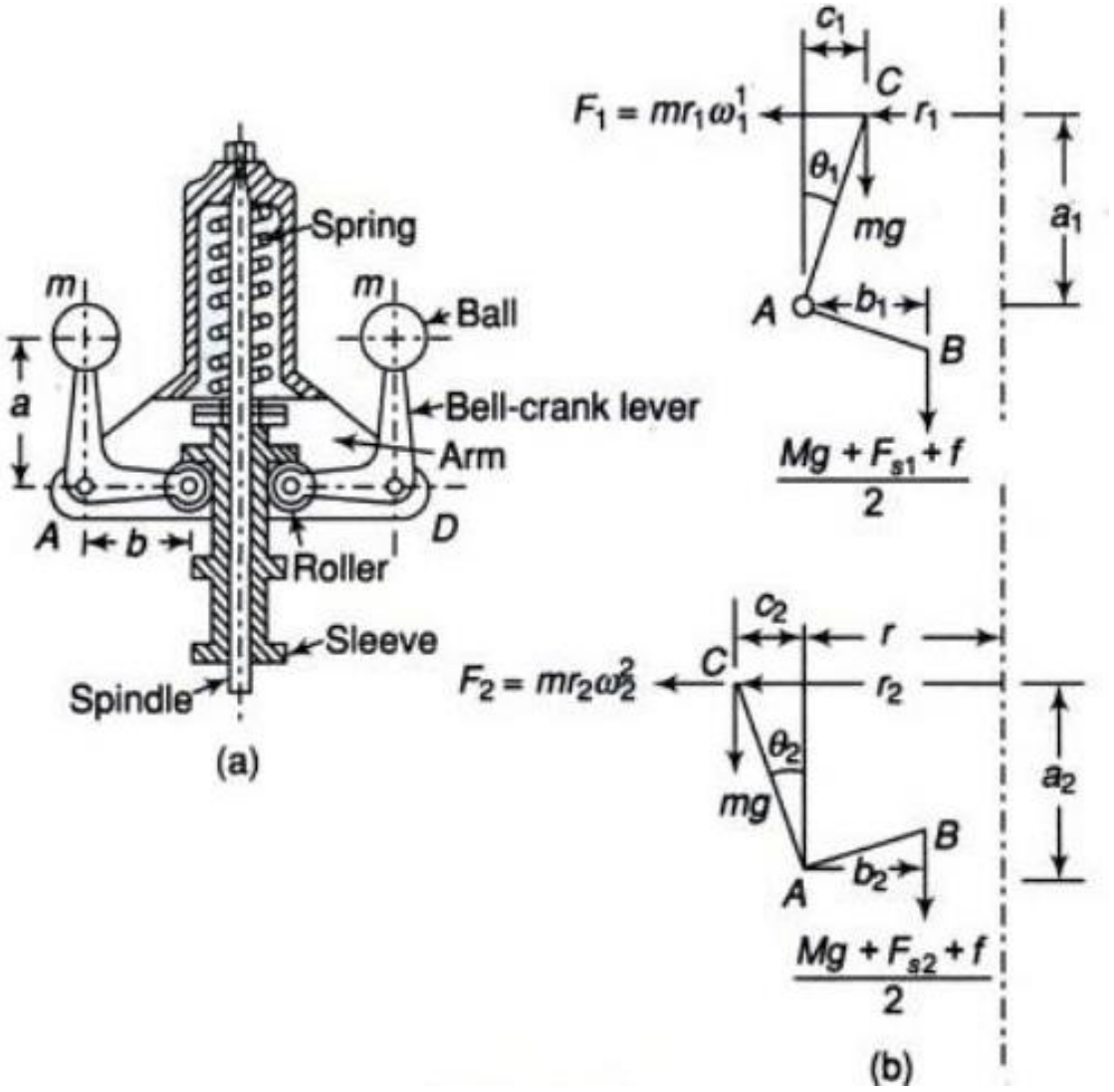


Fig.15.1, Hartnell Governor

Procedure:

1) Switch on the Hartnell governor.
2) Increase the speed to get a particular value of sleeve lift.
3) When the governor gets stabilized at a particular sleeve lift note down the sleeve lift and the corresponding governor speed using a tachometer.
4) Continue to increase the speed to get different sleeve lifts and note down the governor speed.
5) Similar readings for different sleeve lifts are taken for the downward movement of the sleeve by decreasing the speed.
6) Tabulate all the values and calculate the spring stiffness.

Observations:

1) Length of the ball arm, a =_____mm=_____m
2) Length of the sleeve arm, b =_____mm=_____m
3) Mass on each ball, m =_____g =_____kg
4) Initial radius of rotation of the ball, r_0 =_____mm =_____m

Tabular Column:

Sl. No	Lift(x) mm		Speed(n) rpm		Angular Velocity rad/s		Radius of rotation (m)		Controlling Force F_c (N)		Spring Force (N)		Spring Stiffness (N/mm)	
	Up	Down	Up	Down	Up	Down	Up	Down	Up	Down	Up	Down	Up	Down
1														
2														
3														

Specimen Calculations:

$Angular\ Velocity, \omega = \frac{2\pi N}{60} = _____ rad/s$

$$Radius\ of\ rotation, r, for\ sleeve\ lift\ 'x'\ mm = \frac{r - r_0}{x} = \frac{a}{b}$$

$\Rightarrow\gg\ r =$ _____ m

$Controlling\ Force\ (F_c) = m\omega^2 r = _____ N$

$$Spring\ Force\ (F_s):$$

$$F_s \times \frac{b}{2} = F_c \times a$$

$\Rightarrow\gg F_s =$ _____N

$$Spring\ Stiffness, k_1 = \frac{F_{s2} - F_{s1}}{x_2 - x_1} = _____ N/mm$$

$$Initial\ compression\ C:$$

$$C = \frac{Initial\ Spring\ Force}{Spring\ Stiffness}$$

= _____m =_____ mm

Graphs:

1. Displacement vs. Speed
2. Controlling force vs. radius of rotation

Conclusions:

Chapter 16.
Experiment 11: Journal Bearing

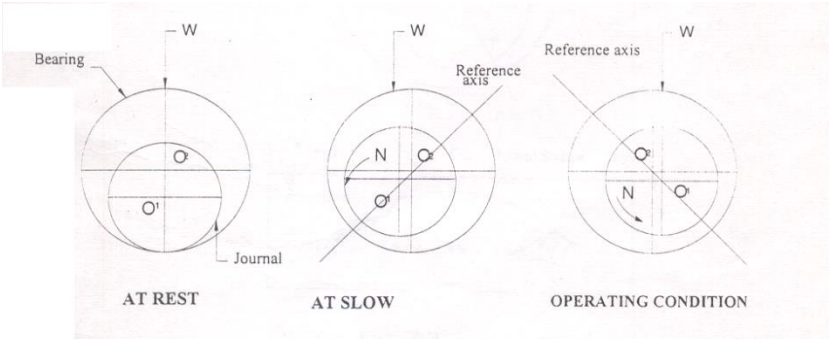
Aim: To study the pressure distribution under different experimental conditions (load, speed, or clearance) and verify the same theoretically.

Apparatus & Materials: journal bearing setup, tachometer, weights and lubricating oil.

Theory:

A journal bearing supports a shaft and permits rotary motion. This rotation, clearance between the surface, load on the bearing, dimension of the bearing and oil leakage from the surface cause wear of surface due to friction between the contact surfaces and heat is generated, resulting in loss of power. To minimize this, lubricating oil is introduced in the clearance between the journal and the bearing. This provides a thin film, separating the contact surface. The amount of separation depends on the thickness of the film formed. The oil film formation of surface thickness, results from the pressure developed in the annular space between the bearing and journal surfaces. The magnitude of pressure is a function of properties of the lubricant, speed of rotation, clearance between the surface, load on the bearing, dimension of the bearing and oil leakage from the surfaces.

The study of pressure distribution and variables associated with the bearing can be used for design purposes.



16.1 Effect of speed on journal bearing

In a **full journal bearing**, usually the journal rotes while the bearing is held fixed. Operating characteristics such as load-carrying capacity, friction resistance and coefficient of friction of a full journal bearing will be discussed in this section. For analysis of an idealized full journal bearing it is assumed that.

(1) There is no end leakage.
(2) The viscosity of the lubricant is constant thought the film.
(3) The flow is laminar.
(4) There is no slip between the lubricant and bearing surface.
(5) The fluid is an incompressible and a Newtonian liquid.

A schematic representation of a full journal bearing is shown in Fig.16.2. The thickness of the converging –diverging film surrounding the journal is a function of the radial clearance and the angle φ which is the angular distance of the film cross section considered from the reference line OO**.** Fig.16.2. shows the unwrapped oil film. The pressure distribution in this sommer field obtained an exact and solution for this equation.

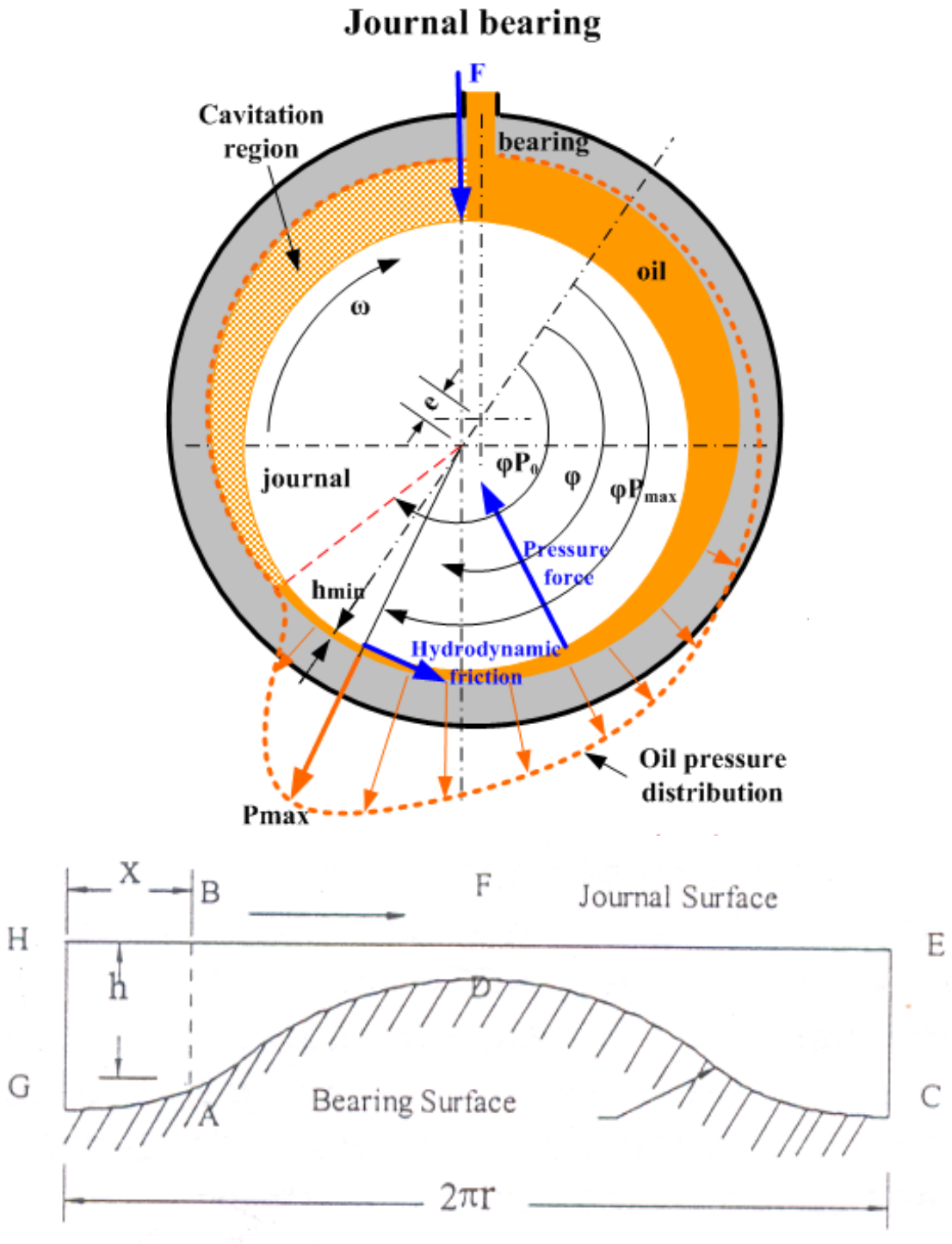


Fig.16.2 and 16.3

The solution is given by the equation:

$$P_\phi - P_0 = \frac{6\mu r U}{c^2(2+n^2)} X \frac{n\sin\phi(2+n\cos\phi)}{(1+n\cos\phi)^2} \quad \textbf{(16.1)}$$

$\mathbf{P}_\phi$ = Pressure at any location

$\mathbf{P}_0$= Pressure at reference point

μ= Viscosity of the oil.

r= radius of the bearing

U= πdN/60 =peripheral velocity.

N =rpm of journal.

C= clearance

n = attitude

φ =angle measured from the inlet value to any given position.

Using this equation, we can find the theoretical pressure at any point of interest.

The load is carrying capacity of an idealized full journal bearing depends on the film thickness which in turn depends upon the clearance and the attitude. A relationship between the external load acting on the journal and the attitude 'n' was first derived by Sommerfield in 1904. For this reason the dimensionless quantity.

$$S = (r/c^2)\mu N' / p = \{(2+n^2)\sqrt{1-n^2}\}/12\pi^2 n \quad \textbf{(16.2)}$$

Is known as **Sommerfield number** which is a function of attitude only. From this it was made possible to determine the minimum film thickness, Sommerfield number for lightly loaded bearings. The Sommerfield number for lightly loaded bearing must be at least equal to or greater than 0.15. Also it was found that when the bearing width becomes about four times greater than the length of the supporting oil film in the direction of motion, the influence of end leakage may be neglected for all practical purposes.

16.4 Setup Details:

It consists of a journal and bearing assembly connected to a D.C.motor through a dimmer stat. By this arrangement speed of varying magnitude can be obtained. The motor and the bearing are fixed on a rigid support. The loading arrangement consists of a vertical rectangular bar (loading bar) fixed to the bearing. The bar carries a hook and pan arrangement into which weights can be placed. A small horizontal bar is attached on to the loading bar which carries balancing weights. By this arrangement the horizontal bar can be made horizontal during testing or the loading arm can be made vertical. An oil tank supplies the lubricating oil to the bearing. The oil fed into the bearing at 45 degrees below the horizontal. The bearing has 16 pressure toppings 12 for circumferential and four for axial pressure readings. These toppings are connected to flexible tubes which are supported vertically. These tubes form manometers for reading the pressure. Each of the tubes is provided with an adjacent scale for measuring the heat of oil. The ends of the tubes are connected to an overflow cylinder and a pipe which conveys the overflows oil to the tank. Fig.16.4 gives the setup details. Any one of the parameters: load speed or clearance can be varied and experiment carried out.

