



Course Code: 25MA1BSC EM /25MA1BS MCS

Course: Mathematical Foundation for Civil, Electrical and Mechanical Streams– 1

Mathematical Foundation for Computer Science Stream -1

UNIT-1: Matrices and System of equations

I. RANK OF MATRICES:

1. Find the rank of the following matrices by reducing them into echelon form.

a) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

Ans: $\rho=2$

e) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

Ans: $\rho=2$

i) $\begin{bmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

Ans: $\rho=2$

f) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$.

Ans: $\rho=2$

j) $\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$.

Ans: $\rho=2$

c) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

Ans: $\rho=3$

g) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

Ans: $\rho=2$

k) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$.

Ans: $\rho=4$

d) $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

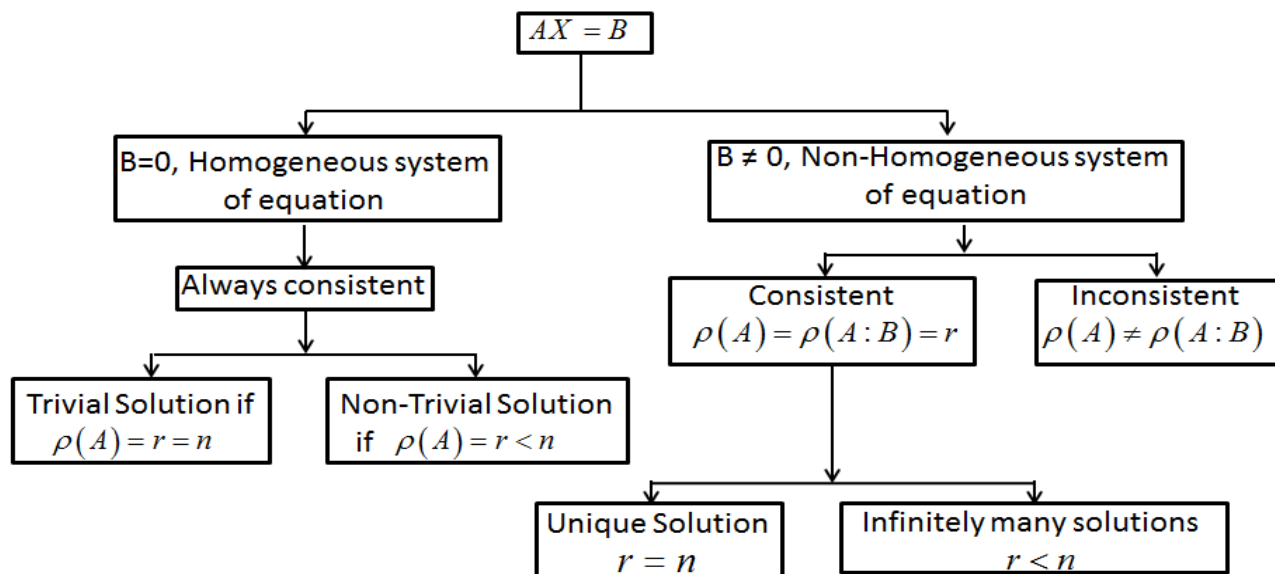
Ans: $\rho=3$

h) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$.

Ans: $\rho=3$

2. Find 'b' if the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3. Ans: $b = 2$ or $b = -6$.

II. Consistency and solution of the linear system of equations:



n = Number of Unknowns

1. Solve the following system of equations:

SL.NO	Questions	Answers
a	$x + 2y + 3z = 0$ $3x + 4y + 4z = 0$ $7x + 10y + 12z = 0$	$x = y = z = 0$
b	$2x_1 + 3x_2 - 4x_3 + x_4 = 0$ $x_1 - x_2 + x_3 + 2x_4 = 0$ $5x_1 - x_3 + 7x_4 = 0$ $7x_1 + 8x_2 - 11x_3 + 5x_4 = 0$	$x_1 = \frac{(k_1 - 7k_2)}{5}, x_2 = \frac{(6k_1 + 3k_2)}{5}, x_3 = k_1, x_4 = k_2$
c	$x_1 - 2x_2 - x_3 + 3x_4 = 0$ $-2x_1 + 4x_2 + 5x_3 - 5x_4 = 0$ $3x_1 - 6x_2 - 6x_3 + 8x_4 = 0$	$x_1 = \frac{(6k_1 - 10k_2)}{3}, x_2 = k_1, x_3 = \frac{-k_2}{2}, x_4 = k_2$
d	$x + 3y - 2z = 0$ $2x - y + 4z = 0$ $x - 11y + 14z = 0$	$x = \frac{-10k}{7}, y = \frac{8k}{7}, z = k$
e	$4x + 2y + z + 3w = 0$ $6x + 3y + 4z + 7w = 0$ $2x + y + w = 0$	$x = \frac{(k_1 + k_2)}{2}, y = k_1, z = -k_2, w = k_2$

