For the Course Code: 23MA1BSCEM
Course: Mathematical Foundation for Civil, Electrical and Mechanical Eng. Stream - 1

ORDINARY DIFFERENTIAL EQUATIONS OF HIGHER ORDER

Linear differential equations are those in which the dependent variable and its derivatives occur only in the first degree and are not multiplied together.

The general n[th] order linear differential equation is of the form

$$p_0 \frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + p_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + p_n y = X \qquad \text{..........(1)}$$

where $p_0, p_1, p_2, \ldots\ldots p_n$ and X are all functions of x only.

In the above differential equation if

i) $X = 0$ Then it is called a homogeneous linear differential equation.
ii) $X \neq 0$ Then it is called a non-homogeneous linear differential equation.
iii) $p_0, p_1, p_2, \ldots \ldots . . p_n$ are constants then it is called a linear ordinary differential equation with constant coefficients.
iv) If $y_0, y_1, \ldots \ldots . . , y_n$ are n independent solutions then its linear combination $y = c_1 y_1, + c_2 y_2 + \cdots + c_n y_n$ is also a solution.

Linear ordinary differential equations with constant coefficients

$$p_0 \frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} p_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots \; p_n y = X$$

Where $p_0, p_1, p_2, \ldots \ldots . . p_n$ are all constants.

Note:

I) Solution: A relation between *x* and *y* which satisfies the given differential equation.

II) General solution: A solution which contains arbitrary constants.
iii) $p_0, p_1, p_2, \ldots \ldots . . p_n$ are functions of x then it is called a linear ordinary differential equation with variable coefficients.
iv) If $y = v$ be any particular solution and $y = u$ be the solution of the homogeneous part of the differential equation then $y = u + v$ is called as the **complete solution**.
The part u is called the **complementary function** (**C.F.**) and the part v is called the **particular integral** (**P.I.**). Thus the complete solution of the above differential equation is $y = \text{C.F} + \text{P.I}$. **Particular integral** (**P.I.**) is a function which satisfies the ODE given by equation **(1)**.

RULES FOR FINDING THE COMPLEMENTARY FUNCTION

Let us consider n[th] order homogeneous linear differential equation with constant coefficients

$$\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1} y}{dx^{n-1}} + k_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + k_n y = 0 \qquad \text{................ (2)}$$

The equation (2) in symbolic form is $(D^n + k_1 D^{n-1} + k_2 D^{n-2} \dots + k_n)y = 0$ ……………. (3)

Further which can be written as $f(D)y = X$

where $f(D) = D^n + k_1 D^{n-1} + k_2 D^{n-2} \dots + k_n$ …….. (4)

Observation:

Auxiliary equation (A.E.): $F(m) = 0$. (Obtained by assuming e^{mx} as a solution of $f(D)y = 0$.

Therefore the auxiliary equation of (2) is $m^n + k_1 m^{n-1} + k_2 m^{n-2} \dots + k_n = 0$ …………….. (5)

Let $m_1, m_2 \dots \dots m_n$ be the roots of equation (5).

Case I: A*ll the roots be real & distinct*

The complete solution of (3) is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots\dots\dots\dots + c_n e^{m_n x}$$

Case II: *The roots are real and repeated* (*equal*)

Suppose if two roots are equal (i.e., $m_1=m_2=m$) then the complete solution of (3) is

$$y = (c_1 + c_2 x)e^{mx} + c_3 e^{m_3 x} + \dots\dots\dots\dots + c_n e^{m_n x}.$$

Similarly if r number of roots are equal (i.e., $m_1=m_2=\dots\dots\dots=m_r=m$) then the complete solution of (3) is

$$y = (c_1 + c_2 x + c_3 x^2 + \dots\dots + c_r x^{r-1})e^{mx} + c_{r+1} e^{m_{r+1} x} + \dots\dots\dots\dots + c_n e^{m_n x}$$

Case III: *The roots are imaginary*

Suppose if one pair of roots be imaginary (*i.e.*, $m_1=\alpha+i\beta$, $m_2=\alpha-i\beta$) then the complete solution of (3) is

$$y = (c_1 \cos\beta x + c_2 \sin\beta x)e^{\alpha x} + c_3 e^{m_3 x} + \dots\dots\dots\dots + c_n e^{m_n x}.$$