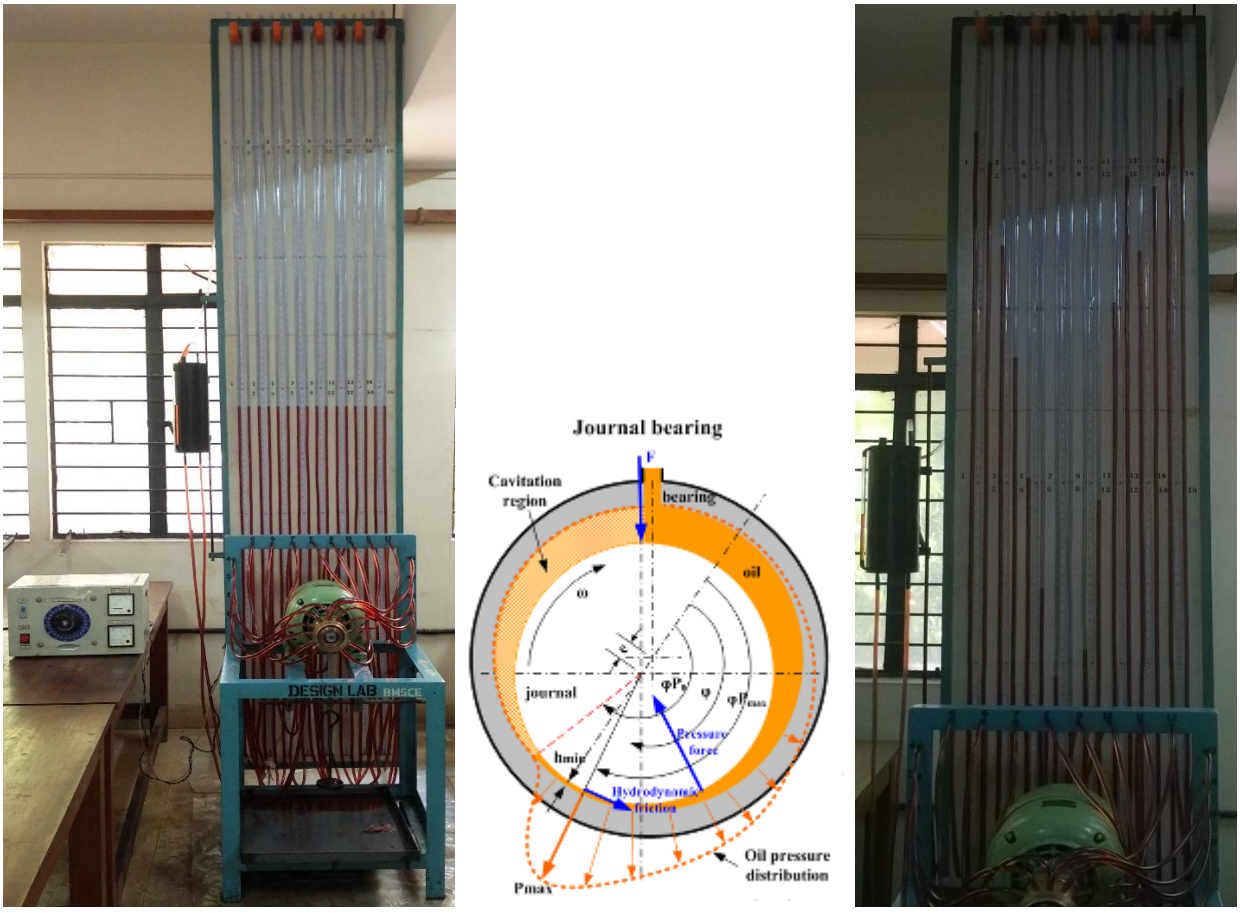


Fig 16.4 Test setup of journal bearing

Procedure:

1. Fill the oil tank with lubricating (say SAE-30) oil under test and position the tank at the desired height.
2. Drain out the air bubbles from all the manometer tubes on the manometer board as well as from the inlet tubes.
3. Check the level of oil in all the manometer tubes. Ensure that the level of oil is the same in the supply tank and all the manometer tubes .Note down the initial Manometer reading.
4. Any minor leakage at the end caps may be neglected .but leakage at any of the joints should be arrested.
5. Check and ensure that the dimmer stat knob is at zero position.
6. Switch on the motor and note down the direction of rotation.
7. Rotate the dimmer stat knob gradually till the desired speed is reached.
8. Add the desired loads and adjust the balancing weights provided so that the loading arm is vertical.
9. Run the set –up at this speed and load till the oil levels in all the manometer tubes are in steady state.
10. Note down the pressure of oil in all the manometer tubes and tabulate them.
11. Change the speed or load or clearance and repeat the experiment if necessary
12. After the experiment is over, remove the load.
13. Bring down the speed to zero (Dimmer stat position to zero) and switch off the motor and the main supply.
14. The difference in manometer pressure at each tapping is plotted as shown.

Setups for plotting pressure distribution graph. (Refer Fig.16.5)

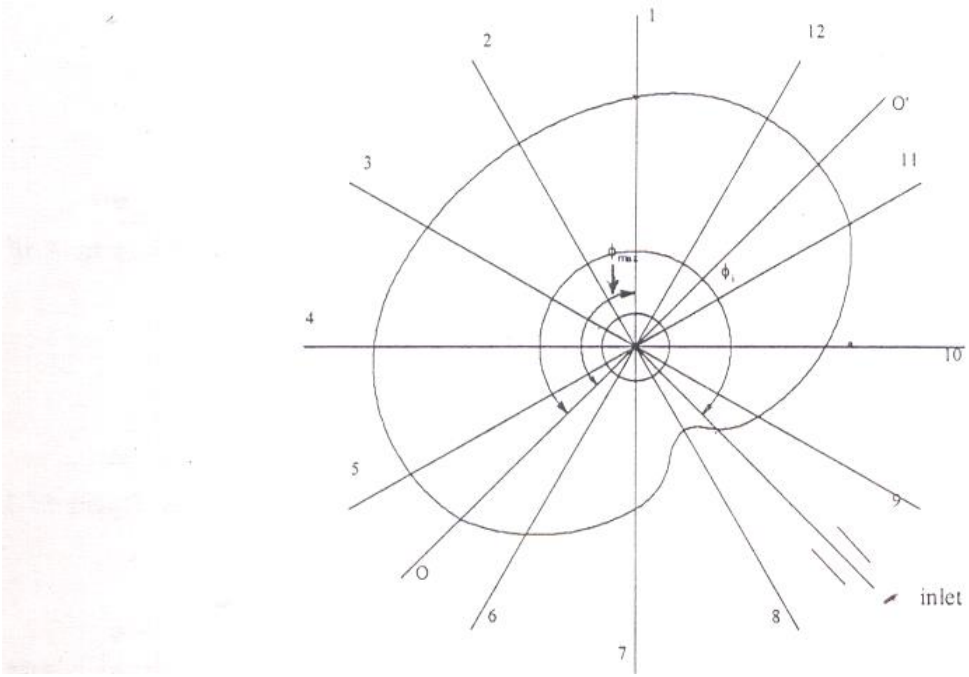


Fig. 16.5 Polar plot

1. Select a suitable scale to plot the pressure distribution curve.
2. With the initial pressure head (difference in pressure head above the center of bearing) as the radius a circle.
3. Divide the circle into12 equal divisions to represent the location of the pressure tapping on the bearing along the circumference.
4. Draw radial lines from the center of the circle along these 12 points and number them Sequentially in the CCW, if the rotation of the shaft is in CW direction and vice versa, starting from the vertical.
5 Mark the pressure heads along these radial lines corresponding to the toppings.
6 Join these points with a smooth curve.
7 Mark then direction of rotation of the journal on the figure.
8 Select two points on the curve having equal pressures (approximate) and 180 degrees apart and join them with a straight line passing through the center of the circle. This is reference line OO'
9 From the inlet hole the point of intersection of this line and the pressure curve, taken in the direction of the rotation of the journal is O and the opposite point is O'.
10 Measure any angle Ø from this point in the direction of rotation of the journal.
11 Comment on the result.

Formulae Used:

i) Calculation of attitude 'n'cos $Ø_m$ = $= \frac{-3n}{2+n^2}$

Where $Ø_m$ is the angel measured from reference line to the line of maximum pressure in the direction of rotation.

ii) The Sommerfield equation for calculation of various pressures

$$p_\phi - P_0 = \frac{6\mu r U}{C^2(2+n^2)} X \frac{n\sin\phi(2+n\cos\phi)}{(1+n\cos\phi)^2}$$

$$p_\phi = \frac{6\mu r U}{C^2(2+n^2)} X \frac{n\sin\phi(2+n\cos\phi)}{(1+n\cos\phi)^2} + P_0 \quad \text{---------- 16.3}$$

Where P_ϕ = pressure of oil film at any point measured clockwise from the line of common.

Centers OO',at an angle of ϕ^o:

at $\phi = 0^o$ and $\phi = 180^o$, $P\phi = P_o$

P_o =Constant manifold pressure in the oil film in journal bearing at t the point when $\phi = 0^o$

c= the angle from the line of the center OO' to any point of interest in the direction of rotation around the journal (Clockwise in our case).

r = angular speed of journal

n = g/c: (e = eccentricity)

iii) Calculation of pressure head of oil:

h = P/w

Where w = specific weight of the oil.

iv) The load charring capacity of the bearing:

$$w = \frac{12\mu r^2 LU\pi}{c^2} \qquad \frac{n}{2+n^2} \qquad \frac{1}{\sqrt{1-n^2}}$$

{Where U = (2πrn/60)m/S}

v) Calculation of Oil film thickness at any position ϕ

h = c(1+n cos ϕ)

Observation:

Diameter of the journal :d = mm

Radial clearance : c = mm

Length of the bearing :l = mm

Lubricating oil used = 20W40

Absolute viscosity of the oil : μ= 0.1184 NS/m^2

Kinematic viscosity of the oil : ϑ= 134.1 mm^2/S

Specific weight of the oil : w = ρg= 883*9.81=7848 N/m^3

Fixed weight = weight of journal + weight of vertical bar
+weight of balancing weight = 32.67 N

Load added =

Total vertical load: w = fixed weight +load added
= N

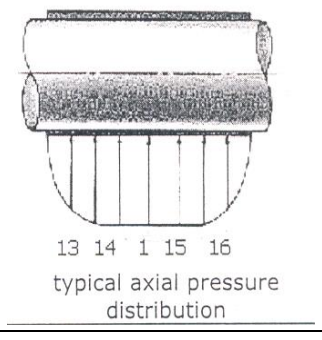
Initial manometer reading: h_i = mm

Speed of journal: N = rpm

Direction of Rotation =

Height of center of bearing from G.L =H_b =670mm

Initial pressure head above the bearing center = h_i - h_b

Tube No.	Final manometer reading (steady state) h_f mm	Head above the center of the bearing $h_{act} = (h_f - h_b)$mm	$P_{act} = h_{act}$ X w Actual pressure N/m^2	ϕ =	P_{theo} = Eq^n 16.3	h_{theo} = $P_{theo/w}$ mm	Oil thickness h= c(1+n) Cos ϕ mm
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							
9.							
10							
11							
12							
13					 typical axial pressure distribution		
14							
15							
16							

Specimen calculation:

Diameter of the journal, = ...m
Inside diameter of bearing, = ...m
Diametral clearance, =.....m
Bearing width, = ...m
Speed of the journal = ...rpm
Speed of the journal, = N/60= .....rps
Lubricating oil used, = 20W40
Viscosity of the oil T around 40^0c, = ...N-s/m^2

Conclusion:

Chapter 17 Strain Gauge

Introduction:

A Strain gage (sometimes referred to as a Strain Gauge) is a sensor whose resistance varies with applied force; it converts force, pressure, tension, weight, etc., into a change in electrical resistance which can then be measured. When external forces are applied to a stationary object, stress and strain are the result.

Principle of Working of Strain Gauges

When force is applied to any metallic wire its length increases due to the strain. The more is the applied force, more is the strain and more is the increase in length of the wire. If L_1 is the initial length of the wire and L_2 is the final length after application of the force, the strain is given as:

$\varepsilon = (L_2-L_1)/L_1$

Further, as the length of the stretched wire increases, its diameter decreases. Now, we know that resistance of the conductor is the inverse function of the length. As the length of the conductor increases its resistance decreases. This change in resistance of the conductor can be measured easily and calibrated against the applied force. Thus strain gauges can be used to measure force and related parameters like displacement and stress. The input and output relationship of the strain gauges can be expressed by the term gauge factor or gauge gradient, which is defined as the change in resistance R for the given value of applied strain ε.

Materials Used for the Strain Gauges

Earlier wire types of strain gauges were used commonly, which are now being replaced by the metal foil types of gauges as shown in the figure below. The metals can be easily cut into the zigzag foils for the formation of the strain gauges. One of the most popular materials used for the strain gauges is the copper-nickel-manganese alloy, which is known by the trade name 'Advance.' Some semiconductor materials can also be used for making the strain gauges. There are 2 types of stain gauges commonly used in practice 1. Metal Foil type (Rosette) (Fig 17.1) 2. Wire Type (Fig 17.2).

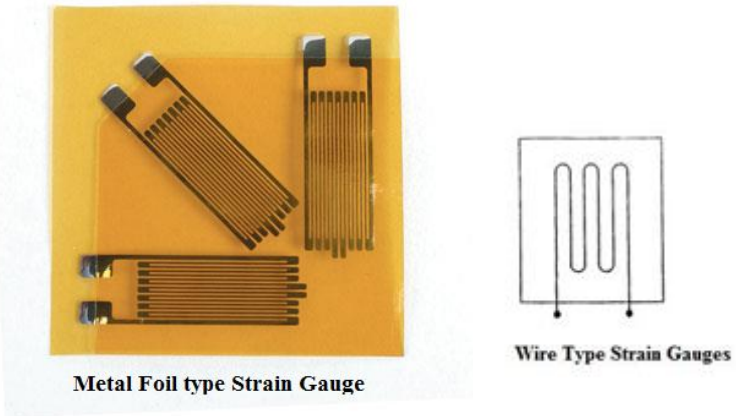


Fig 17.1 (a) and 17.1 (b)

Mounting of Strain gauge on test specimen

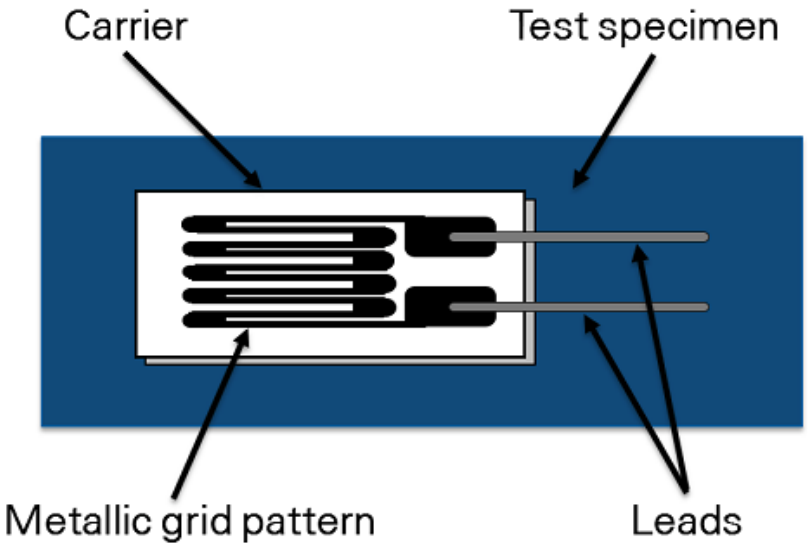


Fig 17.2

Gauge Factor:

A fundamental parameter of the strain gage is its sensitivity to strain, expressed quantitatively as the gage factor (GF). GF is the ratio of the fractional change in electrical resistance to the fractional change in length, or strain:

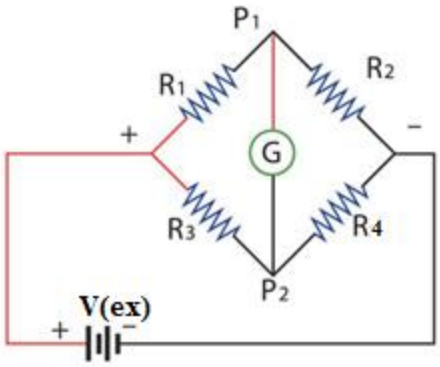
$$GF = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\varepsilon}$$

The GF for metallic strain gages is usually around 2. You can obtain the actual GF of a particular strain gage from the sensor vendor or sensor documentation.

Strain Gauge Measurement

Bridge Circuit:

In practice, the strain measurements rarely involve quantities larger than a few milli-strain ($\varepsilon \times 10^{-3}$). Therefore, to measure the strain requires accurate measurement of very small changes in resistance. For example, suppose a test specimen undergoes a substantial strain of 500 με. A strain gauge with a gauge factor GF = 2 will exhibit a change in electrical resistance of only $2 \bullet (500 \times 10^{-6}) = 0.1\%$. For a 120 Ω gauge, this is a change of only 0.12 Ω. To measure such small changes in resistance, and compensate for the temperature sensitivity discussed in the previous section, strain gauges are almost always used in a bridge configuration with a voltage or current excitation source. The general Wheatstone bridge, illustrated below, consists of four resistive arms with an excitation voltage, V_{EX}, that is applied across the bridge.



The output voltage of the bridge, VO, will be equal to:

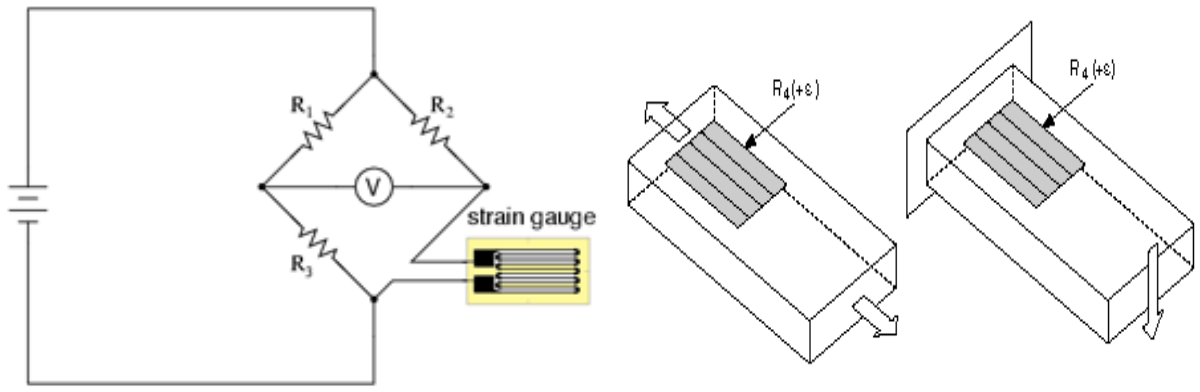
$$V_0 = \left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right) V_{ex}$$

From this equation, it is apparent that when $R_1/R_2 = R_{G1}/R_{G2}$, the voltage output V_O will be zero. Under these conditions, the bridge is said to be *balanced*. Any change in resistance in any arm of the bridge will result in a nonzero output voltage.