2. Discuss the consistency of the following system of linear equations

$$\begin{aligned} 5x + 3y + 7z &= 4 \\ 3x + 26y + 2z &= 9 \\ \text{a) } 7x + 2y + 10z &= 5 \end{aligned}$$

Ans: Consistent and has infinitely many solutions

$$x = \frac{7-16k}{11} \quad y = \frac{k+3}{11} \quad z = k$$

$$4x - 2y + 6z = 8$$

$$x + y - 3z = -1$$

$$\begin{aligned} \text{b) } 15x - 3y + 9z &= 21 \\ \text{Ans: Consistent and has infinitely many solutions} \end{aligned}$$

$$x = 1, y = 3k - 2, z = k$$

$$2x - 3y + 7z = 5$$

$$3x + y - 3z = 13$$

$$\begin{aligned} \text{c) } 2x + 19y - 47z &= 32 \\ \text{Ans: inconsistent} \end{aligned}$$

$$2x_1 + 3x_2 - x_3 = 1$$

$$3x_1 - 4x_2 + 3x_3 = -1$$

$$\text{d) } 2x_1 - x_2 + 2x_3 = -3$$

$$3x_1 + 1x_2 - 2x_3 = 4$$

$$\text{Ans: Inconsistent}$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

$$3x - 5y + 5z = 2$$

$$\begin{aligned} \text{e) } 3x + 9y - z &= 4 \\ \text{Ans: Consistent and has unique solution} \\ x = -1, y = 1, z &= 2 \end{aligned}$$

$$2x + 6y + 11 = 0$$

$$\text{f) } 6x + 20y - 6z + 3 = 0$$

$$6y - 18z + 1 = 0$$

$$\text{Ans: inconsistent}$$

$$2x_1 - 2x_2 + 4x_3 + 3x_4 = 9$$

$$x_1 - x_2 + 2x_3 + 2x_4 = 6$$

$$\text{g) } 2x_1 - 2x_2 + x_3 + 2x_4 = 3$$

$$x_1 - x_2 + x_4 = 2$$

$$\text{Ans: Inconsistent}$$

$$2x + y - z = 0$$

$$2x + 5y + 7z = 52$$

$$\text{h) } x + y + z = 9$$

$$\text{Ans: Consistent and has unique solution}$$

$$x = 1, y = 3, z = 5$$

$$3x + 2y + 2z = 0$$

$$x + 2y = 4$$

$$\text{i) } 10y + 3z = -2$$

$$2x - 3y - z = 5$$

$$\text{Ans: Consistent and has unique solution}$$

$$x = 2, y = 1, z = -4$$

$$x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$$

$$2x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 2$$

$$3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 = 3$$

$$\begin{aligned} \text{j) } \text{Ans: Consistent and} \\ \text{has infinitely many solutions} \end{aligned}$$

$$x_1 = 1, x_2 = 2a, x_3 = a, x_4 = -3b, x_5 = b$$

$$2x + 3y + 5z = 9$$

3. Investigate the values of λ and μ so that the equations $7x + 3y - 2z = 8$ have

$$2x + 3y + \lambda z = \mu$$

(i) no solution (ii) unique solution (iii) infinite number of solutions.

Ans: (i) If $\lambda = 5$ and $\mu \neq 9$ (ii) $\lambda \neq 5$ and μ can be any value (iii) $\lambda = 5$ and $\mu = 9$.

$$x + 2y + 3z = 6$$

4. Determine the values of a and b for which the system $x + 3y + 5z = 9$ have (i) no solution (ii) a

$$2x + 5y + az = b$$

unique solution (iii) an infinite number of solutions.