Suppose if two points of imaginary roots are equal (*i.e.*, $m_1 = m_2 = \alpha + i\beta, m_3 = m_4 = \alpha - i\beta$) then the complete solution of (3) is

$$y = \left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right]e^{\alpha x} + c_5 e^{m_5 x} \dots\dots\dots\dots + c_n e^{m_n x}.$$

Solve the following Homogeneous Linear Differential equations with constant coefficients:

1. $y'' - y' - 12y = 0$ **Ans:** $y = c_1 e^{-3x} + c_2 e^{4x}$
2. $y'' - 4y' + 4y = 0$, $y(0) = 3,\ y'(0) = 1$ **Ans:** $y = (3 - 5x)e^{2x}$
3. $4y''' + 4y'' + y' = 0$ **Ans:** $y = c_1 + (c_2 + c_3 x)e^{-x/2}$
4. $\dfrac{d^3y}{dx^3} + y = 0$ **Ans:** $y = c_1 e^{-x} + \left(c_2 \cos\dfrac{\sqrt{3}x}{2} + c_3 \sin\dfrac{\sqrt{3}x}{2}\right)e^{x/2}$
5. $\dfrac{d^4y}{dx^4} + 8\dfrac{d^2y}{dx^2} + 16y = 0$ **Ans:** $y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x$

6. $4y''-12y'+5y=0$ **Ans:** $y=c_1e^{x/2}+c_2e^{5x/2}$

7. $y'''-4y''+y'+6y=0$ **Ans:** $y=c_1e^{-x}+c_2e^{2x}+c_3e^{3x}$

8. $y''+6y''+9y=0$ **Ans:** $y=(c_1+c_2x)e^{-3x}$.

9. $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=0$ **Ans:** $y=(c_1+c_2x+c_3x^2)e^{x}$.

10. $(D^2+1)^3y=0$ **Ans:** $y=(c_1+c_2x+c_3x^2)\cos x+(c_4+c_5x+c_6x^2)\sin x$.

Rules for finding the particular integral (P. I.) of

The nth order non-homogeneous linear differential equation with constant coefficients can be expressed as

$$\frac{d^ny}{dx^n}+k_1\frac{d^{n-1}y}{dx^{n-1}}k_2\frac{d^{n-2}y}{dx^{n-2}}+\cdots k_ny=X \quad \text{.. ....(6)}$$

The equation (6) in symbolic form is

$$(D^n+k_1D^{n-1}+k_2D^{n-2}+\cdots+k_n)y=X \quad \text{..(7)}$$

Further which can be written as $f(D)y=X$ where

$$f(D)=D^n+k_1D^{n-1}+k_2D^{n-2}+\cdots+k_n$$

Particular integral of is given by □.□. $=\frac{1}{f(D)}X$

Case I: When $X=e^{ax}$

$$\text{P.I.}=\frac{1}{f(D)}X=\frac{1}{f(a)}e^{ax}, \quad \text{provided } f(a)\neq 0$$

If $f(a)=0$, $\text{P.I.}=x\frac{1}{f'(a)}e^{ax}, \quad \text{provided } f'(a)\neq 0$

If $f'(a)=0$, $\text{P.I.}=x^2\frac{1}{f''(a)}e^{ax}, \quad \text{provided } f''(a)\neq 0$ and so on.

Case II: When $X=\cos(ax+b)$ or $\sin(axb)$

$$\text{P.I.}=\frac{1}{f(-a^2)}[\cos(ax+b)\,\text{or}\,\sin(ax+b)], \quad \text{provided } f(-a^2)\neq 0.$$

If $f(-a^2)=0$, $\text{P.I.}=x\frac{1}{f'(-a^2)}[\cos(ax+b)\,\text{or}\,\sin(ax+b)]$, provided $f'(-a^2)\neq 0$.