Quarter-Bridge Strain Gage

If we replace R4 in Figure with an active strain gauge, any changes in the strain gauge resistance will unbalance the bridge and produce a nonzero output voltage. If the nominal resistance of the strain gauge is designated as R_G, then the strain-induced change in resistance, ΔR, can be expressed as $\Delta R = R_G \cdot GF \cdot \varepsilon$. Assuming that $R_1 = R_2$ and $R_3 = R_G$, the bridge equation above can be rewritten to express V_O/V_{EX} as a function of strain

Quarter-bridge strain gauge circuit



$$\frac{V_0}{V_{ex}} = -\frac{GF * \varepsilon}{4}\left(\frac{1}{1 + GF * \frac{\varepsilon}{2}}\right)$$

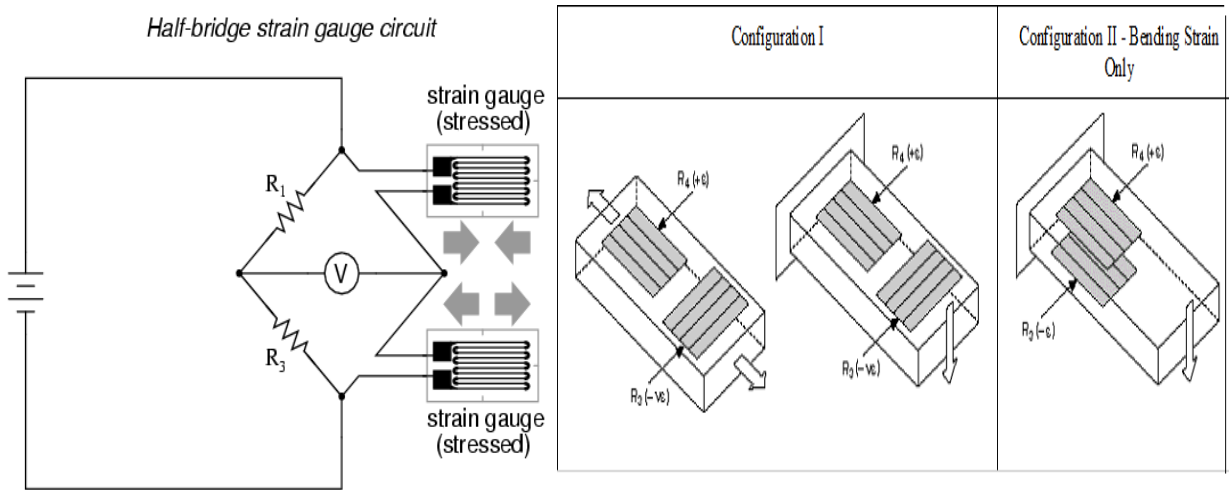
Configuration Type I

- Measures axial or bending strain
- Requires a passive quarter-bridge completion resistor known as a dummy resistor
- Requires half-bridge completion resistors to complete the Wheatstone bridge
- R4 is an active strain gage measuring the tensile strain (+ε)

Half-Bridge Strain Gage

By using two strain gauges in the bridge, the effect of temperature can be avoided. For example, Figure illustrates a strain gauge configuration where one gauge is active ($R_G + \Delta R$), and a second gauge is placed transverse to the applied strain. Therefore, the strain has little effect on the second gauge, called the *dummy* gauge. However, any changes in temperature will affect both gauges in the same way. Because the temperature changes are identical in the two gauges, the ratio of their resistance does not change, the voltage V_O does not change, and the effects of the temperature change are minimized.

Alternatively, you can double the sensitivity of the bridge to strain by making both gauges active, although in different directions. For example, in bending beam application with one bridge mounted in tension ($R_G + \Delta R$) and the other mounted in compression ($R_G - \Delta R$). This half-bridge configuration yields an output voltage that is linear and approximately doubles the output of the quarter-bridge circuit.



$$\frac{V_0}{V_{ex}} = -\frac{GF * \varepsilon}{2}$$

Configuration Type I

- Measures axial or bending strain
- Requires half-bridge completion resistors to complete the Wheatstone bridge
- R4 is an active strain gage measuring the tensile strain (+ε)
- R3 is an active strain gage compensating for Poisson's effect (-νε)

This configuration is commonly confused with the quarter-bridge type II configuration, but type I has an active R3 element that is bonded to the strain specimen.

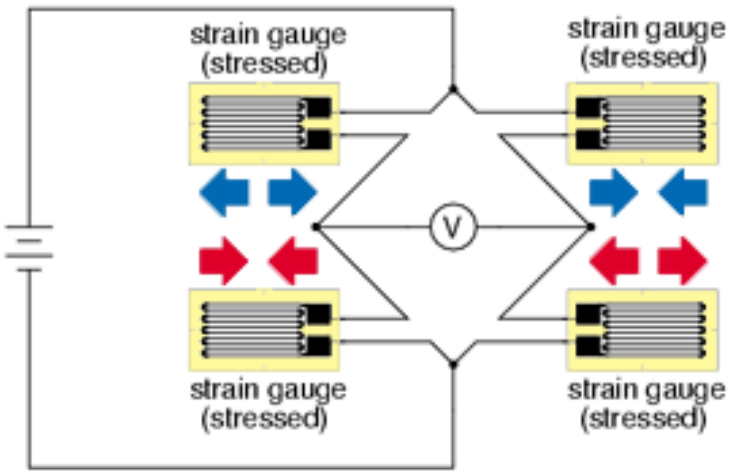
Configuration Type II

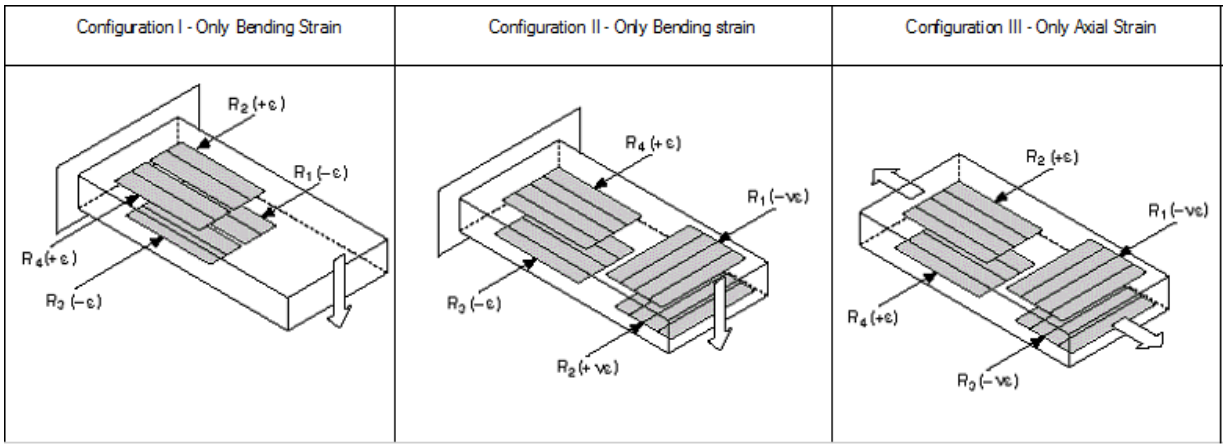
- Measures bending strain only
- Requires half-bridge completion resistors to complete the Wheatstone bridge
- R4 is an active strain gage measuring the tensile strain (+ε)
- R3 is an active strain gage measuring the compressive strain (-ε)

Full-Bridge Strain Gage

Finally, you can further increase the sensitivity of the circuit by making all four of the arms of the bridge active strain gauges, and mounting two gauges in tension and two gauges in compression. The full-bridge circuit is shown in Figure below.

Full-bridge strain gauge circuit





$$\frac{V_0}{V_{ex}} = -GF * \varepsilon$$

A full-bridge strain gage configuration has four active strain gages and is available in three different types. Types 1 and 2 measure bending strain and type 3 measures axial strain. Only types 2 and 3 compensate for the Poisson effect, but all three types minimize the effects of temperature.

Configuration Type I

- Highly sensitive to bending strain only
- R1 and R3 are active strain gages measuring compressive strain (–e)
- R2 and R4 are active strain gages measuring tensile strain (+e)

Configuration Type II

- Sensitive to bending strain only
- R1 is an active strain gage measuring the compressive Poisson effect (–νe)
- R2 is an active strain gage measuring the tensile Poisson effect (+νe)
- R3 is an active strain gage measuring the compressive strain (–e)
- R4 is an active strain gage measuring the tensile strain (+e)

Configuration Type III

- Measures axial strain
- R1 and R3 are active strain gages measuring the compressive Poisson effect (–ve)
- R2 and R4 are active strain gages measuring the tensile strain (+e)

The equations given here for the Wheatstone bridge circuits assume an initially balanced bridge that generates zero output when no strain is applied. In practice however, resistance tolerances and strain induced by gauge application will generate some initial offset voltage. This initial offset voltage is typically handled in two ways. First, you can use a special offset-nulling, or balancing, circuit to adjust the resistance in the bridge to rebalance the bridge to zero output.

Chapter-18
Experiment – 12. Strain Rosettes

Aim:

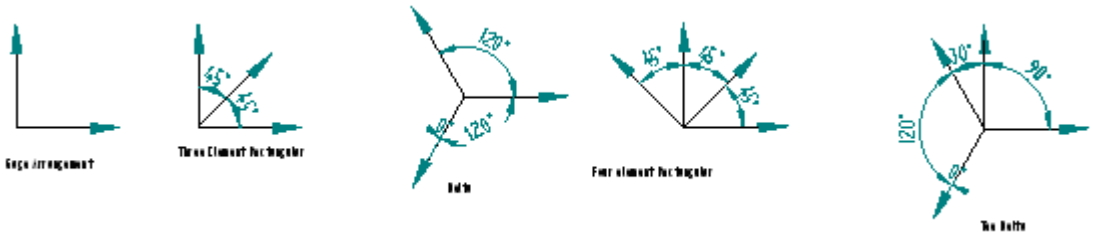
1. To determine strains induced the specimen when subjected to torque.
2. To determine principal stress and principal strain when subjected to combined torque and bending moment.

Apparatus and Material:

Experimental set up, strain gauges, specimen weights and strain indicators

Theory:

Strain gauges can be used for the measurement of strains on the free surface of the body and finds application in experimental stress analysis. In general, it is necessary to measure 3 strains at a point to completely define either the stress or the strain field. In terms of principle strains, it is necessary to measure ϵ_1, ϵ_2, and the direction θ of ϵ_1 relative to the X- axis. When less is known beforehand regarding the state of stress in the specimen, it is necessary to employ multiple element strain gauges to establish magnitude of the stress field. The strain gauges can be arranged in combination to get three elemental rectangular rosette or 3-element delta rosette or 4 elemental rectangular rosette etc.



Rectangular Rosette:

For three element rectangular rosette with gauges A, B and C with angles of Θ_A, Θ_B & Θ_C, respectively, the strains induced are

$$\varepsilon_A = \varepsilon_{xx}\text{Cos}^2\Theta_A + \varepsilon_{yy}\text{Sin}^2\Theta_A + \gamma_{xy}\text{Sin}\Theta_A.\text{Cos}\Theta_A \quad (18.1)$$

$$\varepsilon_B = \varepsilon_{xx}\text{Cos}^2\Theta_B + \varepsilon_{yy}\text{Sin}^2\Theta_B + \gamma_{xy}\text{Sin}\Theta_B.\text{Cos}\Theta_B \quad (18.2)$$

$$\varepsilon_C = \varepsilon_{xx}\text{Cos}^2\Theta_C + \varepsilon_{yy}\text{Sin}^2\Theta_C + \gamma_{xy}\text{Sin}\Theta_C.\text{Cos}\Theta_C \quad (18.3)$$

Normally $\Theta_A = O^0$, $\Theta_B = 45^0$, $\Theta_C = 90^0$

If $\Theta_A = O^0$, $\varepsilon_A = \varepsilon_{xx}$, $\text{Cos}2\Theta = \varepsilon_{xx}$ (18.4)

$$\theta_B = 45^0, \varepsilon_B = \varepsilon_{xx}\left(\frac{1}{\sqrt{2}}\right)^2 + \varepsilon_{yy}\left(\frac{1}{\sqrt{2}}\right)^2 \gamma_{xy}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\varepsilon_{xx}+\varepsilon_{yy}+\gamma_{xy}}{2} \quad (18.5)$$

$$\theta = 90^0 \varepsilon_c = \varepsilon_{xx}(0) + \varepsilon_{yy}(1) + \gamma_{xy}(0) = \varepsilon_{yy} \quad (18.6)$$

From (17.5) $\gamma_{xy} = 2\varepsilon_B - 2\varepsilon_{yy} - \varepsilon_{yy}$ (18.7)

$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_c$

Principle strains $\varepsilon_{1,2}$ is given by

$$\varepsilon_{1,2} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2} + \gamma^2{}_{xy}$$

$$\therefore \varepsilon_{1,2} = \frac{\varepsilon_A + \varepsilon_C}{2} \pm \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 (2\varepsilon_B - \varepsilon_A - \varepsilon_C)}^2 \quad (18.8)$$

And the orientation is given by

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2\varepsilon_B - \varepsilon_A - \varepsilon_c}{\varepsilon_A - \varepsilon_B}\right) \quad (18.9)$$

$0 < \theta_1 < 90^0 \, when \varepsilon_b > \frac{\varepsilon_A + \varepsilon_B}{2}, -90 < \theta_1 < 0^0 \, when \quad \varepsilon_b < \frac{\varepsilon_A + \varepsilon_B}{2}$

Principal stresses

$\sigma_1 = \frac{E}{1-\gamma^2}(\varepsilon_1 + \gamma\varepsilon_2)$ Substituting $\varepsilon_1 and \varepsilon_2$

$$\sigma_1 = \frac{E}{2(1-\gamma)}(\varepsilon_A + \varepsilon_C)^2 + \frac{E}{2(1+\gamma)}\sqrt{(\varepsilon_A + \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2} \quad (18.10)$$

Similarly

$$\sigma_2 = \frac{E}{(1-\gamma^2)}(\varepsilon_2 + \gamma\varepsilon_1) \quad (18.11)$$

$$\sigma_2 = \frac{E}{2(1-\gamma\ )}(\varepsilon_A + \varepsilon_C) - \frac{E}{2(1-\gamma\ )}\sqrt{(\varepsilon_A + \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

Setup Details:

The experimental setup is as shown in the Fig.18.1. It consists of a rigid frame which houses the specimen in the form of a circular bar and a loading arrangement. The specimen is held by a clamp at one end and is supported at the other end by a prop. Strain gauges are mounted on the specimen as rectangular rosette at 0^0, 45^0 and 90^0 orientation, to sense the strain. To the prop end of the specimen a torsion arm is provided the torsion arm is a rectangular cross section bar with slots in through which loads can be hung. The lever arm formed between the center of the specimen and the center of the slot through which the load is hung gives the moment arm. Behind the specimen rod and slightly above it, a hinged rod is fixed onto the frame. It carries a rectangular cross section bar with slots for load attachment and acts as a bending arm. This bar can be moved along the axis of the hinged rod such that bending moment on the specimen can be induced at any point, of different magnitude. This bending load arms can be independently applied onto the specimen as required.

Only torsion arm is used for creating torsion in the specimen whereas torsion and bending load arm are used for inducing torsion and bending moment in the specimen.