Ans: (i) If $a = 8$ and $b \neq 15$ (ii) $a \neq 8$ and for any b (iii) $a = 8$ and $b = 15$.

5. Find the value of a for which the system $x + 2y + z = 3$; $ay + 5z = 10$; $2x + 7y + az = b$ has a unique solution. Also, find the pair of values (a, b) for which it has infinitely many solutions.

Ans.: For $a \neq -3, 5$ system has unique solution. For $(a, b) = (-3, -4)$ and $(a, b) = (5, 12)$ has infinite number of solutions

$$x + ay + z = 3$$

6. Find the values of a and b for which the system of equations $x + 2y + 2z = b$ is consistent.

$$x + 5y + 3z = 9$$

Ans: If $a = -1$ and $b = 6$ equations will be consistent and have an infinite number of solutions.

If $a \neq -1$ and b has any value, equations will be consistent and have a unique solution.

$$3x + 4y + 5z = a$$

7. Show that the equations $4x + 5y + 6z = b$ do not have a solution unless $a + c = 2b$.

$$5x + 6y + 7z = c$$

$$-2x + y + z = a$$

8. Test for consistency $x - 2y + z = b$ where a, b and c are constants.

$$x + y - 2z = c$$

Ans: if $a + b + c \neq 0$ inconsistent, if $a + b + c = 0$, then infinitely many solutions.

III. Gauss elimination method:

1. Solve the following system of equations by Gauss elimination method

SL NO	Questions	Answers
a	$2x_1 + x_2 + x_3 = 10$ $3x_1 + 2x_2 + 3x_3 = 18$ $x_1 + 4x_2 + 9x_3 = 16$	$x_1 = 7, x_2 = -9, x_3 = 5.$
b	$2x + 2y + z = 12$ $3x + 2y + 2z = 8$ $5x + 10y - 8z = 10$	$x = \frac{-51}{4}, y = \frac{115}{8}, z = \frac{35}{4}$
c	$2x_1 + 4x_2 + x_3 = 3$ $3x_1 + 2x_2 - 2x_3 = -2$ $x_1 - x_2 + x_3 = 6$	$x_1 = 2, x_2 = -1, x_3 = 3$
d	$10x + 2y + z = 9$ $2x + 20y - 2z = -44$ $-2x + 3y + 10z = 22$	$x = 1, y = -2, z = 3$
e	$2x_1 + x_2 + 4x_3 = 12$ $8x_1 - 3x_2 + 2x_3 = 20$ $4x_1 + 11x_2 - x_3 = 33$	$x_1 = 3, x_2 = 2, x_3 = 1.$

SL NO	Questions	Answers
f	$x_1 + 4x_2 - x_3 = -5$ $x_1 + x_2 - 6x_3 = -12$ $3x_1 - x_2 - x_3 = 4$	$x_1 = \frac{117}{71}, x_2 = -\frac{81}{71},$ $x_3 = \frac{148}{71}$
g	$2x_1 + x_2 + 3x_3 = 1$ $4x_1 + 4x_2 + 7x_3 = 1$ $2x_1 + 5x_2 + 9x_3 = 3$	$x_1 = \frac{-1}{2}, x_2 = -1, x_3 = 1.$
h	$2x_1 - 7x_2 + 4x_3 = 9$ $x_1 + 9x_2 - 6x_3 = 1$ $-3x_1 + 8x_2 + 5x_3 = 6$	$x_1 = 4, x_2 = 1, x_3 = 2$
i	$5x_1 + x_2 + x_3 + x_4 = 4$ $x_1 + 7x_2 + x_3 + x_4 = 12$ $x_1 + x_2 + 6x_3 + x_4 = -5$ $x_1 + x_2 + x_3 + 4x_4 = -6$	$x_1 = 1, x_2 = 2,$ $x_3 = -1, x_4 = -2$

$$3x - y + 4z = 3$$

2. Show that if $\lambda \neq -5$ the system of equations $x + 2y - 3z = -2$ have a unique solution. If $\lambda = -5$

$$6x + 5y + \lambda z = -3$$

show that the equations are consistent. Determine the solution in each case.