If $f'(-a^2)=0$, $\text{P.I.}=x^2\frac{1}{f''(-a^2)}[\cos(ax+b)\,\text{or}\,\sin(ax+b)]$, provided $f''(-a^2)\neq 0$

Case III: When $X=x^m$

$$\text{P.I.}=\frac{1}{f(D)}x^m=[f(D)]^{-1}x^m.$$

Expand $[f(D)]^{-1}$ in ascending powers of D as far as the term in D^m and operate on x^m term by term. Since the $D^nx^m=0$ for $n>m$, we need not consider terms beyond D^m.

Solve the following non-Homogeneous Linear Differential equations with constant coefficients:
Problems on Case I

1. $y_2 - 6y_1 + 9y = 6e^{3x} + 7e^{-2x} - \log 2$. **Ans:** $y = (c_1 + c_2 x)e^{3x} + 3x^2 e^{3x} + \frac{7}{25}e^{-2x} - \frac{1}{9}\log 2$

2. $(D-2)^3 y = e^{2x}$. **Ans:** $y = (c_1 + c_2 x + c_3 x^2)e^{2x} + \frac{x^3}{6}e^{2x}$

3. $(D^2+4)y = e^{3x}$. **Ans:** $y = (c_1 e^{2ix} + c_2 e^{-2ix}) + \frac{1}{13}e^{3x}$

4. $(D^2 - D + 1)y = \sinh x$. **Ans:** $y = \left(c_1 \cos\frac{\sqrt{3}x}{2} + c_2 \sin\frac{\sqrt{3}x}{2}\right)e^{x/2} + \frac{1}{6}(3e^x - e^{-x})$

5. $D(D+1)^2 y = 12e^{-x}$. **Ans:** $y = c_1 + (c_2 + c_3 x)e^{-x} - 6x^2 e^{-x}$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -\cosh x, \quad y = 0, \ \frac{dy}{dx} = 1 \ \text{at} \ x = 0$. **Ans:** $y = \frac{3}{5}e^{-2x}(\cos x + 3\sin x) - \frac{e^x}{10} - \frac{e^{-x}}{2}$.

7. $(D^2 + 2D + 1)y = 2e^{3x}$. **Ans:** $y = (c_1 + c_2 x)e^{-x} + \frac{e^{3x}}{8}$

8. $(D-2)^3 y = e^{2x}$. **Ans:** $y = (c_1 + c_2 x + c_3 x^2)e^{2x} + \frac{x^3 e^{2x}}{6}$

9. $(D+2)(D-1)^2 y = e^{-2x} + 2\sinh x$. **Ans:** $y = c_1 e^{-2x} + (c_2 + c_3 x)e^x + \frac{xe^{-2x}}{9} + \frac{x^2 e^x}{6} + \frac{1}{4}e^{-x}$

Problems on Case II

1. $(D^2+4)y = \sin 3x + \cos 2x$. **Ans:** $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{5}\sin 3x + \frac{x}{4}\sin 2x$

2. $(D^2 + 5D - 6)y = \sin(4x)\sin x$ **Ans:** $y = c_1 e^x + c_2 e^{-6x} + \frac{1}{2}\left[\frac{\sin 3x - \cos 3x}{30} + \frac{31\cos 5x - 25\sin 5x}{1586}\right]$.

3. $\frac{d^2x}{dt^2} + n^2 x = k\cos(nt + \alpha)$. **Ans:** $x = c_1 \cos nt + c_2 \sin nt + \frac{kt}{2n}\sin(nt + \alpha)$

4. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$. **Ans:** $y = (c_1 + c_2 x)e^{-x} + \frac{e^{2x}}{9} - \frac{1}{2} - \frac{2}{25}\sin 2x + \frac{3}{50}\cos 2x$

5. $(D^3 + 4D)y = \sin 2x$. **Ans:** $y = c_1 + c_2\cos 2x + +c_3 \sin 2x - \frac{x}{8}\sin 2x$

6. $y'' + 4y' + 4y = 3\sin x + 4\cos x, y(0) = 1 \text{and } y'(0) = 0$ **Ans:** $y = (1+x)e^{-2x} + \sin x$

7. Find the PI of $(D^3+1)y = \cos(2x-1)$. **Ans:** $\text{P.I.} = \frac{1}{65}[\cos(2x-1) - 8\sin(2x-1)]$

8. $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$. **Ans:** $y = c_1 + (c_2 + c_3 x)e^{-x} - \frac{x^2 e^{-x}}{2} + \frac{3}{50}\cos 2x - \frac{2}{25}\sin 2x$

Problems on Case III

1. $(D^2 - 1)y = 2x^4 - 3x + 1$ **Ans:** $y = c_1 e^x + c_2 e^{-x} - [2x^4 + 24x^2 - 3x + 49]$.