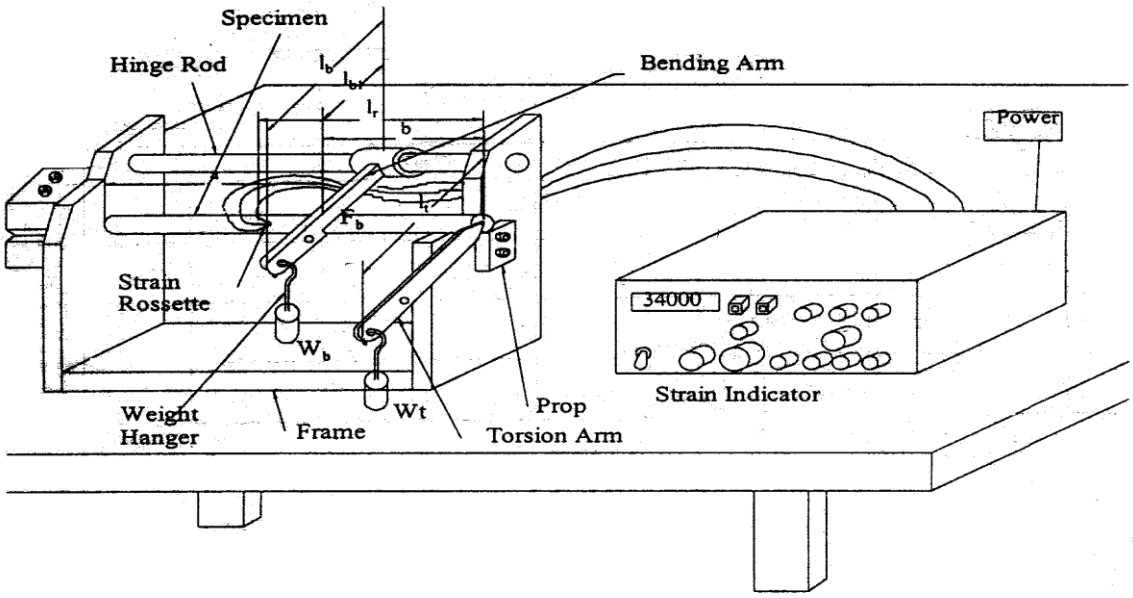


Fig 18.1 Experimental Setup with Rectangular rosette

Part I : Torsion only

Part I – To find the strains induced

Procedure:

1. Check the ohms and bridge arm.
2. Connect the leads of the strain gauges to the strain indicators as shown in Picture.
3. Set the arm to 2 for half bride and select the channel in the indicator.
4. Set the gauge factor to the desired value on the indicators.
5. set initial reading on the strain indicator to zero (with torsion arm and pan in loading position)
6. Apply torsion load W_T at a distance of l_t.
7. Note down the reading on the three channels (ϵ_A, ϵ_B & ϵ_C)(one channel should read zero (ϵ_B) and the other two should have the same magnitude but opposite sense)
8. Compare with theoretical values and tabulate the results

Observation:

Material of the specimen (shaft): Mild steel

Diameter of the specimen: d = mm

Length of the torque arm: l_T = mm

Modulus of rigidity of the material of the specimen (steel): G = 80 Gpa

Type of strain gauge =

Strain gauge resistance =

Bridge configuration =

Tabular column

Sl. No.	Load (W_T)		Torque (T)	Measured Strain(µS)			Theoretical Strain (µS)		
	Kg	N	N-m	ε_A	ε_B	ε_C	ε_A	ε_B	ε_C
1.									
2.									
3.									
4.									

Specimen calculation (theory) (Torsional only)

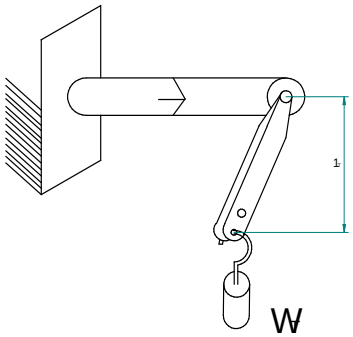


Fig 18.2 LOADING THE SPECIMEN IN TORSION

Specimen Calculation for Torsion

Load W_T = Kg = N

Torque (Mt) = W_T x l_T

= Nm

Shear stress $\tau = 16Mt / \pi d^3$ = Mpa
Shear strain $\gamma = \tau/G$ = μ S

$\varepsilon_A = \gamma/2$ =

$\varepsilon_B = 0.0$ =

$\varepsilon_C = -\gamma/2$ =

% Error = $\frac{\text{Measured Strain – Theoretical Strain}}{\text{Theoretical Strain}}$ x 100

Part II – to find the principal stresses and strains under combined torsion and bending loads & compare with theoretical values.

Procedure:

1. Check the ohms and bridge arm.
2. Connect leads of strain gauges to strain indicator.
3. Set gauge factor to the desired value in the indicator.
4. Set initial reading of the strain indicator to zero at no load condition (With torque and bending arm in position with pans)
5. Apply torsion load W_T at a distance of L_T on torque arm at propped end and bending load W_b on bending arm.
6. Note down the corresponding strains from the indicator.
7. Repeat the experiments for different W_T and W_b values.
8. Calculate theoretical and experimental values of strain and compare them.

Observation:

Length of the bending arm l_b = mm
l_{b1} = mm
Length of the torque l_T = mm
Bending arm position a = mm
b = mm
Length of the shaft L = (a+b) = mm

Distance of rosette from propped end l_r = mm
Diameter of the shaft: mild steel d=25.4 mm
Young's modulus of the shaft E =210 GPa G = 80GPa
Poisson's ratio: $\gamma = 0.3$
Type of strain gauge =
Strain gauge resistance =
Bridge configuration =

Analytical Method

Torque Mt = W_T x l_T (refer fig. 18.2)

Force exerted F_b by the bending arm on the specimen

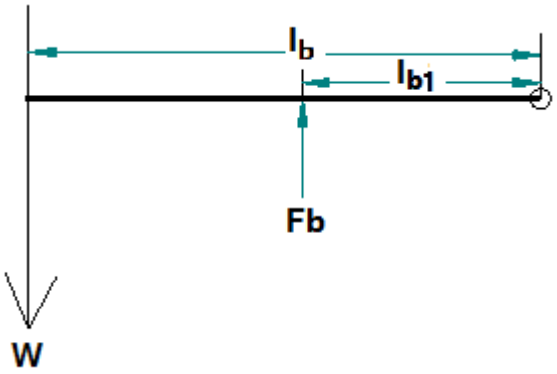


Fig. 18.3 Force F_b on the specimen with bending arm

$F_b \times l_{b1} = W_b \times l_b$ $\quad F_b = \frac{W_b \times l_b}{l_{b1}}$ (18.12)

Bending Moment M_b:

Consider the propped cantilever beam (statically indeterminate) fig.18.4

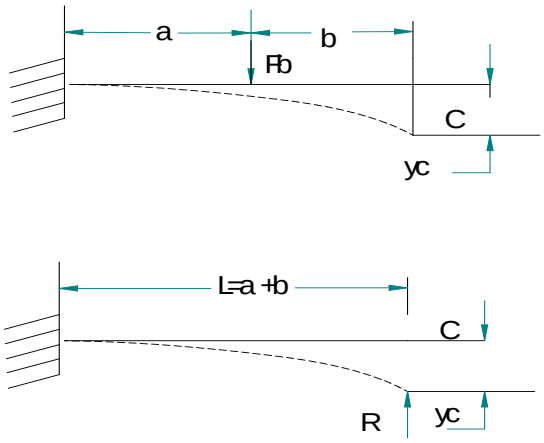


Fig. 18.4 Deflection of the specimen

Downward deflection at C due to the load F_b = Upward deflection at C due to the reaction R_c.

Deflection at the point C due reaction R_c $y_e = \frac{R_c L^3}{3El}$ (18.13)

Deflection at C due to F_b $= \frac{F_b a^3}{3El} + \frac{F_b a^2 b}{2El}$ (18.14)

Equating (18.13) and (18.14) we get

$$R_c = \left[\frac{F_b a^3}{3} + \frac{F_b a^3 b}{2} \right] . \frac{3}{L^3} \quad (18.15)$$

$$R_c = \left(2F_b a^3 + 3F_b a^2 b\right) \frac{1}{2L^3}$$

If $a = b = L/2$ *then* $R_c = \frac{5F_b a^3}{2L^3} = \frac{5F_b}{16}$ (18.16)

BM at section B (across rosette)

$M_b = R_b * L_r$ if $b > l_r$

$M_b = R_b * L_r - F_b)\ (l_r - b)$ if $b < l_r$ (18.17)

Section B is subjected to shear stress and bending stress

Shear stress $\tau = \frac{16M_t}{\pi d^3}$ (18.18)

Bending stress $\sigma_b = \frac{32M_b}{\pi d^3}$ (18.19)

<u>Principle stresses</u>

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} \quad (18.20)$$

<u>Principle strains</u>

$\varepsilon_1 = (\sigma_1 - \gamma\sigma_2)\frac{1}{E}$ and $\varepsilon_2 = (\sigma_2 - \gamma\sigma_1)\frac{1}{E}$ (18.21)

Direction of principal stress is given by

$$\theta_{1,2} = \frac{1}{2} \tan^{-1}\left(\frac{2\tau}{\sigma_b}\right) \quad (18.22)$$

Tabular Column:

Sl.No.	Bending Load (W_b)	(F_b)		TorqueLoad (W_t)		Strain indicator reading			Torque (M_t)	Reaction (R_b)	Bending moment (M_b)
	Kg	N	N	Kg	N	ε_A	ε_B	ε_C	Nm	N	Nm
1.											
2.											
3.											
4.											
5.											

Sl.No.	Bending stress σ_b(MPa)	Shear stress τ(MPa)	Measured values				Theoretical value			
			Principal strains		Principal stress (MPa)		Principal strains		Principal stress (MPa)	
			ε_1	ε_2	σ_1	σ_2	ε_1	ε_2	σ_1	σ_2
1.										
2.										
3.										
4.										
5.										

Specimen calculation

Load applied to the torque arm w_t = Kg

= X 9.81 N

= N

Torque $M_t = W_t$ x l_T = Nm

Load applied to the bending arm w_b = Kg

= X 9.81 N

= N

Effective bending load F_b on the specimen (shaft)

$\therefore F_b = \dfrac{w_b x l_b}{l_{b1}}$ = N

Reaction R_c (from equation 7.16)

Bending moment Mb (from equation 7.17)

Bending stress $\sigma_b = \dfrac{32M_b}{\pi d^3}$ Mpa =

Shear stress $\tau = \dfrac{16M_T}{\pi d^3} Mpa =$

Calculated values (Theoretical)

Principle stresses $\sigma_1 = \dfrac{\sigma_b}{2} + \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau^2}$

$$\sigma_2 = \frac{\sigma_b}{2} - \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

Principle strains $\varepsilon_1 = \frac{\sigma_1}{E} - \gamma\frac{\sigma_2}{E}$

$\varepsilon_2 = \frac{\sigma_{21}}{E} - \gamma\frac{\sigma_1}{E}$

Measured principle stress =

$$\sigma_{12} = \frac{E(\varepsilon_A + \varepsilon_c)}{2(1-\gamma)} \pm \frac{E}{2(1-\gamma)}\sqrt{(\varepsilon_A + \varepsilon_c)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_c)^2}$$

Compare experimental and theoretical value of $\sigma_{1,}\sigma_{2,}\varepsilon_{1,}\varepsilon_{2,}$

Graphical solution Mohr Circle

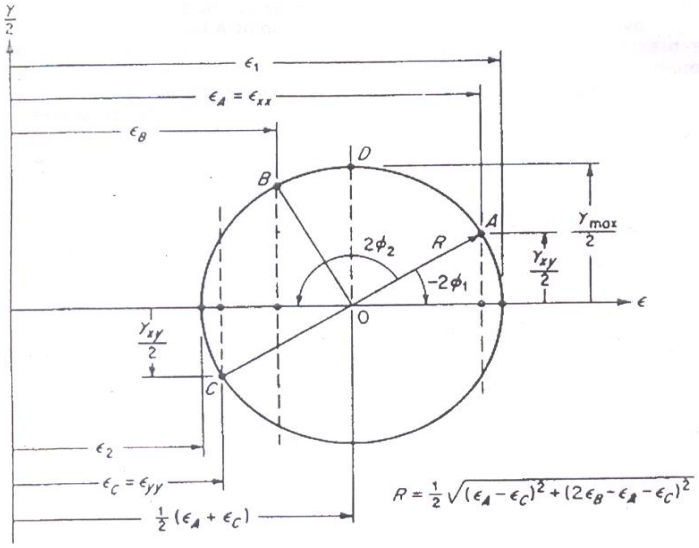


Fig.18.5 Graphical solution for the principal strain

It is also possible to determine these quantities with a graphical approach, as illustrated in fig 18.5. A Mohr's strain circle is initiated by laying out the Є (abscissa) and the 1/2 γ (ordinate) axes. The three strains Є. a, Є b and Є c are then plotted as points on the abscissa. Vertical lines are then drawn through these three points. The shearing strains γ xy is computed from eqn 18.6. and 1/2 γ xy is plotted positive downward or negative upward along the vertical line drawn through Є a to establish point A. This shearing strain may also be plotted as positive upward or negative downward along the vertical line through Є c to establish point C. The diameter of the circle is then determined by drawing a line between A and C which intersects the abscissa and defines the center of the circle at a distance 1/2(Є a + Є c) from the origin.

A circle is then drawn from this center passing through points A and C. The circle will intersect the vertical line drawn through Є b, and this point of intersection is labeled as B. A straight line through the center of the circle and point B should be a perpendicular bisector of the diameter

AC. The values of the principle strains ϵ_1 and ϵ_2 are given by the intersections of the circle with the abscissa. The principal angle $2\varphi_1$ is given by angle AO ϵ_1 and is negative if point A lies above the ϵ axis. The principal angle $2\Phi_2$ is given by the angle AO ϵ_2 and is positive if point A lies above the ϵ axis. The maximum shearing strain is established by a vertical line drawn through the center of the circle to give point D at the intersection. The projection of point D onto the $1/2\ \gamma$ axis determines the value of $1/2\ \gamma_{max.}$

The principal stresses can be determined directly from the principal strains. By employing eqn. 18.10.& 18.11.

Chapter 19.

Experiment - 13 **Curved beams**

Aim: Determination of stresses in Curved beam using strain gauge.

Objective

1. To experimentally observe the stress/strain distributions in a curved beam by using strain gauges and to compare it with analytically obtained values.

Background

Strain Gauges: The most common method of measuring mechanical strain in an object is with strain gauges. The basic principle underlying the operation of strain gauges is that the resistivity of wire changes when it undergoes mechanical strain. If the resistance element is attached directly to the object being strained it will undergo an equal strain and in this way, the measured change in resistance can be correlated to the strain in the object.

Curved beam theory

Introduction

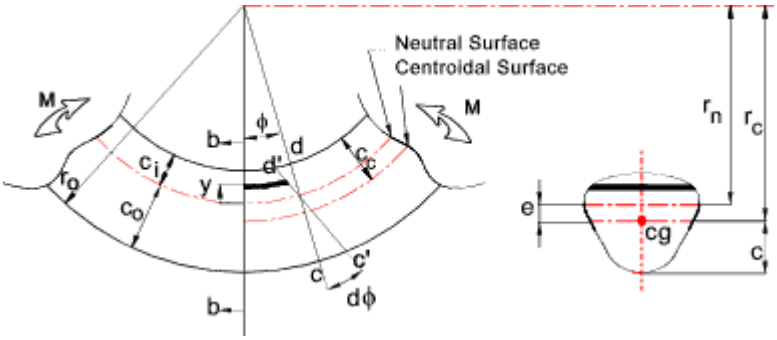
The beam theory can also be applied to curved beams allowing the stress to be determined for shapes including crane hooks and rings. When the dimensions of the cross section are small compared to the radius of curvature of the longitudinal axis the bending theory can be relatively accurate. When this is not the case even using the modified Bernoulli-Euler only provides approximate solutions

Symbols

ε = strain
e = eccentricity ($r_c - r_n$) (m)
c_c = Distance from centroid axis to inner surface. (m)
c_i = Distance from neutral axis to inner surface. (m)
c_o = Distance from neutral axis to outer surface. (m)
$d\varphi$= Surface rotation resulting from bending stress
σ = stress (N/m^2)
E = Young's Modulus = σ /e (N/m^2)
y = distance of surface from neutral surface (m).
r_n = Radius of neutral axis (m).
r_c = Radius of centroid (m).
r = Radius of axis under consideration (m).
I = Moment of Inertia (m^4 - more normally cm^4)
Z = section modulus = I/y_{max}(m^3 - more normally cm^3)

Theory

The sketch below shows a curved member subject to a bending moment M. The neutral axis r_n and the centroid r_c are not the same. This is the primary difference between a straight beam and a curved beam.



The strain at a radius r =

$$\varepsilon = (r - r_n)\frac{d\phi}{r\,\phi}$$

The strain is clearly 0 when r = at the neutral axis and is maximum when r = the outer radius of the beam ($r = r_o$)

Using the relationship of stress/strain = E the normal stress is simply.