Ans: when $\lambda \neq -5$, $x = \frac{4}{7}$, $y = \frac{-9}{7}$, $z = 0$, when $\lambda = -5$, $x = \frac{4-5k}{7}$, $y = \frac{13k-9}{7}$, $z = k$.

$$5x + 3y + 2z = 12$$

3. Prove that the equations $2x + 4y + 5z = 2$ are incompatible unless $c = 74$; and in that case the equations are satisfied by $x = 2 + t$, $y = 2 - 3t$, $z = -2 + 2t$, where t is any arbitrary quantity.

$$x + y + z = 1$$

4. For what values of k , the equations $2x + y + 4z = k$ have a solution and solve them completely in each case.

$$4x + y + 10z = k^2$$

Ans: When $k = 1$, $x = -3z$, $y = 2z + 1$, when $k = 2$, $x = 1 - 3z$, $y = 2z$.

IV. Gauss-Seidel Iteration method:

If the system is diagonally dominant, then the method is known to give convergent solution.

Diagonal dominance is not a necessary condition.

Suppose $AX = B$ is diagonally dominant with $A = (a_{ij})$, $X = (x_i)$ and $B = (b_i)$.

$$x_1^{n+1} = \frac{1}{a_{11}}(b_1 - a_{12}x_2^n - a_{13}x_3^n \dots)$$

Then the iterative formula is $x_2^{n+1} = \frac{1}{a_{22}}(b_2 - a_{21}x_1^{n+1} - a_{23}x_3^n \dots)$

$$x_3^{n+1} = \frac{1}{a_{33}}(b_3 - a_{31}x_1^{n+1} - a_{32}x_2^{n+1} \dots)$$

$$\vdots$$

Solve the following system of equations by Gauss-Seidel method.

SL.NO.	Questions	Answers
1	$20x + y - 2z = 17$ $3x + 20y - z = -18$ $2x - 3y + 20z = 25$	$x = 1, y = -1, z = 1$
2	$5x + 2y + z = 12$ $x + 4y + 2z = 15$ $x + 2y + 5z = 20$	$x = 0.996, y = 2, z = 3$
3	$2x + y + 6z = 9$ $8x + 3y + 2z = 13$ $x + 5y + z = 7$	$x = 1, y = 1, z = 1$
4	$28x + 4y - z = 32$ $x + 3y + 10z = 24$ $2x + 17y + 4z = 35$	$x = 0.9876, y = 1.5090, z = 1.8485$
5	$10x + 2y + z = 9$ $2x + 20y - 2z = -44$ $-2x + 3y + 10z = 22$	$x = 1, y = -2, z = 3$

SL.NO	Questions	Answers
6	$83x + 11y - 4z = 95$ $7x + 52y + 13z = 104$ $3x + 8y + 29z = 71$	$x = 1.06, y = 1.37, z = 1.96$
7	$54x + y + z = 110$ $2x + 15y + 6z = 72$ $-x + 6y + 27z = 85$	$x = 1.926, y = 3.573, z = 2.425$
8	$5x_1 - x_2 = 9$ $-x_1 + 5x_2 - x_3 = 4$ $-x_2 + 5x_3 = -6$	$x_1 = 1.99, x_2 = 0.99, x_3 = -1$
9	$8x_1 + x_2 - x_3 = 8$ $2x_1 + x_2 + 9x_3 = 12$ $x_1 - 7x_2 + 2x_3 = -4$	$x_1 = 1, x_2 = 1, x_3 = 1$
10	$4x_1 + 2x_2 + x_3 = 11$ $-x_1 + 2x_2 = 3$ $2x_1 + x_2 + 4x_3 = 16$	$x_1 = 1, x_2 = 2, x_3 = 3$
11	Start with $(2, 2, -1)$ and solve $5x_1 - x_2 + x_3 = 10$ $2x_1 + 4x_2 = 12$ $x_1 + x_2 + 5x_3 = -1$	$x_1 = 2.5555, x_2 = 1.7222, x_3 = -1.0555$
12	$10x_1 + x_2 + x_3 = 12$ $2x_1 + 10x_2 + x_3 = 13$ $2x_1 + 2x_2 + 10x_3 = 14$	$x_1 = x_2 = x_3 = 1$
13	$27x_1 + 6x_2 - x_3 = 85$ $6x_1 + 15x_2 + 2x_3 = 72$ $x_1 + x_2 + 54x_3 = 110$	$x_1 = 2.4255, x_2 = 3.573, x_3 = 1.926$
14	$x_1 - 8x_2 + 3x_3 = -4$ $2x_1 + x_2 + 9x_3 = 12$ $8x_1 + 2x_2 - 2x_3 = 8$	$x_1 = x_2 = x_3 = 1$
15	$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$	$x = 1, y = 2, z = 3, u = 4$