2. $(D^6 - D^4)y = x^2$ **Ans:** $y = (c_1 + c_2 x + c_3 x^2 + c_4 x^3) + c_5 e^x + c_6 e^{-x} - \left[\frac{x^6}{360} + \frac{x^4}{12} + x^2 + 2\right]$.

3. $\left(D^2+4D+4\right)y=x^2+2x,\ y(0)=0,\ y'(0)=0$ **Ans:** $y=-\frac{3}{8}(1+2x)e^{-2x}+\frac{1}{8}\left(2x^2+3\right)$.

4. $\left(D-2\right)^2 y=8x^2$. **Ans:** $y=(c_1+c_2x)e^{2x}+2x^2+4x+3$.

5. $\left(D^2+2\right)y=x^3+e^{-2x}+\cos 3x$ **Ans:** $y=(c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x)+\frac{1}{2}\left[x^3-3x\right]+\frac{1}{6}e^{-2x}-\frac{1}{7}\cos 3x$.

6. $\left(D^2+D+1\right)y=x^3$. **Ans:** $y=e^{\frac{x}{2}}(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x)+x^3-3x^2+6$

Alternative method for finding Particular Integral - Method of variation of parameters

This method is quite general and applied to equations of the form

$y''+Py'+Qy=R$ where P, Q and R are functions of x.

Let $\text{C.F.}=c_1u(x)+c_2v(x)$

Then $\text{P.I.}=A(x)u(x)+B(x)v(x)$

Where $A(x)=-\int\frac{vR}{W}dx$, $B(x)=\int\frac{uR}{W}dx$ and $W=\begin{vmatrix}u & v\\ u' & v'\end{vmatrix}$ is called the Wronskian of u, v. Hence the complete solution is given by $y=\text{C.F.}+\text{P.I.}$

Problems

1. $y''+y=\sec x$ **Ans:** $y=c_1\cos x+c_2\sin x+\cos x\log(\cos x)+x\sin x$

2. $\frac{d^2y}{dx^2}+y=\frac{1}{1+\sin x}$ **Ans:** $y=c_1\cos x+c_2\sin x+\sin x\log(1+\sin x)-x\cos x-1$

3. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$ **Ans:** $y=c_1+c_2e^{2x}-\frac{1}{2}e^x\sin x$

4. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{1}{1+e^{-x}}$ **Ans:** $y=\left(e^x+e^{2x}\right)\log\left(1+e^x\right)+\left(c_1-1-x\right)e^x+\left(c_2-x\right)e^{2x}$.

5. $\left(D^2+2D+1\right)y=e^{-x}\log x$. **Ans:** $y=c_1e^{-x}+c_2xe^{-x}+\frac{x^2}{2}\left(\frac{1}{2}-\log x\right)e^{-x}+e^{-2x}\left(x\log x-x\right)$

6. $\frac{d^2y}{dx^2}-y=\frac{2}{\left(1+e^x\right)}$ **Ans:** $y=c_1e^x+c_2e^{-x}-1+e^x\log(e^{-x}+1)-e^{-x}\log\left(e^x+1\right)$.

7. $y''-6y'+9y=\frac{e^{3x}}{x^2}$ **Ans:** $y=\left(c_1+c_2x\right)e^{3x}-e^{3x}\log\left(x+1\right)$.

8. $y''-2y'+2y=e^x\tan x$. **Ans:** $y=\left(c_1\cos x+c_2\sin x\right)e^x-e^x\cos x\log\left(\sec x+\tan x\right)$.

9. $\frac{d^2y}{dx^2}+a^2y=\text{cosec}\,ax$. **Ans:** $y=\left(c_1-\frac{x}{a}\right)\cos ax+\left[c_2+\left(\frac{1}{a^2}\right)\log\sin ax\right]\sin ax$.