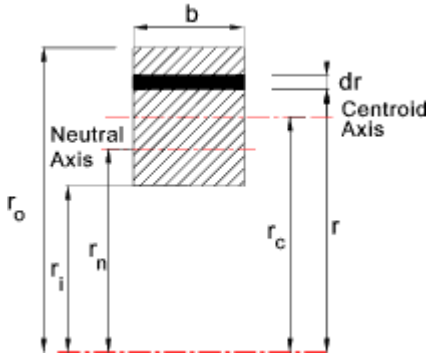
$$\sigma = E\varepsilon = E(r - r_n)\frac{d\phi}{r\,\phi}$$

The location of the neutral axis is obtained from summing the product of the normal stress and the area elements over the whole area to 0

$$\int_A \sigma\, dA = \frac{E\, d\,\phi}{\phi}\int_A \frac{r - r_n}{r}\, dA = 0 \quad \text{Reduces to} \quad A - r_n\int_A \frac{dA}{r} = 0$$

$$\text{Therefore} \quad r_n = \frac{A}{\int_A \left(\frac{dA}{r}\right)}$$

Neutral Axis for a Rectangular Section



$$r_n = \frac{A}{\int_A \left(\frac{dA}{r}\right)} = \frac{b\,(r_o - r_i)}{\int_{r_i}^{r_o} b\left(\frac{dr}{r}\right)} = \frac{(r_o - r_i)}{\ln (r_o/r_i)}$$

$$\text{eccentricity } e = (r_c - r_n) = \frac{(r_o - r_i)}{2} - \frac{(r_o - r_i)}{\ln (r_o/r_i)}$$

Curved Beam in Bending

The stress resulting from an applied bending moment is derived from the fact that the resisting moment is simple the integral over the whole section of the moment arm from the neutral axis (y) multiplied by σdA (= dF). Moment equilibrium is achieved if

$$M = \int_A (r - r_n)\,\sigma\, dA = \frac{E\,d\phi}{\phi}\int_A \frac{(r - r_n)^2 dA}{r} = \frac{E\,d\phi}{\phi}\int_A \frac{(r^2 - 2r\,r_n + r_n^2)\,dA}{r}$$

$$r_n = \frac{A}{\int_A \left(\frac{dA}{r}\right)}$$

$$= \frac{E\,d\phi}{\phi}\left(\int_A r\,dA - r_n A - r_n A + r_n^2 \int_A \frac{dA}{r}\right) \text{ reduces to } \frac{E\,d\phi}{\phi}\left(\int_A r\,dA - r_n A\right)$$

The centroid of a section = $r_c = \frac{1}{A}\int_A r.dA$

Therefore $M = E\frac{d\phi}{\phi} A\,e$ ..... $(e = r_c - r_n)$

$$M = \frac{r\,\sigma\,Ae}{(r - r_n)}$$

$$\sigma = \frac{M.(r - r_n)}{A\,e\,r} = \frac{-M.y}{A\,e\,(r_n - y)} \quad \ldots (y = r_n - r)$$

The maximum stress occurs at either the inner or out surface.

inner $\sigma_i = \frac{M.c_i}{A\,e\,r_i}$ outer $\sigma_o = \frac{M.c_o}{A\,e\,r_o}$

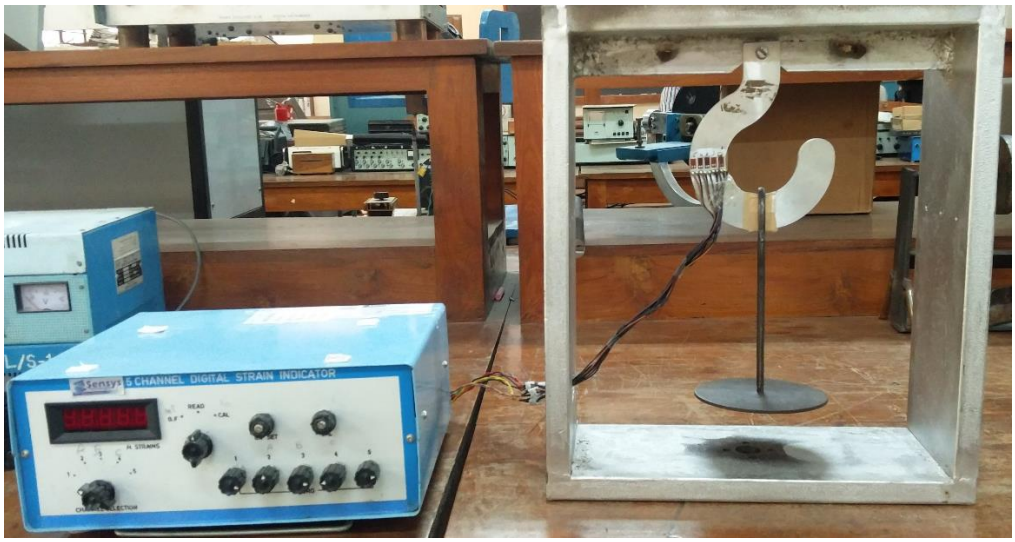
The curved beam flexure formula is in reasonable agreement for beams with a ratio of curvature to beam depth of r_c/h of > 5 (rectangular section). As the beam curvature/depth radius increases the difference between the maximum stress calculated by curved beam formula and the normal beam formula reduces. If the ratio is about 8 then a maximum stress error of only about 5% results from using the straight beam formulae.

Note:

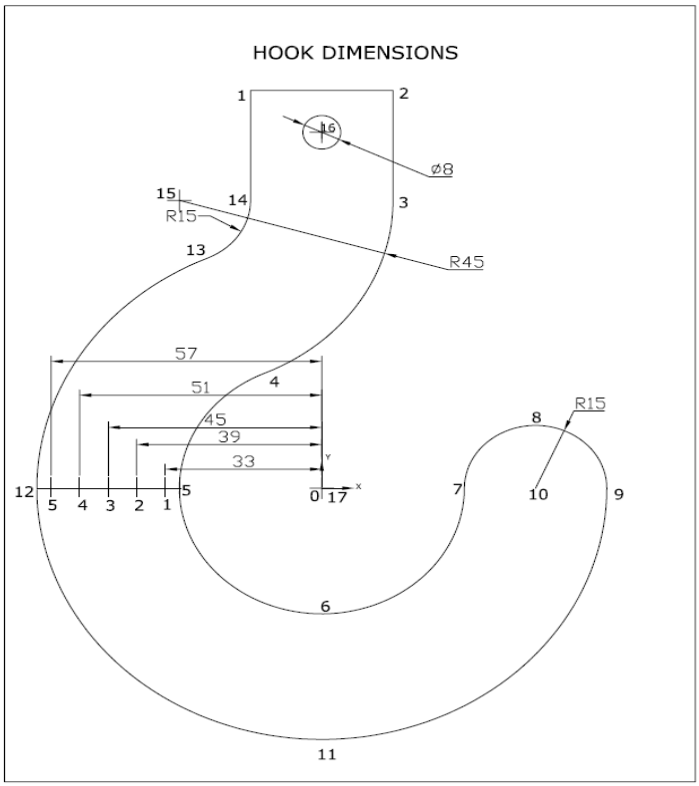
The above equations are valid for pure bending. The bending moment, in this case has to be taken about the centroidal axis, not the neutral axis and the additional tensile or compressive stresses have to be considered to obtain the resultant stresses on the section.

Apparatus

The apparatus used in this lab is as follows:
1. Curved beam specimen with strain gauges fixed to the specimen
2. Support frame
3. Loading apparatus
4. Weights
5. Digital strain indicator



Procedure



Part A: Strain Gauge Analysis

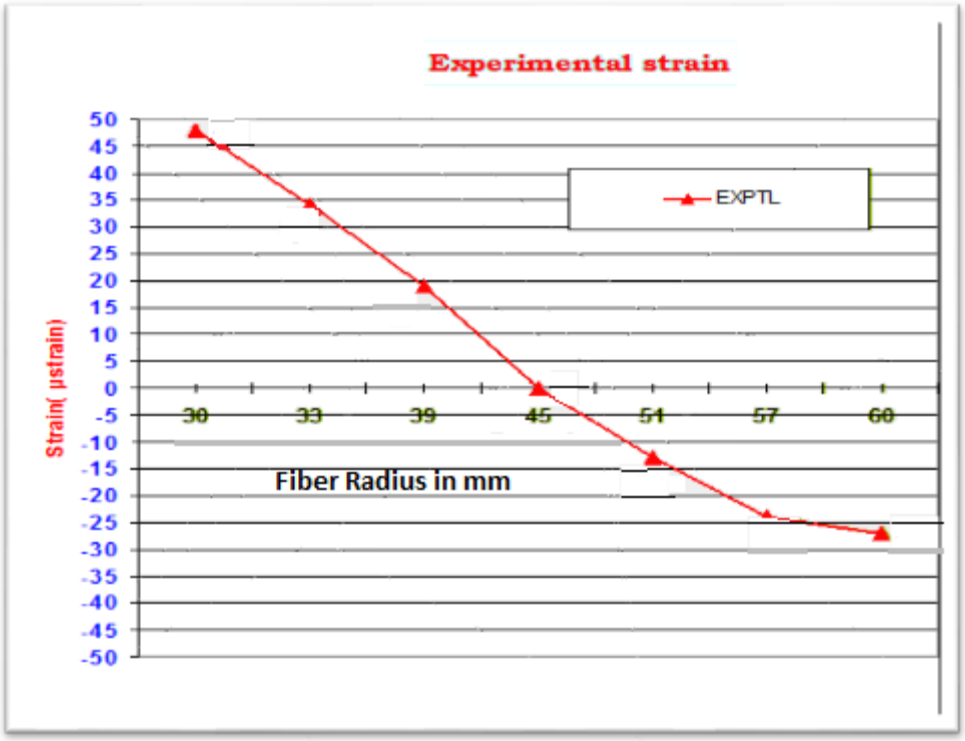
1. Make a sketch of the apparatus. Record all appropriate dimensions and identify the locations of the strain gauges as precisely as possible.
2. Check the resistance and bridge arm of circuit.
3. Ensure that the strain gauges are properly connected and zeroed under no load conditions.
4. For a load, record the strain readings.
5. Plot the strain distribution across the specimen and extrapolate it to inner and outer boundaries.
6. Compute corresponding stress values.
7. Compare experimental and theoretical values of stress and strains.
8. From the experimental results determine the location of the neutral axis

Observations

Material of curved beam: Aluminium
E= 69.97 GPa, ν=0.25
Inner Radius r_i= 30 mm
Outer radius= r_o = 60 mm
Thickness t= 6 mm
Width d=r_o-r_i =30 mm

Tabular Column

Sl.No	Weight		Strain gauge location and Readings				
	Weight W		@r=33 mm	@r=39 mm	@r=45 mm	@r=51 mm	@r=57 mm
Sl.No.	kgf	N	μ ε	μ ε	μ ε	μ ε	μ ε
1	5						
2	10						
3	15						



Graph of strain vs location

***(extrapolate the values to inner and outer fibers and note the values and enter in tabular column)

Specimen calculation:

EXPERIMENTAL

Resultant stress at any radius $\sigma_R = \varepsilon * E$

Theoretical Analysis of Stress And Strain on Curved Beam:

Model: crane hook,

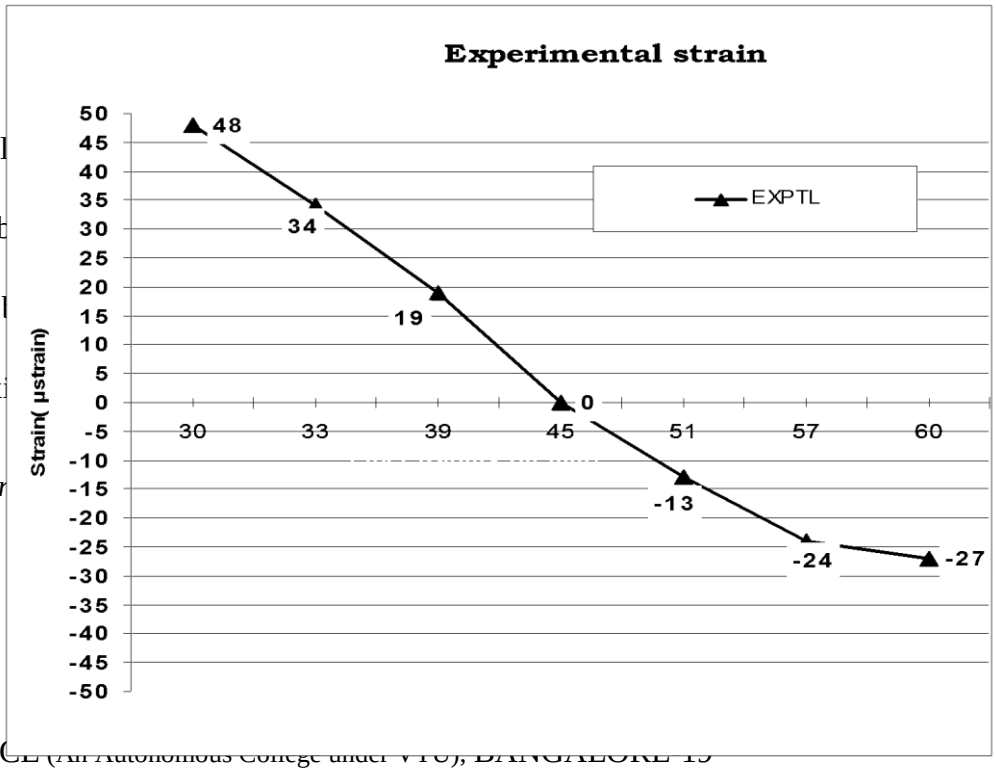
Material: Aluminum all

Inner radius of curved b

Outer radius of curved b

Radius of centroidal axi

Radius of neutral axis r



Eccentricity **(e)**= (distance of material axis to centroidal axis)

$= r_c - r_n =$ mm

Distance of neutral axis to inner radius =$\mathbf{C_i}$

$$C_i = r_n - r_i$$

$$= \quad \text{mm}$$

Distance of neutral axis to outer radius **(c_o):**

$$C_o = r_o - r_n$$

$$= \quad \text{mm}$$

Area of cross section **A =** h x b = mm²

Bending moment about centroidal axis

Mb = F x l ……N-mm.

Distance from centroidal axis to force:

L = r_c = mm

(1) For 5kg load:

$$M_b = F \times l$$

$$= \quad \text{N-mm.}$$

Combined stress at inner fiber

At radius = 30, $\sigma_{bi} = \frac{F}{A} + \frac{M_b C_i}{Aer_i} =$ N/mm^2

Combined stress at outer fiber:

At radius = mm , $\sigma_{bo} = \frac{F}{A} - \frac{M_b C_o}{Aer_o} =$ N/mm^2

Stresses at required fiber:

Combined stresses at radius 33, 39,45,51,57 mm.

$$\sigma_{33} = \frac{F}{A} - \frac{M_b(r_n - r)}{Aer} = N/mm^2$$

$$\sigma_{39} = \frac{F}{A} - \frac{M_b(r_n - r)}{Aer} = N/mm^2$$

$$\sigma_{45} = \frac{F}{A} - \frac{M_b(r_n - r)}{Aer} = N/mm^2$$

$$\sigma_{51} = \frac{F}{A} - \frac{M_b(r_n - r)}{Aer} = N/mm^2$$

$$\sigma_{57} = \frac{F}{A} - \frac{M_b(r_n - r)}{Aer} = N/mm^2$$

Tabulation of results

Sl. No	Load		Fiber radius mm	Experimental		Theoretical	
	Kg	N		Strain x10e-6	Stress $\sigma = E \times €$ (N/mm²)	Stress	Strain x10e-6
1							
2							
3							

The tensile stress at the inner surface is calculated at _____ N/mm^2 and the compressive stress at the outer surface is calculated at _____ N/mm^2...This section profile results in a tensile stress _____times greater than the compressive stress

Note

Part A: Strain Gauge Analysis

1. From the experimental results determine the location of the neutral axis for both beams.
2. Plot the strain distribution across the specimen. Is this linear? why or why not?
3. From the axial strain readings, determine the axial stress values and plot these with respect to their location on the component.

Include at least these plots in your results section:
Experimental stress *vs.* gauge location (straight beam)
Experimental stress *vs.* gauge location (curved beam)
Theoretical stress *vs.* gauge location (straight beam)
Theoretical stress *vs.* gauge location (curved beam)

Supplemental Questions

1. Do your experimental results for the strain distribution agree with the theoretical values? Why or why not?
2. Perform an uncertainty analysis on your theory and results. Note that for the theoretical component it is only necessary to perform the analysis on the straight beam as it is very tedious for the non-linear case.
3. Determine the loading that would produce failure of the components.

Chapter 20
Experiment – 14, GYROSCOPE
Aim: To demonstrate the principle of gyroscope
Apparatus used: gyroscope equipment, Dimmer stat, tachometer, stop watch, weights.

Theory: a gyroscope is an instrument used for stabilizing sea borne ships, gyroscopic compasses, control of aero planes and guided missile.

Principles of gyroscope:

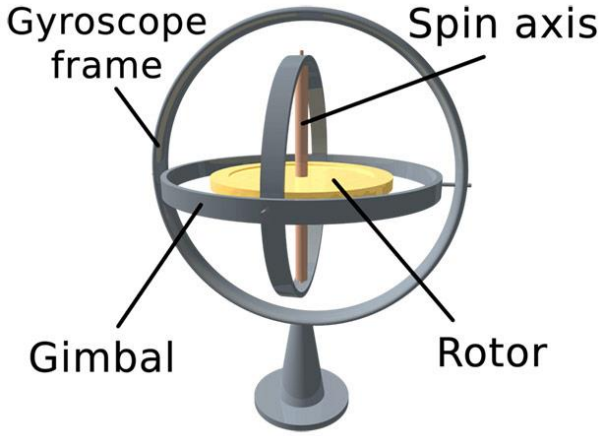


Fig.20.1: Principle of Gyroscope

The principle of gyroscope can be understood from the figure 20.1 & 20.2. a rotating disc supported on gimbal rings, rotates with an angular velocity called the velocity of spin denoted by 'ω' gimbal ring has a projected rod, which is supported on a frictionless surface e, the axis OY with an angular velocity denoted by 'ωρ' it is called velocity of precession. If the friction is considered zero, the axis of the rotating disc will revolve in the horizontal plane XOZ. But normally the energy is dissipated in friction. Which is always there and energy from the loss of potential energy of the disc,. Thus the rotating disc will gradually stop.