V. Characteristic values (Eigenvalues) and characteristic vectors (Eigenvectors):

If A is a square matrix, then λ is said to be an eigenvalue of the matrix if there exists a non-zero vector X such that $AX = \lambda X$. X is called the eigenvector corresponding to the eigenvalue λ .

$\lambda X = \lambda IX \Rightarrow (A - \lambda I)X = 0$. We seek non-trivial solutions of $(A - \lambda I)X = 0$.

X is non-trivial if $\rho(A - \lambda I) < n \Rightarrow |A - \lambda I| = 0$.

If A is a matrix of size 3×3 then $-\lambda^3 + \text{Tr}(A)\lambda^2 - \sum M_{ii}^{2 \times 2} \lambda + |A| = 0$.

Find the eigenvalues and its corresponding eigenvectors for the following matrices:

a. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, Ans: $\lambda = -2, 3, 6$; $x_1 = [-k, 0, k]$, $x_2 = [k, -k, k]$, $x_3 = [k, 2k, k]$

b. $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, Ans: $\lambda = 2, 3, 5$; $x_1 = k_1[1, -1, 0]$, $x_2 = k_2[1, 0, 0]$, $x_3 = k_3[3, 2, 1]$.

c. $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, Ans: $\lambda = 0, 3, 15$; $x_1 = [1, 2, 2]$, $x_2 = [2, 1, -2]$, $x_3 = [2, -2, 1]$

d. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, Ans: $\lambda = 1, 2, 3$; $x_1 = [1, 0, -1]$, $x_2 = [0, 1, 0]$, $x_3 = [1, 0, 1]$

e. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, Ans: $\lambda = 5, -3, -3$; $x_1 = k[1, 2, -1]$, $x_2 = [3k_1 - 2k_2, k_2, k_1]$

f. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, Ans: $\lambda = 8, 2, 2$; $x_1 = [2, -1, 1]$, $x_2 = [1, 0, -2]$, $x_3 = [1, 2, 0]$

g. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$, Ans: $\lambda = 1, -1, 4$; $x_1 = [2, -1, 1]$, $x_2 = [0, 1, 1]$, $x_3 = [-1, -1, 1]$

h. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$, Ans: $\lambda = 1, 2, 2$; $x_1 = k_1[1, 1, -1]$, $x_2 = k_2[2, 1, 0]$, $x_3 = k_3[2, 1, 0]$

i. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, Ans: $\lambda = 5, 1, 1$; $x_1 = [1, 1, 1]$, $x_2 = [1, 0, -1]$, $x_3 = [2, -1, 0]$

j. $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, Ans: $\lambda = 2, 2, 3$; $x_1 = x_2 = [5, 2, -3]$, $x_3 = [1, 1, -2]$

$$\text{k. } \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}, \text{ Ans: } \lambda = 2, 2, -2; x_1 = x_2 = [0, 1, 1], x_3 = [-4, -1, 1]$$

$$\text{l. } \begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}, \text{ Ans: } \lambda = -5, 2, 2; x_1 = [3, 2, 4], x_2 = x_3 = [1, 3, -1]$$

VI. Rayleigh Power Method:

Find AX_i and express $AX_i = |\lambda_i| X_{i+1}$ where $|\lambda_i|$ is the numerically largest element of X_i . Iterate until desired accuracy.

The largest/dominant eigenvalue is given by $\lambda = \frac{\langle AX_n, X_n \rangle}{\langle X_n, X_n \rangle} = \frac{X_n^T (AX_n)}{X_n^T X_n}$.