10. $\left(D^2-2D+1\right)y=e^x\log x$ **Ans:** $y=\left(c_1+c_2x\right)e^x+\frac{x^2}{4}e^x\left(2\log x-3\right)$.

LINEAR DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

The linear differential equations with variable coefficients can be reduced to linear differential equations with constant coefficients by suitable substitutions.

Cauchy's linear differential equation:

An equation of the form

$$x^n \frac{d^n y}{dx^n} + k_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + k_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + k_{n-1} x \frac{dy}{dx} + k_n y = X \quad \text{.............(1)}$$

Using the substitution $x = e^t$ in equation (1) we get $xDy = D_1 y$, $x^2 D^2 y = D_1(D_1 - 1)y$,.

Similarly $x^3 D^3 y = D_1(D_1 - 1)(D_1 - 2)y$ and so on, where $D = d/dx$, $D_1 = d/dt$

After making these substitutions in equation (1), there results a linear equation with constant coefficients which can be solved as before.

Problems

1. $x\frac{d^2y}{dx^2} - \frac{2y}{x} = x + \frac{1}{x^2}$ **Ans:** $y = c_1 x^2 + \frac{c_2}{x} + \frac{1}{3}\left(x^2 - \frac{1}{x}\right)\log x$
2. $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 4x - 6$ **Ans:** $y = c_1 x^2 + c_2 x^3 + 2x - 1$
3. $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$. **Ans:** $y = \left\{c_1 + c_2 \log x + \log\frac{x}{x-1}\right\}\frac{1}{x}$
4. $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^{e^x}$ **Ans:** $y = \frac{c_1}{x} + \frac{\left(c_2 + e^x\right)}{x^2}$
5. $x^2 y'' + xy' + y = \log(x)\sin(\log x)$

   Ans. y=$c_1\cos(\log x) + c_2\sin(\log x) - \frac{1}{4}(\log x)^2\cos(\log x) + \frac{1}{4}\log(\log x)\sin(\log x)$

6. $x^2 y'' - 3xy' + 5y = x^2 \sin(\log x)$ **Ans.** $y = x^2(c_1\cos\log x + c_2\sin logx) - \log x.\frac{x^2}{2}.\cos\log x$

Legendre's linear differential equation:

An equation of the form

$$(ax+b)^n \frac{d^n y}{dx^n} + k_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + k_2 (ax+b)^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + k_{n-1}(ax+b)\frac{dy}{dx} + k_n y = X$$ …….(2) is

called the Legendre's linear differential equation. Using the substitution $ax + b = e^t$ in equation (1) we get

$(ax+b)Dy = aD_1 y$, $(ax+b)^2 D^2 y = a^2 D_1(D_1 - 1)y$, $(ax+b)^3 D^3 y = a^3 D_1(D_1-1)(D_1-2)y$ and so on where $D = \frac{d}{dx}, D_1 = \frac{d}{dt}$. After making these substitutions in (1), there results a linear equation with constant coefficients, which can be solved as before.

Problems

7. $(x+2)^2 y'' + 3(x+2)y' - 3y = 0$ **Ans:** $y = c_1(x+2) + c_2(x+2)^{-3}$

8. $(2x+3)^2 y'' - (2x+3) y' - 12y = 6x$

Ans: $y = c_1 (2x+3)^a + c_2 (2x+3)^b - \frac{3}{14}(2x+3) + \frac{3}{4}, \quad \text{where } a, b = \frac{3 \pm \sqrt{57}}{4}$

9. $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin[2\log(1+x)]$

Ans: $y = c_1 \cos\log(1+x) + c_2 \sin\log(1+x) - \frac{1}{3}\sin[2\log(1+x)]$

10. $(x-1)^3 \frac{d^3y}{dx^3} + 2(x-1)^2 \frac{d^2y}{dx^2} - 4(x-1)\frac{dy}{dx} + 4y = 4\log(x-1)$

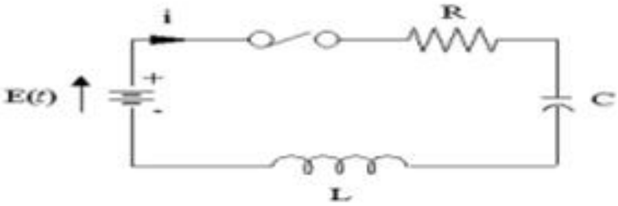
Ans: $y = c_1 (x-1) + c_2 (x-1)^2 + c_3 (x-1)^{-3} + \log(x+1) + 1$.