This is the case of natural precession, in case of vehicles, ships and other such devices, the spinning masses may be forced to process in the desired direction. The couples will be applied to the shaft carrying such spinning masses when the axis is forced to process in the given direction.
Gyroscopic couple:

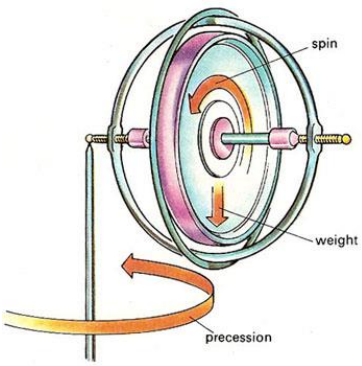


Fig. 20.2: Gyroscopic couple

The plane in which the disc is rotating is parallel to the plane YOZ and therefore, it is called plane of spin and the axis of the rotor is along OX as shown in the figure. Therefore OX is called axis of spin and the rotor is spinning with angular velocity 'ω' along the axis of spin OX. XOZ is the horizontal plane, and the axis of spin is rotating in the plane parallel to the horizontal about an axis OY or it is set to be processing about axis OY with an angular velocity 'ω_p ' thus XYZ is the plane of precession. As shown in figure, the rotor is rotating in clockwise direction. By applying the right handed screw rule, the vector of angular momentum is represented by vector oa since the axis o9f spin is also rotating anticlockwise from the top, about axis OY, let the new position occupied by the rotor be shown dotted in the figure, the angular momentum is now shown by dotted Ob. The change in angular momentum can result from the application of a couple to the rotor.

Thus from newton's II law,

Couple applied = rate of change of momentum

$= \delta(I\omega) / \delta t$

Where, I is the moment of inertia of the disc

$(I\omega)$ is the angular momentum

$\delta(I\omega)$ is the change is angular momentum

δt is the time interval in which the change takes place.

We know, ω is the angle in radians through by the axis of spin in time δt.

Therefore, $C - Oa(\delta\theta / \delta t)$

$= I\omega (\delta\theta / \delta t)$

For infinitesimal value of δθ

$C = I\omega (\delta\theta / \delta t)$

$= I\omega \omega_P$

Therefore, the vector representing Change in angular momentum ab lies in the lies in the XOZ plane or horizontal plane. For a very small displacement dq, ab will be perpendicular to the vertical plane XOY. Therefore, the couple causing this change in angular momentum will lie in XOY plane. By right-handed screw rule the sense ab that is the vector pointing away from the sight represents a clockwise couple in the plane XOY. Therefore OZ, an axis perpendicular to the plane XOY, about which the couple acts, is called the axis of couple and there will be equal and opposite reactions couple. Gyroscopic couple causes change in direction of the angular velocity keeping its magnitude constant.

It is therefore concluded that the change in direction of the axis of rotation is caused by a gyroscopic couple applied to it. Gyroscopic couple applied to it. Gyroscopic couple is usually applied through bearing s which supports the shaft. The bearings will resist equal and opposite couple. Thus the precession of spin is also accompanied by gyroscopic reaction couple, but in opposite sense. The axis of reaction couple can be represented by 'OZ'

Fig.20.3: Experimental setup for Gyroscope

Type 1 Free Precision:

Procedure:

Initial Observation

The spinning body exerts a torque or a couple in such a direction which tends to make the axis of spin coincides with that of the precession. To study the phenomenon of forced precession following procedure is adopted.

1. Balance the initial horizontal position of rotor.
2. Start the motor and adjust the voltage to get the constant speed.
3. Press the yoke frame about the vertical axis by applying the necessary force by hand in the clockwise direction viewed from the Top.
4. It will be observed that rotor frame swing about the horizontal axis so that the motor side moves upwards.
5. Rotating the yoke axis in the opposite direction caused the rotor frame to move in the opposite direction.

The spinning body processes in such a way that to make the axis of spin to coincide with that of the applied couple.

The direction is verified by following the procedure given below and using the apparatus as well as the relation for the magnitude of the couple.

1. Balance the rotor in the horizontal plane.
2. Start the motor and adjust the speed with the help of voltage regulation. The speed is measured using a tachometer.
3. Put weights on the side opposite to the motor.
4. The yoke start processing.
5. Note down the direction of precession.
6. Verify this direction.
7. Measure the velocity of precession using the pointer provided the yoke and stop watch.
8. Verify the relation given for gyroscopic couple, G.

Observation:

Moment arm distance L= mm

I= $mr^2/2$=

Tabular column:

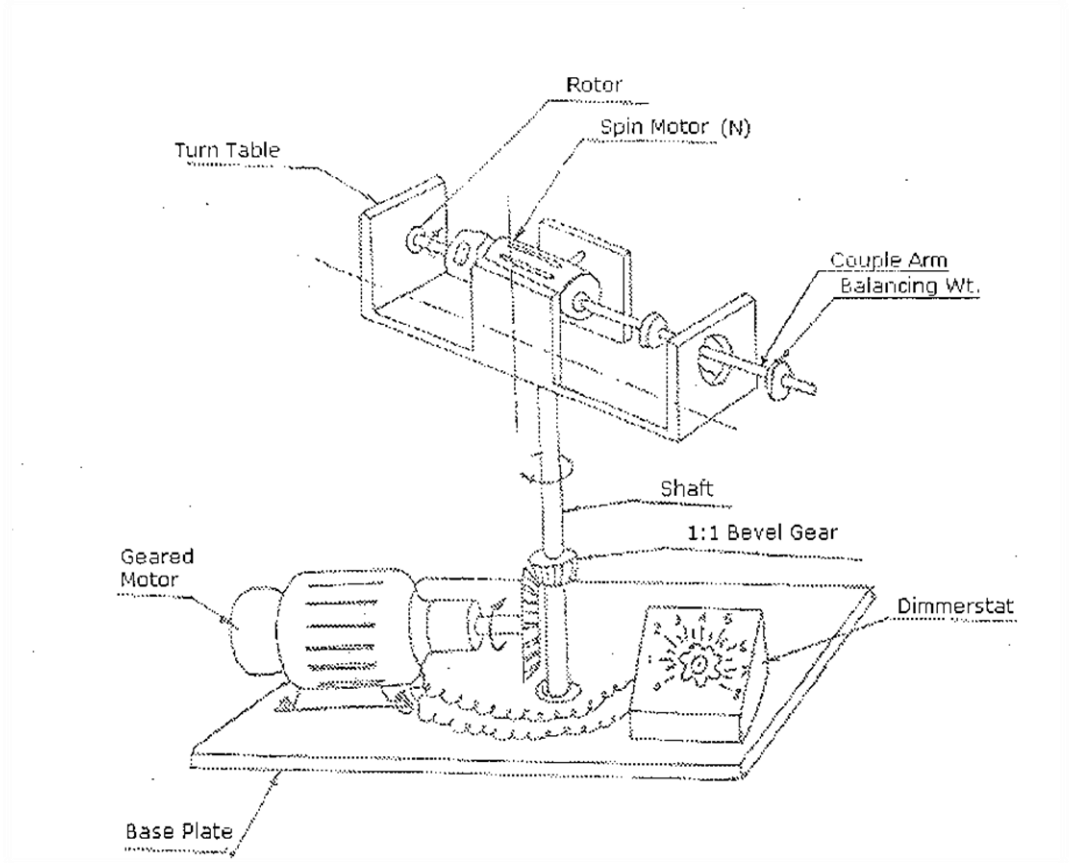
Sl No	Speed Of rotor in 'rpm'		Weight		Precision time 't' in sec (t_p)	Precision angle in degrees θ_p	Angular velocity of Precision $\omega_p = \frac{\theta_p}{t_p} * \frac{\pi}{180}$ rad/sec	Couple	
	N	ω	kg	N				C_{act}	C_{the}

Calculation:

C_{the}= I ω ω_p N-m

C_{act}= W*L

Type 2 Forced Precision



Procedure:

1. The Gyroscopic setup is as shown in the figure 20.3. One of the dimmer stats is used to drive the spin motor and other is used to drive the spin motor and the other is used to process (geared motor).
2. A known weight is added to the couple arm.
3. The power supply is switched on.
4. By varying the dimmer stat, the spin motor is set at a certain speed. It is allowed to attain speed (N).
5. Speed of the rotor is measured by using a tachometer.
6. By varying the dimmer stat connected to the geared motor (precession) the loading arm is brought to a horizontal position so that it balances the applied couple.
7. Time taken 10 revolutions of the turn table assembly is noted down with the help of stopwatch.
8. The values are tabulated as shown.

Observation:

Moment arm distance L= mm

I= $mr^2/2$=

Tabular column:

Sl No	Speed Of rotor in 'rpm'		Weight		Precision time 't' in sec (t_p)	Precision angle in degrees θ_p	Angular velocity of Precision $\omega_p = \frac{\theta_p}{t_p} * \frac{\pi}{180}$ rad/sec	Couple	
	N	ω	kg	N				C_{act}	C_{the}

Calculation:

$C_{the} = I\ \omega\ \omega_p$ N-m

$C_{act} = W*L$

Conclusion:

Chapter 20. INTRODUCTION TO BALANCING

The high speed of engines and other machines is a common phenomenon now a day. It is therefore, very essential that all the rotating and reciprocating parts should be completely balanced as for as possible. If these parts are not properly balanced, the dynamic forces are setup. These forces not only increase the loads on bearing and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Balancing and its classification: Balancing of machines has been receiving increasing attention for more than one reason. Mankind's quest for higher speeds for means of transport, inherent high speeds of rotation of a steam as well as gas turbines, rotary compressors, centrifugal pumps electrification can be cited as some of them.

To illustrate the above remarks, imagine a 50 KN rotor of a steam turbine running at 9000 rpm, out to the extent of 2 mm due to imperfect machining, inaccurate pitch of blades, non – homogeneity of materials etc. then the resultant centrifugal force will be equal to,

$$F = m\omega^2 r = \frac{50000*(2\pi*9000)^2*0.002}{9.81*60^2} = 905.7032 \text{ kN}$$

This force of 905.7032 KN not only produces a hammering action but also has a tendency to lift the turbine from its foundation, 9000 times a minute. It will be realized that no foundation will normally be able to withstand this. Hence to avoid the unpleasant effects, precise balancing is essential.

Causes of unbalance

Very few rotors do not require balancing. Where then does the unbalance come from?

Unbalance is always present if the mass distribution of the rotor relative to its shaft axis is not symmetrical (i.e. one of its central principal axis is not coincident with the shaft axis). It will be somewhat less in simple assembled components, e.g. wound electric motor armatures, and very small in turned parts. It may be made worse by adding rolling element bearings. Even in ground components rotating at high speed in plain bearings unbalance will be so significant that balancing becomes essential.

The main causes of unbalance may be divided into three groups

- Design flaw
- Material faults
- Machining or assembly errors.

Conditions for balancing

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action these centrifugal forces must be the same. In other words, the center of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing.***
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero The conditions (1) and (2) together give **dynamic balancing**.

Study of balancing is subdivided, for convenience, under the following headings

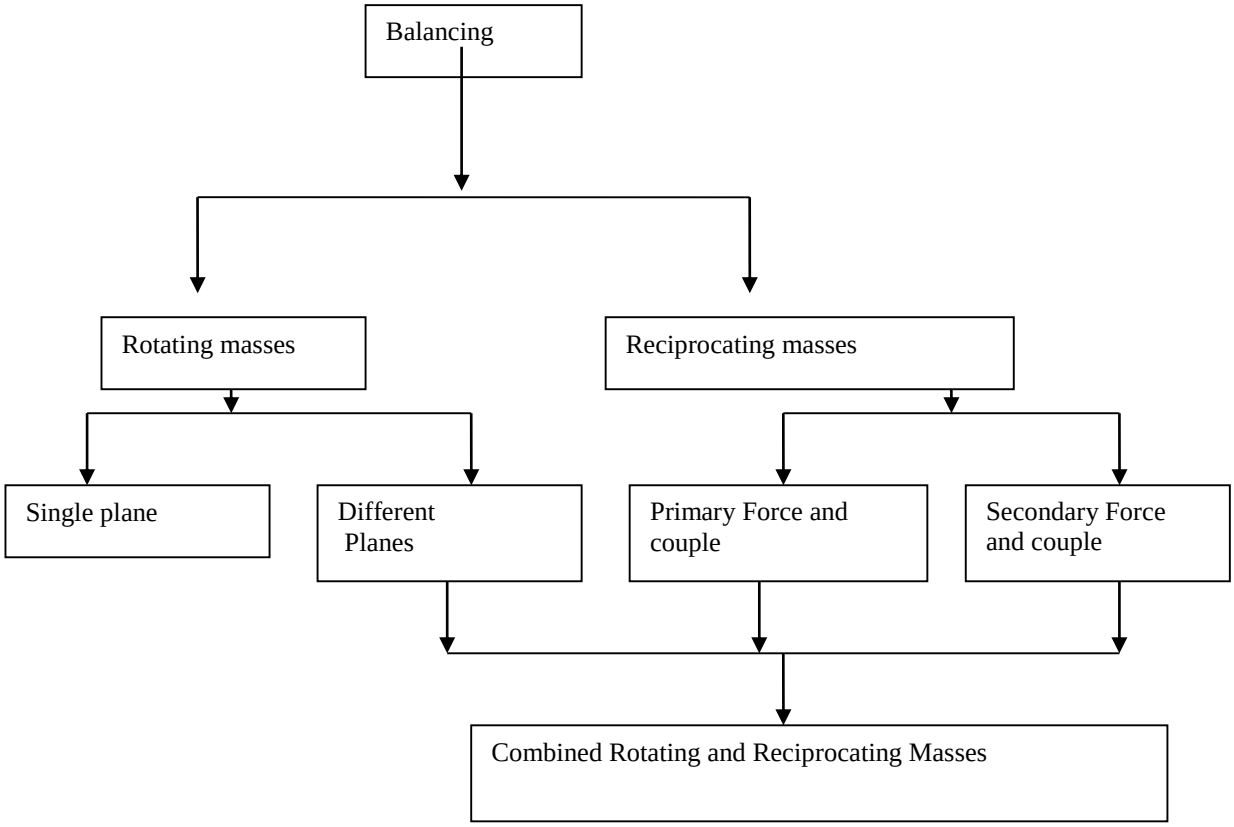


Fig. 20.1 Classification of balancing problems

Chapter 21. Balancing of Single Rotating Mass in Two Different Planes

Experiment 14

Aim: Balancing of Single Rotating Mass in Two Different Planes

Setup: Balancing machine as shown in figure fig. 21.1 along with different weights, tachometer and scale

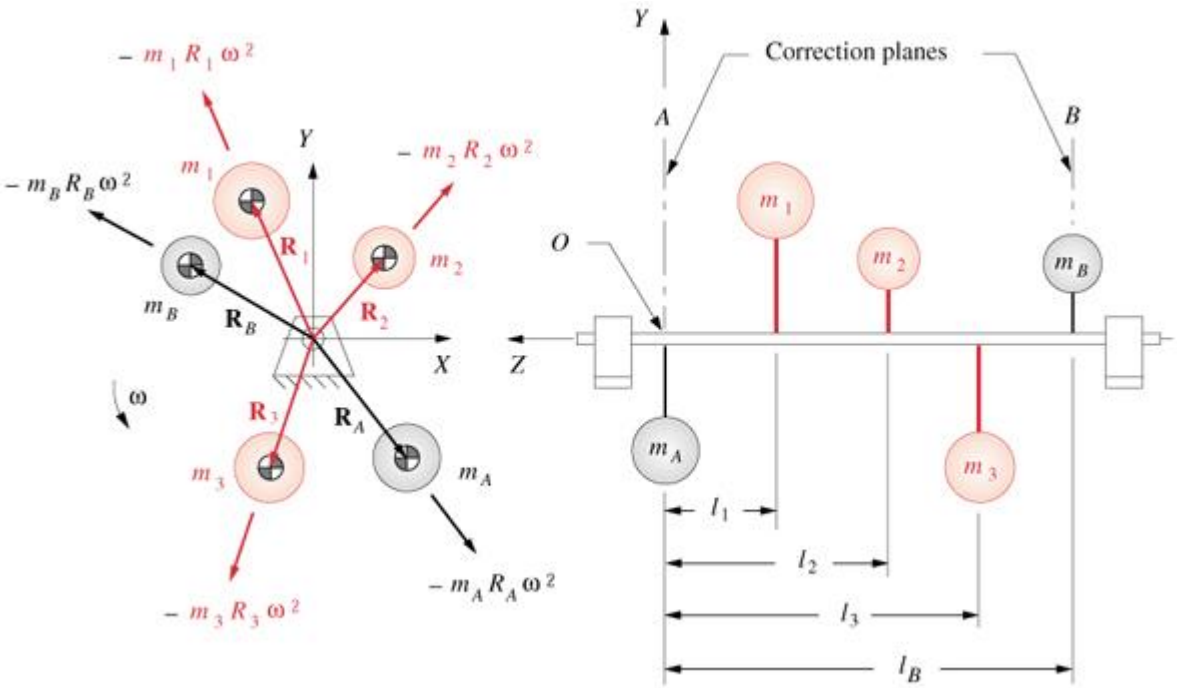


Fig: 21.1 A 5 plane-5 mass balancing machine

Theory:

Balancing of rotating masses

The following cases are considered here:

Type1: Balancing of a single rotating mass by two masses rotating in different planes.