Find the dominant eigenvalue and the corresponding eigenvector of the following matrices.

$$1. \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \text{ by taking the initial vector as } [1 \ 0 \ 0]^T.$$

$$\text{Ans.: } X_1 = \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 0.0444 \\ 0.0667 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ 0.45 \\ 0.0688 \end{bmatrix}, X_4 = \begin{bmatrix} 1 \\ 0.0451 \\ 0.0685 \end{bmatrix}, X_5 = \begin{bmatrix} 1.000 \\ 0.0451 \\ 0.0685 \end{bmatrix}$$

$$\lambda = 25.1522$$

$$2. \begin{bmatrix} -0.5 & 0.5 & -1 \\ 0.5 & -0.5 & 1 \\ -1 & 1 & 0 \end{bmatrix} \text{ by taking the initial vector as } [1 \ 1 \ 1]^T.$$

Ans: $\lambda \approx -2$. Others are 1 and 0.

$$X_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 0.5 \\ -0.5 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} -1 \\ 1 \\ -0.6667 \end{bmatrix}, X_4 = \begin{bmatrix} 0.8333 \\ -0.8333 \\ 1 \end{bmatrix}, X_5 = \begin{bmatrix} -1 \\ 1 \\ -0.9091 \end{bmatrix}$$

$$\lambda = -1.9942$$

$$3. \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \text{ by taking the initial vector as } [1 \ 0 \ 0]^T.$$

$$\text{Ans.: } X_1 = \begin{bmatrix} 0.1667 \\ 0.667 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 0.0208 \\ 0.2917 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} 0.191 \\ 0.0417 \\ 1 \end{bmatrix}, X_4 = \begin{bmatrix} 0.3295 \\ -0.0111 \\ 1 \end{bmatrix}, X_5 = \begin{bmatrix} 0.3403 \\ 0.0394 \\ 1 \end{bmatrix}$$

$$\lambda = 7.1030$$

$$4. \begin{bmatrix} -8 & 6 & -2 \\ 6 & -7 & 4 \\ -2 & 4 & -3 \end{bmatrix} \text{ by taking the initial vector as } [1 \ 1 \ 1]^T. \text{ Ans: } \lambda \approx -15.$$

$$\text{Ans.: } X_1 = \begin{bmatrix} 1 \\ -0.75 \\ 0.25 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ -0.9423 \\ 0.4423 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ -0.9881 \\ 0.4881 \end{bmatrix}, X_4 = \begin{bmatrix} 1 \\ -0.9976 \\ 0.4976 \end{bmatrix},$$

$$X_5 = \begin{bmatrix} 1 \\ -0.9995 \\ 0.4995 \end{bmatrix}, \lambda = -15$$

5. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by taking the initial vector as $[1 \ 0 \ 0]^T$.

Ans.: $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ 0 \\ 0.9286 \end{bmatrix}, X_4 = \begin{bmatrix} 1 \\ 0 \\ 0.9756 \end{bmatrix}, X_5 = \begin{bmatrix} 1 \\ 0 \\ 0.9918 \end{bmatrix}, \lambda = 3$

6. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking the initial vector as $[1 \ 1 \ 1]^T$.

Ans.: $X_1 = \begin{bmatrix} 0.25 \\ 0 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 0.1429 \\ -0.3571 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} 0.1765 \\ -0.5098 \\ 1 \end{bmatrix}, X_4 = \begin{bmatrix} 0.2234 \\ -0.5685 \\ 1 \end{bmatrix}, X_5 = \begin{bmatrix} 0.2528 \\ -0.5879 \\ 1 \end{bmatrix}$
 $\lambda = 4.1132$

7. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking the initial vector as $[1 \ 1 \ 1]^T$.