11. $(3x+2)^2 \frac{d^2y}{dx^2} + 5(3x+2)\frac{dy}{dx} - 3y = x^2 + x + 1$

Ans: $y = c_1 (3x+2)^{1/3} + c_2 (3x+2)^{-1} + \frac{1}{27}\left[\frac{1}{15}(3x+2)^2 + \frac{1}{4}(3x+2) - 7\right]$

APPLICATIONS

LRC- series circuit



Kirchhoff's voltage law (KVL): The algebraic sum of all the instantaneous voltage drops across the three elements inductor, resistor and capacitance is equal to the resultant electromotive force (external source) in the circuit. Thus the LRC-circuit is modeled as:

$$L\frac{dI}{dt} + RI + \frac{1}{c}Q = E(t).$$

Since $I = \frac{dQ}{dt}$ the above equation can be written as

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{c}Q = E(t) \quad \text{.........(1)}$$

Differentiating (10) w.r.t. 't', we obtain

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{c}I = \frac{dE}{dt} \quad \text{...............(2)}$$

Thus the charge Q and current I at any time in the LRC-circuit are obtained by solving the equations (1) and (2) respectively.

Alternately, If equation (1) is solved, then current can be obtained by $I = \frac{dQ}{dt}$ and if equation (2) is solved then Charge can be obtained by integrating the solution of (2) with respect to t.

Problems

1. A condenser of capacity C discharged through an inductance L and resistance R in series and the charge q at time t satisfies the equation $L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$. Given that L=0.25henries, R=250 ohms, $C = 2\times10^{-6}$ farads and that when $t = 0$, charge q is 0.002 coulombs and the current $\frac{dq}{dt} = 0$, obtain the value of q in terms of t.

2. Determine Q and i in the LRC-circuit with $L=0.5H, R=6\Omega, C=0.02F$, $E(t)=24\sin(10t)$ and initial conditions $Q=i=0$ at t=0.
3. Find the charge on the capacitor in an LRC-series circuit at $t=0.01s$ when $L=0.05H$, $R=2\Omega$, $C=0.01F$, $E(t)=0V$, $q(0)=5C$ and $i(0)=0A$. Determine the first time when the charge on the capacitor is zero.
4. Find the charge on the capacitor in an LRC-series circuit when $L=\frac{1}{4}H$, $R=20\Omega$, $C=\frac{1}{300}F$, $E(t)=0V$, $q(0)=4C$ and $i(0)=0A$. Is the charge on the capacitor ever equal to zero?
5. Find the charge on the capacitor in an LRC-series circuit when $L=1H$, $R=100\Omega$, $C=0.0004F$, $E(t)=30V$, $q(0)=0C$ and $i(0)=2A$. Determine the first time when the charge on the capacitor is zero.
6. Find the steady-state charge and the steady-state current in an LRC-series circuit when $L=1H$, $R=2\Omega$, $C=0.25F$, $E(t)=50\cos(t)V$.
7. Find the steady-state charge and the steady-state current in an LRC-series circuit when $L=\frac{1}{2}H$, $R=20\Omega$, $C=0.001F$, $E(t)=100\sin(60t)+200\cos(40t)V$.
8. Find the steady-state charge and the steady-state current in an LRC-series circuit when $L=\frac{1}{2}H$, $R=20\Omega$, $C=0.001F$, $E(t)=100\sin(60t)$.
9. Find the current in the RLC circuit connected in series for the given data. Assume zero initial current and charge
   a. $R=200\Omega$, $L=0.1H$, $C=0.006F$ and $E=te^{-t}$ *volts*.
   b. $R=400\Omega$, $L=0.12H$, $C=0.04F$ and $E=120\sin(20t)$ *volts*.