Type2: Balancing different masses rotating in different planes.

Case A: When the plane of the disturbing mass lies in between the planes of the two balancing planes

Consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in figure 12.2 let r,r_1 and r_2 be the radii of rotation of the masses in planes A, Land M respectively

Let l_1= Distance between the planes A and L, l_2= Distance between the planes A and M and
l = Distance between the planes L and M.

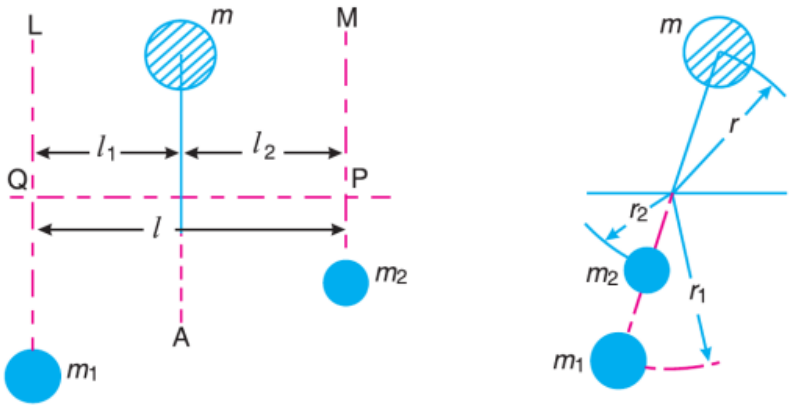


Fig. 21.2: Balancing of a single rotating mass by two rotating masses in different planes when the plane of single rotating masses lies in between the planes of two balancing masses.

We know that the centrifugal forces exerted by masses m in plane A,

$F_{c1} = m.w^2.r$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$F_{c1} = m_1.w^2.r_1$

And, the centrifugal force exerted by the mass m_2 in the plane M

$F_{c2} = m_2.w^2.r_2$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$F_c = F_{c1} + F_{c2}$ or $m.w^2.r = m_1.w^2.r_1 + m_2.w^2.r_2$

$m.r = m_1 r_1 + m_2.r_2$ ------21.1

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$F_{C1}.l = F_C.l_2$ or $m_1.\omega^2.r_1.l = m.\omega^2.r.l_2$

$m_1.r_1.l = m.r.l_2$ or $m_1.r_1 = m.r.\ (l2/l)$ ------21.2

Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$F_{C2}.l = F_C.l_1$ or $m_2.\omega^2.r_2.l = m.\omega^2.r.l_1$

$M_2.r_2.l = m.r.l_1$ or $m_2.r_2 = m.r.\ (l1/l)$ ------21.3

It may be noted that equation 21.1 represents the condition for static balance, but in order to achieve dynamic balance, equation 21.2 or 21.3 must also be satisfied.

Case B: When the plane of the disturbing mass lies on one end of the planes of the two balancing planes.

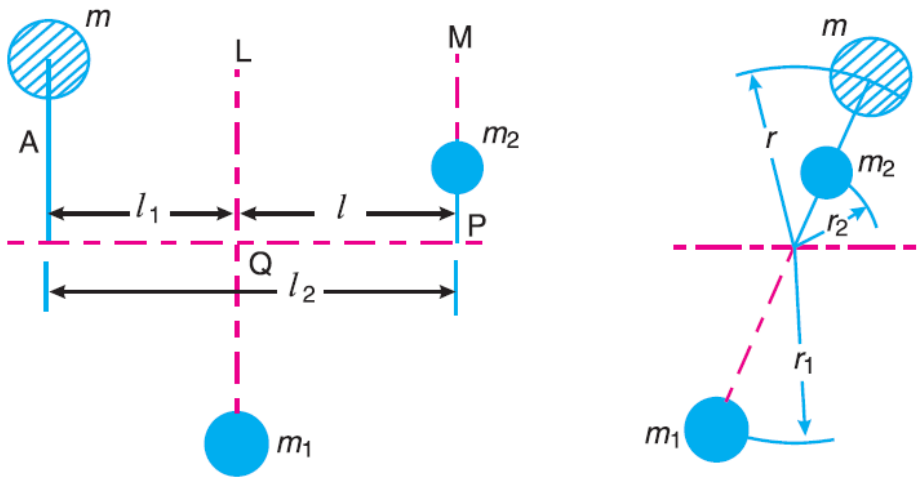


Fig. 21.3: Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of two balancing masses.

In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in figure 21.3 as discussed in case A, the following conditions must be satisfied in order to balance the system i.e.

$F_C + F_{C2} + F_{C1}$ or $m.\omega^2.r + m_2.\omega^2.r_1 = m_1.\omega^2.r_1$

$m.r + m_2.r_2 = m_1.r_1$ ------21.4

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$F_{C1}.L = F_C.l_2$ or $m_1.\omega^2.r_1.l = m.\omega^2.r.l_2$

$m_1.r_1.l = m.r.l_2$ or $m_1.r_1 = m.r.(l_2/l)$ ------21.5

Similarly, to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$F_{C2}.l = F_C.l_1$ or $m_2.\omega^2.r_2.l = m.\omega^2.r.l_1$

$M_2.r_2.l = m.r.l_1$ or $m_2.r_2 = m.r.(l_1/l)$ ------21.6

Procedure

1. Insert mass m at radius r in a plane and switch on the motor. The cradle oscillates violently because of imbalance. Switch it off
2. Insert masses m1 and m2 at radius r1 and r2 in l and m planes selected suitably, Switch on the motor. The oscillations will be minimum (Theoretically zero) indicating that the system is balanced.

Observation and Tabulation
Experiment (9a) and (9b)

Particulars	**Case A**: When the plane of the disturbing mass lies in between the planes of the two balancing planes ($l = l_1 + l_2$)	**Case B**: When the plane of the disturbing mass lies on one end of the planes of the two balancing planes ($l = l_1 \sim l_2$)
m in kg.		
r in mm		
l in mm		
l_1 in mm		
l_2 in mm		
$m_1 = mrl_2/r_1l$		
$m_2 = mrl_1/r_2l$		

Demonstrate how the cradle oscillates without balancing mass and with the introduction of balance mass the oscillation reduces.

Conclusion:

Chapter 22. Balancing of several masses rotating in different planes

Experiment 15:

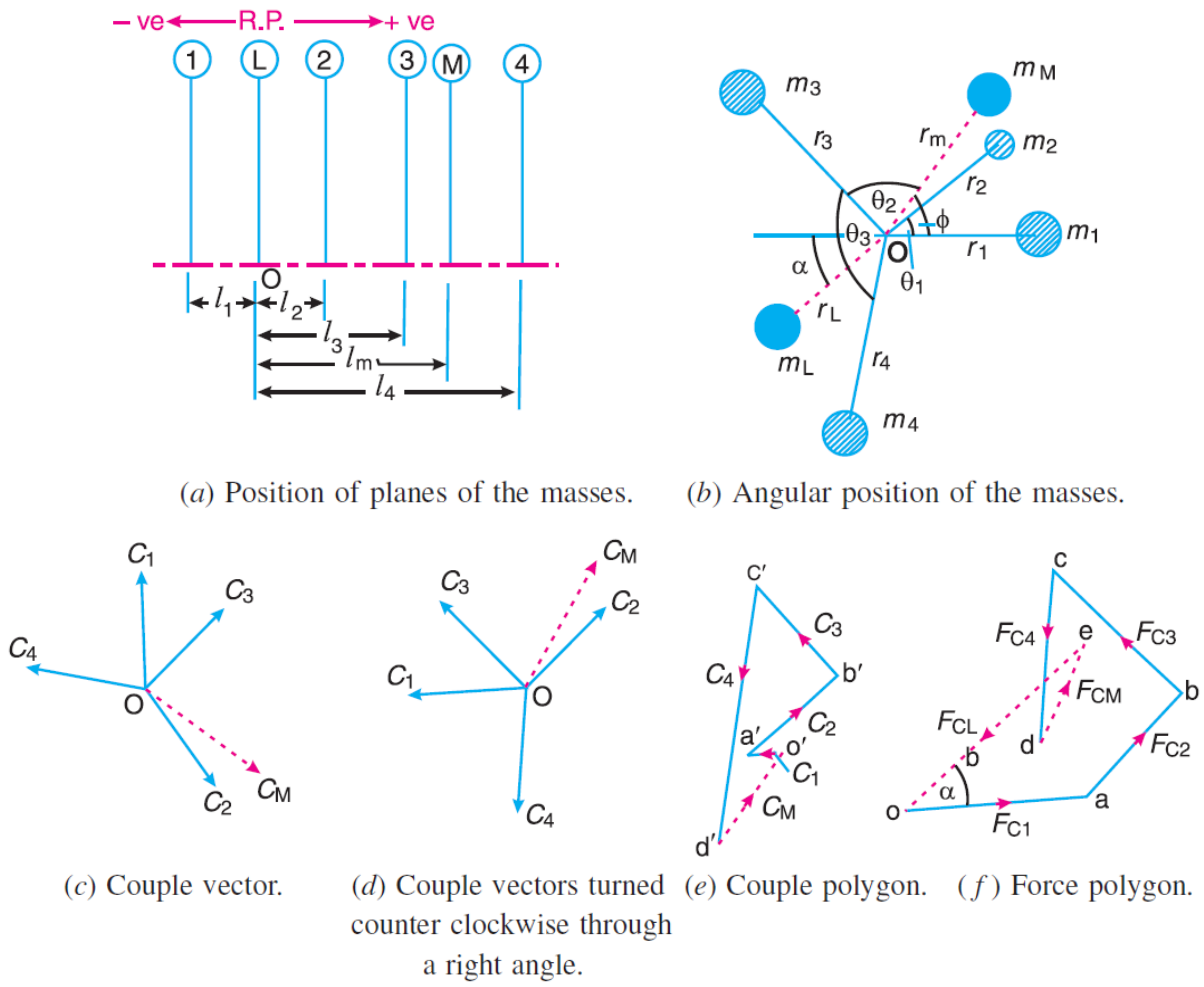
Aim: To determine the magnitude of balancing mass along with angular and axial location for the problem of Balancing of several masses rotating in different planes.

Setup: Balancing machine as shown in figure fig. 12.1 along with different weights, tachometer and scale

Theory:

When several masses revolve in different planes, they may be transferred to a reference plane, which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass in one plane to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to product of the force and the distance between the plane of rotation and reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied:

- The forces in the reference plane must balance, i.e. the resultant force must be zero



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.

(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig: 22.1 Configuration and couple and force polygon

Let us now consider several masses of weight W_1, W_2, W_3 and W_4 etc., revolving in planes 1, 2, 3 and 4 respectively as shown in fig. 22.1(a). The relative angular positions of these masses are shown in the end view fig. 22.1(b). The magnitude of the balancing weights W_L and W_M in planes L and M may be obtained as discussed below:

- Take one of the planes; say L as the reference plane (R.P). The distance of all the other planes to the left of the reference plane may be regarded as negative, as those to the right as positive.
- Tabulate the data as shown in table 22.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Tabular Column

PLANE	WEIGHT (W)	RADIUS (r)	CENTRIFUGAL FORCE ω^2/g (W.r)	DISTANCE FROM PLANE L (L)	COUPLE ω^2/g (W.R.L)
1	W_1	r_1	$W_1 r_1$	$-l_1$	$-W_1 r_1 l_1$
L(R.P)	W_L	r_L	$W_L r_L$	0	0
2	W_2	r_2	$W_2 r_2$	L_2	$W_2 r_2 L_2$
3	W_3	r_3	$W_3 r_3$	L_3	$W_3 r_3 L_3$
M	W_M	r_M	$W_M r_M$	L_M	$W_M r_M L_M$
4	W_4	r_4	$W_4 r_4$	l_4	$W_4 r_4 l_4$

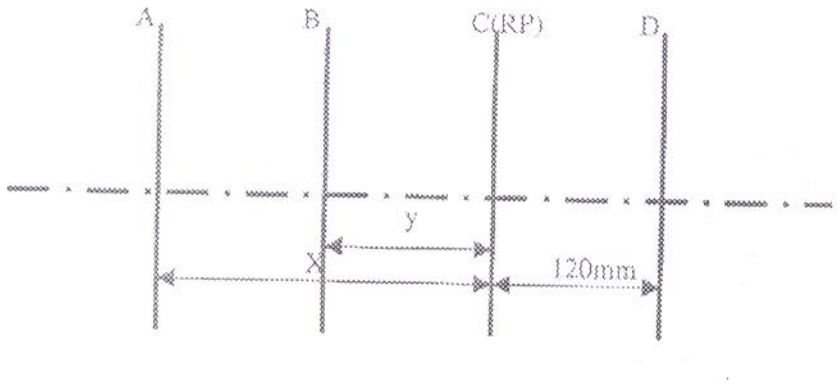
- A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring W_1 to the reference plane through OW_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to OW_1 as shown by OC_1 in fig. 22.1(c). Similarly the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to OW_2, OW_3 and WO_4 respectively and in the plane of the paper.
- The couple vectors as discussed above are turned counter clockwise through a right angle for convenience of drawing as shown 22.1(d). We see that their relative positions remain unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as OW_2, OW_3 and WO_4, while the vectors to OC_1 is parallel OW_2 but in opposite direction. Hence the couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.
- Now draw the couple polygon as shown in fig. 22.1(e). The vector 'do' represent the balanced couple. Since the balanced couple C_M is proportional to $W_M r_M l_M$ therefore
  $C_M = W_M r_M l_M$ = vector 'do'
  W_M = vector 'do'/ $r_M l_M$
  From this equation, the value of the balancing mass W_M in the plane M may be obtained, and the angle of inclination 'Φ' of this mass may be measured.
- Now draw the force polygon as shown in fig. 22.1(f). The vector 'eo' (in the direction from 'e' to'o') represents the balanced force. Since the balanced force is proportional to $W_L r_L$, therefore
  $W_L r_L$ = vector 'eo'

$\therefore W_L$ = vector 'eo'/ r_L

From this expression the value of the balancing mass W_L in the plane L may be obtained, and the angle of inclination 'α' of this mass with the horizontal may be measured.

Problem

With reference to the figure shown and data in the tabular column determine the values of x and y and the inclinations of plane A and B mass D makes and angle of 30° with C



Tabulation:

Plane	MASS (m) (g)	Radius (r) (mm)	Force Mr (g-mm)	Distance from ref plane'I' (mm)	Couple Mrl (g-mm^2)
A	206	50			
B	206	35			
C(R.P)	206	45			
D	206	40			

Note: determination of ' mr ' values:

For the current dynamic blanching machine, a combination of 8 split masses was made with different "r' values i.e. the distance between the central axis of the shaft and the balancing weights. The weight of each split mass is 156g, the balance weights will be added to this weight and the total weight will be multiplied with the radius of that respective mass to give the 'mr' values. Hence the split masses are to be selected according to the given 'mr' values,

'mr' value = (balancing weight + split mass weight) X radius

Steps:

1. Complete the tabular column.
2. Draw the force polygon and determine θ
3. Draw couple polygon selecting C as a reference plane. From this determine X and Y.
4. Arrange the balancing machine with the masses at respective radius, planes and angular position as determined and run the machine, if the system is balanced correctly then the oscillations of the cradle in the setup should be nil. (Since the values are found from graphical technique which is an approximate one mild oscillations may exists.

Force polygon:

The procedure to draw the force polygon for the given problem is as given below.

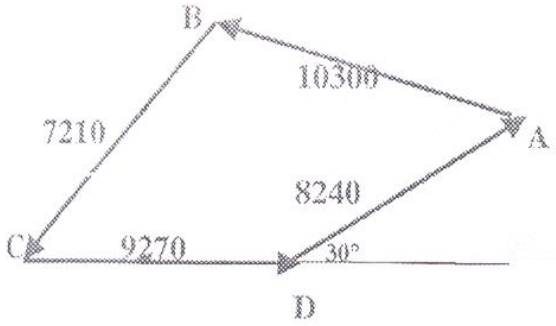
Select a suitable scale to the 'mr' value,

Since C is the reference plane, draw a horizontal line equal to the 'mr' value of ___c.