Ans.: $X_1 = \begin{bmatrix} 1 \\ 0 \\ 0.6667 \end{bmatrix}, X_2 = \begin{bmatrix} 1 \\ -0.3636 \\ 0.5455 \end{bmatrix}, X_3 = \begin{bmatrix} 1 \\ -0.4651 \\ 0.5116 \end{bmatrix}, X_4 = \begin{bmatrix} 1 \\ -0.4912 \\ 0.5029 \end{bmatrix}, X_5 = \begin{bmatrix} 1 \\ -0.4978 \\ 0.5007 \end{bmatrix}$
 $\lambda = 8$

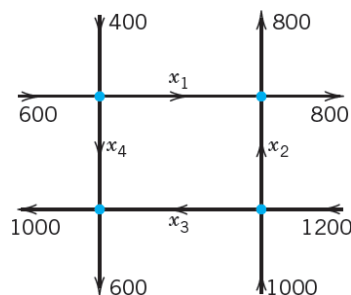
8. $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ by taking the initial vector as $[1 \ 1 \ 0]^T$.

Ans.: $X_1 = \begin{bmatrix} 0.8 \\ 1 \\ 0.6 \end{bmatrix}, X_2 = \begin{bmatrix} 0.3478 \\ 0.7391 \\ 1 \end{bmatrix}, X_3 = \begin{bmatrix} 0.1241 \\ 0.5172 \\ 1 \end{bmatrix}, X_4 = \begin{bmatrix} 0.0566 \\ 0.4527 \\ 1 \end{bmatrix}, X_5 = \begin{bmatrix} 0.0353 \\ 0.4318 \\ 1 \end{bmatrix}$
 $\lambda = 10.8008$.

VII. APPLICATIONS:

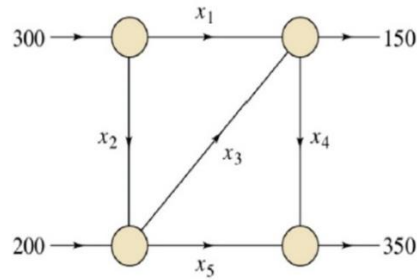
TRAFFIC FLOW PROBLEMS

1. Find the traffic flow in the net of one-way street directions shown in the figure:



Ans.: $x_3 = 1600 - x_4, x_2 = 600 + x_4, x_1 = 1000 - x_4$

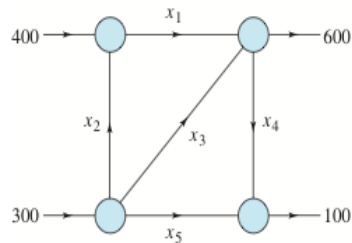
2. Find the traffic flow in the net of one-way street directions shown in the figure:



Ans.: $x_1 = 500 - x_5$, $x_2 = x_3 + x_5 - 200$, $x_4 = 350 - x_5$

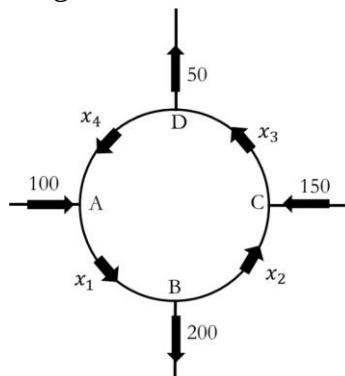
3. The flow of traffic (in vehicles per hour) through a network of streets is shown in the figure.

- Solve this system for x_i , $i=1, 2, \dots, 5$
- Find the traffic flow when $x_3=0$ and $x_5=0$.
- Find the traffic flow when $x_3=x_5=100$



Ans.: i. $x_1 = 700 - x_5$, $x_2 = 300 - x_3 - x_5$, $x_4 = 100 - x_5$
 ii. $x_1 = 700$, $x_2 = 300$, $x_4 = 100$
 iii. $x_1 = 600$, $x_2 = 100$, $x_4 = 0$

4. Following figure represents traffic entering and leaving a round of road junction, such junctions are common in Basavangudi.



- Construct a mathematical model that describe the flow of traffic along various branches.
- What is the minimum flow theoretically possible along branch AB.? What are the other flows at that time?
- What is the minimum flow theoretically possible along branch DA if the branch BC is under repair? What is the other flow at that time?
- If it possible to repair branch CD without having another alternative road.

Ans.: $x_1 = 100 + x_4$, $x_2 = x_4 - 100$, $x_3 = x_4 + 50$

(ii) $x_1 = 200$ (iii) $x_4 = 100$ (iv) No

BALANCING CHEMICAL EQUATIONS

Balance the following chemical equations using matrix method.