Angle between C and D is 30°, therefore from the end of the previous line draw a line at an angle 30° whose length is equal to the 'mr' value of d.

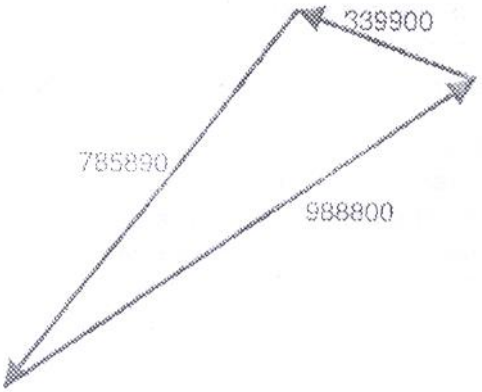
Now the angle of planes A and B is known. Only 'mr' value is known, therefore from the end of line D draw an arc equal to the 'mr' value of A, draw one more arc to cut the previous arc from the starting point of C with radius to the 'mr' value of B,

5. join these lines to get the force polygon as shown in the fig and determine θ and θ



Couple polygon:

- Select a suitable scale to the 'mr' value.
- Draw a line at an angle 30° equal to the 'mr' value of D
- From the end of this line, a line parallel to DA of the force polygon is draw,
- From the starting point of the first line, a line parallel to AB of the force polygon is drawn to intersect the previous line,
- Join these lines to get the couple polygon as shown is the fig.



Calculations:

We have "mr' value of A= -10300x (minus sign. Because mass A is to the left side of the ref plane C)

From the couple polygon we have A= 340000

$$\text{i.e. } 340000 = -10300x$$

$$x = -(34000/10300)$$

$\therefore x$ =33mm to the left of C

We have 'mrl' value of B = -7210y (minus sign, because mass B also is to the left side of the ref. plane) from the couple polygon we have B = 784000

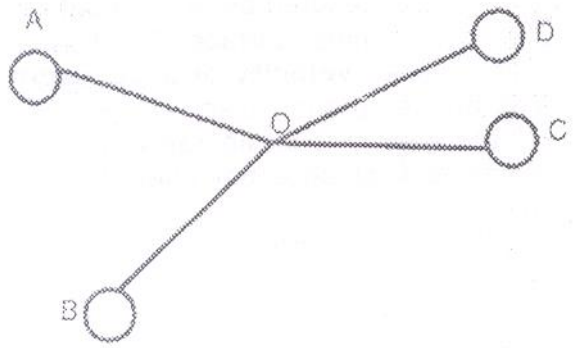
$$\text{i.e. } 784000 = -7210y$$

$$y = -(340000/10300)$$

$\therefore$ y = 109 mm to the left of C

Position of masses:

The angular positions of the balancing masses are obtained from the force polygon and the axial distance the planes are obtained from the above calculations. The angular positions and the distances between the planes for the given problem is as shown below.



Demonstrate how the cradle oscillates without balancing mass and with the introduction of balance mass the oscillation reduces.

Note:

Work out some more problems of balancing of rotating masses in different planes

Chapter 23.
Experiment 16. WHIRLING OF SHAFT

Aim: To determine experimentally and theoretically the critical speed of the given shaft with the given end conditions

The fundamental objectives are:

1) Observe the whirling phenomenon
2) Measure the natural frequency of steel shaft
3) Compare the measured natural frequency to that obtained theoretically
4) Discuss the sources of error.

APPARATUS REQUIRED

Whirling of shafts apparatus, Motor speed control unit, Shafts, Tachometer and Ruler.

Theory

Concept of whirling:

Machine components at a standstill may behave very differently when they are moving, even at relatively low speeds. A solid shaft able to support a hundred times its own weight plus the weight of the components mounted on it may, when rotating at certain speeds, bend and vibrate. The speeds are called 'critical speeds and the bending and the vibration is known as 'whirling'. If this 'critical speed of whirling' is maintained then the resulting amplitude becomes sufficient to cause buckling and failure. However if the speed is rapidly increased before such deleterious effects occur then the shaft is seen to re-stabilize and run true again until at another specific speed a double bow is produced.

Whirling is usually associated with fast-rotating shafts. When a shaft rotates it is subjected to radial or centrifugal forces, which cause the shaft to deflect from its rest position. These centrifugal forces are unavoidable, since material in homogeneities and assembly difficulties ensure that the center of gravity of the shaft or its attached masses cannot coincide with the axis of rotation. Dunkerley first investigated the centrifugal forces involved and determined that the only re-stabilizing or restoring force was that due to the elastic properties or stiffness of the shaft. Hence, he was able to deduce the speed at which the shaft would suffer an infinite deflection due to whirling.

When the speed of rotation is increased the centrifugal force also increases and so does the restoring force. Below the critical speeds, the restoring forces increase with increasing shaft deflection faster than the centrifugal forces, so the deflection is held in check. At the critical speeds, the restoring forces increase at the same rate as the unbalance forces, so they cancel each other out. Shaft deflection is unchecked and the shaft behaves as though it is very flexible. Above the critical speeds the unbalance forces hold sway, and the shaft rotates about the center of mass of the assembly (which is very close to the center of the shaft).

Shaft carrying a mass with eccentric C.O.G

If we examine the simplest case of a single, heavy rotor rigidly attached to a light (inertia-less) spindle, then the physical situation can be expressed in Fig. 23.1.

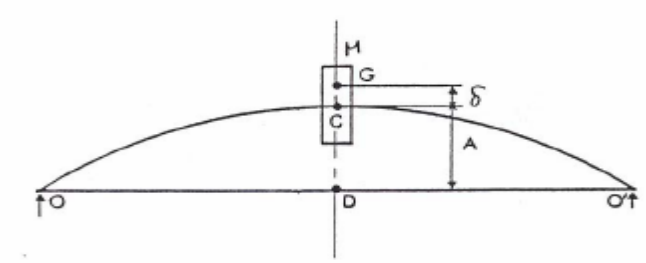


Fig. 23.1. Whirling of shaft due to unbalance

The system consists of a disc of mass M located on a shaft simply supported by two bearings. The center of gravity G of the disc is at a radial distance δ from the geometric center, C. The centerline of the bearings OO' intersect the plane of the disc at D, at which point the disc center C is deflected a distance A.

The center of gravity G thus revolves around point D, describing a circle radius (A+δ) and the centrifugal reaction thus produced is: $M\omega^2(A+\delta)$ for any given speed ω. This force, according to Dunkerley, is balanced by the elastic restoring force of the shaft at point D equal to KA where K is the stiffness. Therefore, we have

$M\omega^2 (A+\delta)= KA$

Then,

$$A = \frac{M\omega^2\delta}{K - M\omega^2} \qquad \ldots(23.1)$$

This equation will become infinite when $K-M\omega^2 = 0$ or $\omega^2 = \frac{K}{M}$. Therefore, if we denote the critical whirling speed, by $\omega_c = \frac{K}{M}^{0.5}$, and substituting that in equation (1), we obtain:

$$A = \frac{\omega^2}{\omega_c^{\,2} - \omega^2}\delta \qquad \ldots 23.2$$

Therefore, at $\omega<\omega_c$, then A and δ have the same sign i.e. the center of gravity G is situated as shown in Fig. At $\omega = \omega_c$, the deflection of A becomes infinite as described above. At $\omega > \omega_c$, A and δ are of opposite signs and hence the center of gravity now lies between C and D, inferring

that the disc has rotated through 180° from its rest position. For very high speeds where $\omega >> \omega_c$, the amplitudes A tend to −δ, hence the disc rotates about G with perfect stability.

If equation 14.2 is compared with the equation of motion for a single load W, undergoing a simple harmonic vibration, it may be noted that similarity exists. A full analysis of the problem demonstrates that at the whirling speed, A, the radius of the shaft rotation about the bearing center line, and δ, the radius of G from the geometric center of the disc, are perpendicular which is analogous to the resonant conditions which exist for a forced vibration where the disturbing force vector is 90 degrees in advance of the displacement vector.

Dunkerley deduced that the whirling speeds were equal to the natural frequencies of transverse vibration, there being the same number of whirling speeds as natural frequencies for a given system. Thus a theoretical value for the critical speed may be obtained from the formula for the fundamental frequency of transverse vibrations:

$$f = \left(\frac{EIg}{WL^4} \right)^{0.5} C \qquad ...(23.3)$$

where f = natural frequency of transverse vibration (Hz)
E = Young's Modulus
I = second moment of area of shaft
W = weight per unit length of shaft
g = acceleration due to gravity
C = constant dependent upon the end conditions

Note: W is weight per unit length, not mass per unit length

The value of C is that resultant from beam theory and for various end conditions; the values are shown in Table 1.

Case	Ends	C1	C2
1	Free-free	1.572	6.3
2	Fixed-fixed	3.75	8.82
3	Cantilever	0.56	-
4	Fixed-free	2.459	7.96

The value C_1 is the constant for use in calculating the first natural frequency and C_2 is that necessary for the second mode.

Description of the apparatus

The apparatus used to analyze the whirling effect is the whirling of shafts apparatus shown in Fig.23. 2.

Fig. 23.2. Whirling of shaft apparatus

The shaft I is of the form shown in schematic Fig. 23.3 and is located in chuck F and phosphor bronze retainer N. The diameter of the shaft used in this experiment is 3.310 mm (approx.) and the length, l, of the specimen is 0.9144 m. The material is AISI 4130 steel with E = 200 GPa and $\rho = 7850\ kg/m^3$.

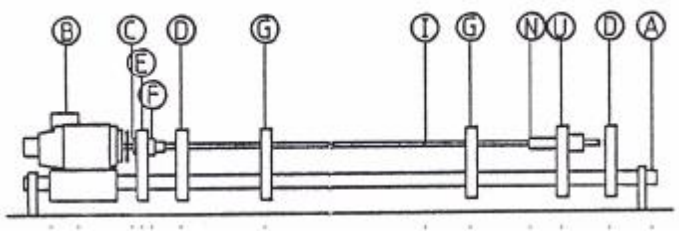


Figure 23.3. Diagrammatic representation.

The support chucks F and N have been so designed as to allow the shaft movement in a longitudinal direction, for the purpose of location before tightening, and also provide directional clamping of the shaft end. With the standard apparatus, chuck N provides directional fixing to the end of the shaft, although an interchangeable sliding chuck T is available which provides a directionally free support.

A movable support E is provided with chuck F which, when moved to the right from the position shown in the diagram, provides the motor end support with directional freedom identical to that of chuck T. Thus, by selection of the required supports, any combination of fixed or free end conditions may be selected.

The shaft is driven by a fractional horsepower, 6000-rpm, direct current motor, B, via the kinematic coupling C, shown in Fig. 23.1. Motor speed is controlled by a motor speed control unit.

Because of the possibility of excessive amplitudes and possible shaft failure, guards G are provided and are adjustable along the length of the apparatus. Each guard contains bushes, which are designed to limit amplitudes whilst not damaging the whirling shaft. Support U may be moved to enable various shaft lengths to be accommodated. A transparent guard enclosing the full length of the shaft is incorporated in the machine.

Two unique features are incorporated, which allow the shaft to adopt its actual whirling configuration predicted by elastic theory. The first is a kinematic coupling located at the driven end of the shaft, which is designed to prevent the transmission of any restraining forces by the motor to the shaft. The second feature is a sliding bushed end, which affords sliding motion of the shaft on a longitudinal phosphor bronze bearing, whilst revolving in a radial ball bearing. The apparatus thus allows an accurate analysis of the critical whirling speeds for a range of shaft geometry, both loaded and unloaded, and with various combinations of end conditions.

An aluminum disc is attached to the shaft. Some markers were on the disc. The stationary images of these markers can be used to determine the rotating speed using the Tachometer.

Experimental procedure

Whirling of an unloaded shaft:

In this lab, only one boundary situation (rigidly fixed at both ends) will be tested for the specimen described previously.

1) Measure the dimension of the specimen using ruler and caliper.

2) Mount the shaft on to the machine by tightening it in the chuck F by means of the setscrew provided with the chuck, with the shaft running through the guides, G, positioned evenly along its length. The adjustable support, U, containing retainer N may then be brought up to locate the threaded portion of the test shaft in the central hole of the retainer. Once located, the shaft may be retained by a locknut, which runs on the threaded portion of the shaft. Both supports, D, should be slid into position.

   At this point, it is thus crucial to ensure that the setscrew is tightened and that the guides and supports are rigidly fixed to the main frame, by tightening the hand wheels located beneath each. Most shaft failures are produced because of inadequate support, which results from insufficient tightening up of the apparatus prior to testing

3) Switch on the speed control and rotate the control knob slowly in a clockwise direction until the first natural frequency is reached, which is indicated by the formation of a single bow as shown in Fig.23.4. When the speed is increased further the shaft begins to vibrate violently as it nears the critical speed.

NEVER KEEP THE SHAFT ROTATING AT ITS CRITICAL SPEED. THE SPEED SHOULD ONLY BE APPROACHED OR PASSED QUICKLY; OTHERWISE, THE SPECIMEN MAY BE DAMAGED.

Once the critical speed is passed the shaft restabilizes and on further increase of the speed the second natural frequency is reached which is indicated by the formation of a double bow as shown in Fig.23.5

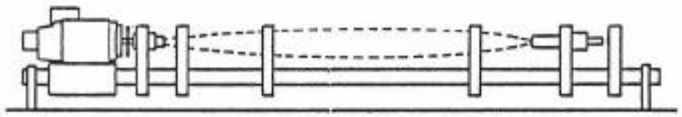


Fig. 23.4 First mode – Single bow

Fig. 23.5 Second mode – Double bow

4) Measure the speeds of rotation of the shaft at its first and second natural frequencies directly with the Tachometer.

5) Measure the speeds three times and use the average value for the calculations.

Results:

Calculate the theoretical frequencies using equation 23.3 and compare it with the measured natural frequencies of the shaft. Calculate the percentage of error between the theoretical and measured natural frequencies and also discuss the reasons for the deviation.

VIVA Questions

Vibration

1. Define vibration. What are the components of a vibration system?
2. What type of stress is induced because of vibration?
3. What is the requirement for a system to vibrate?
4. Classify the different types of vibrations? Explain with examples.
5. What is damping? Explain the different types of damping.
6. Define the followings:
   Cycle, Time period, damping factor, damping co-efficient, DOF, Stiffness, Logarithmic decrement, Resonance, Transmissibility, Whirling speed
7. What do you mean by natural frequency, forced frequency, resonant frequency?
8. What is the importance of critical damping? Give one example for critically damped system.
9. Give any two practical applications of damping.
10. Define base excitation, vibration isolation, and transmissibility.
11. What are the materials used to isolate vibrations in machineries?
12. What are the vibration measuring instruments?
13. Plot a graph of amplitude v/s time for natural vibration, damped vibration and define logarithmic decrement.
14. Write a differential equation for spring-damper-mass system.
15. Define critical speed.
16. What is the effect diameter of the shaft on critical speed?
17. Define node, mode and anti-node.
18. How do you eliminate critical speeds?
19. Give the formulae for finding maximum deflection of shaft with different end conditions.

Balancing

20. What is the need for balancing?
21. Differentiate between static balancing and dynamic balancing.
22. What is the unit of an unbalanced force and unbalanced couple?
23. How is balancing achieved practically?
24. How do you avoid critical speeds?
25. Why cast iron is used for machine tool beds?

Journal Bearing

26. Classify different types of bearings, with their applications.
27. What do you mean by anti-friction and sliding contact bearings? Give examples.
28. If the pressure distribution is negative what is your action to correct the same.
29. Why do you think the pressure is different at different points in journal bearing?
30. Write a co-efficient of friction formula and explain the terms.
31. Define viscosity. What are the methods used to determine viscosity?
32. What are the standard grades of oil available?
33. What does SAE stand for?

34. What is journal bearing?
35. What is viscous friction?
36. What is hydrodynamic and hydrostatic lubrication?
37. What are the material used for Slider contact Bearings and Ball & Roller contact bearings
38. Define the following with respect to journal bearing
    a. clearance
    b. eccentricity
    c. attitude

Gyroscope

39. Explain the gyroscopic principle.
40. Write an expression for gyroscopic couple and explain each term.
41. What are the applications of gyroscope?

Governors

42. Differentiate between a governor and a flywheel.
43. What are the types of governors? Classify them.
44. Define the following terms:

Lift, Height of a governor, Sensitivity, Range of speed, Effort, Power, Hunting, Isochronism, Stability,

45. What are the applications of governor?
46. When do you consider the friction value to be positive or negative?
47. Explain controlling force diagram. What is the importance of the same?