BMS COLLEGE OF ENGINEERING, BENGALURU
DEPARTMENT OF MATHEMATICS

UNIT 3: LINEAR ALGEBRA

I. RANK OF MATRICES

1. Find the rank of the following matrices by reducing them into echelon form.

a) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

Ans: ρ=2

b) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

Ans: ρ=2

c) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

Ans: ρ=3

d) $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

Ans: ρ=3

e) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$.

Ans: ρ=2

f) $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$.

Ans: ρ=2

g) $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$.

Ans: ρ=2

h) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$.

Ans: ρ=3

i) $\begin{bmatrix} 1 & -2 \\ 3 & -6 \\ 7 & -1 \\ 4 & 5 \end{bmatrix}$

j) $\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$.

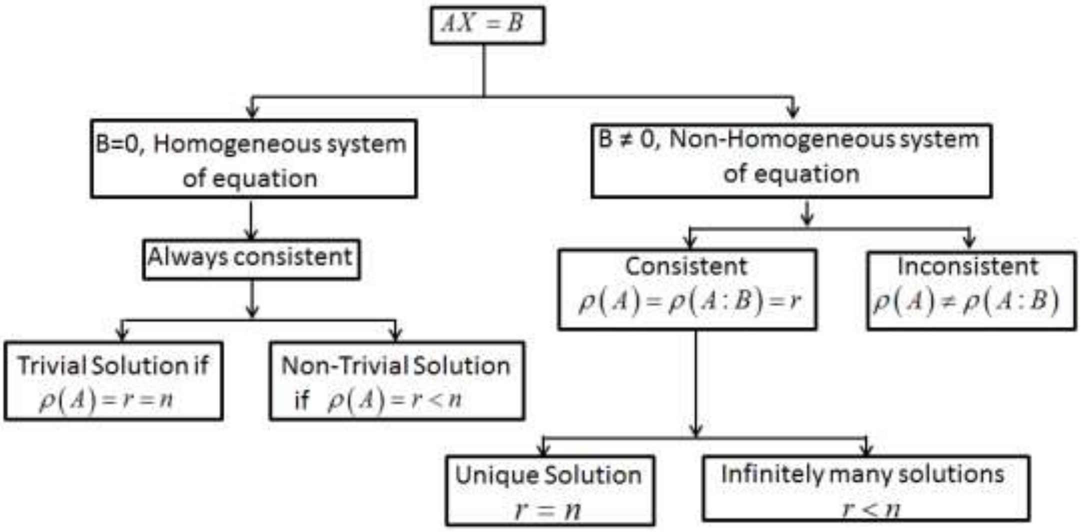
Ans: ρ=2

k) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$.

Ans: ρ=4

2. Find 'b' if the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3. Ans: $b=-6$

II. Consistency and solution of linear system of equations:



$n =$ Number of Unknowns

1. Discuss the solution of the following system of linear equations

1. $x+2y+3z=0$, $3x+4y+4z=0$, $7x+10y+12z=0$.

2. $2x_1+3x_2-4x_3+x_4=0$, $x_1-x_2+x_3+2x_4=0$, $5x_1-x_3+7x_4=0$, $7x_1+8x_2-11x_3+5x_4=0$.
   Ans: $x_1=$ $x_2=$ $x_3=$ $x_4=$

3. $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$.
   $Ans: x=\frac{7-16k}{11}$ $y=\frac{k+3}{11}$ $z=k$

4. $4x-2y+6z=8$, $x+y-3z=-1$, $15x-3y+9z=21$.
   $Ans: x=1,\ y=3k-2,\ z=k$

5. $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$.
   Ans: inconsistent

6. $2x_1+3x_2-x_3=1$, $3x_1-4x_2+3x_3=-1$, $2x_1-x_2+2x_3=-3$, $3x_1+1x_2-2x_3=5$.
   Ans: Inconsistent

7. $x+2y+z=3$, $2x+3y+2z=5$, $3x-5y+5z=2$, $3x+9y-z=4$.
   $Ans: x=-1,\ y=1,\ z=2$

8. $2x+6y+11=0$, $6x+20y-6z+3=0$, $6y-18z+1=0$.
   Ans: inconsistent

9. $2x_1-2x_2+4x_3+3x_4=9$, $x_1-x_2+2x_3+2x_4=6$, $2x_1-2x_2+x_3+2x_4=3$, $x_1-x_2+x_4=2$.
   Ans: Inconsistent

10. $2x+y-z=0$, $2x+5y+7z=52$, $x+y+z=9$.
    Ans: $x=1, y=3, z=5$

11. $3x+2y+2z=1$, $x+2y=4$, $10y+3z=-2$, $2x-3y-z=5$.
    Ans: $x=2, y=1, z=-4$

12. $x_1+x_2-2x_3+x_4+3x_5=1$, $2x_1-x_2+2x_3+2x_4+6x_5=2$, $3x_1+2x_2-4x_3-3x_4-9x_5=3$.
    Ans: $x_1=1,\ x_2=2a,\ x_3=a$
    $x_4=-3b,\ x_5=b$

2. Investigate the values of λ and μ so that the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have (i) no solution (ii) unique solution (iii) infinite number of solutions.

   Ans: (i) If $\lambda=5$ and $\mu\neq 9$ (ii) $\lambda\neq 5$ and μ can be any value (iii) $\lambda=5$ and $\mu=9$.

3. Determine the values of a and b for which the system $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+az=b$ have(i) no solution (ii) unique solution (iii) infinite number of solutions.

4. Find the values of a and b for which the system of equations $\begin{matrix} x+ay+z=3 \\ x+2y+2z=b \\ x+5y+3z=9 \end{matrix}$ is consistent.

Ans: If $a=-1$ and b =6 equations will be consistent and have infinite number of solutions.
If $a \neq -1$ and b has any value, equations will be consistent and have a unique solution.

5. Show that if $\lambda \neq -5$ the system of equations $\begin{matrix} 3x-y+4z=3 \\ x+2y-3z=-2 \\ 6x+5y+\lambda z=-3 \end{matrix}$ have a unique solution. If $\lambda=-5$ show that the equations are consistent. Determine the solution in each case.

Ans: when $\lambda \neq -5,\ x=\frac{4}{7},\ y=\frac{-9}{7},\ z=0$, when $\lambda=-5,\ x=\frac{4-5k}{7},\ y=\frac{13k-9}{7},\ z=k$.

6. Show that the equations $\begin{matrix} 3x+4y+5z=a \\ 4x+5y+6z=b \\ 5x+6y+7z=c \end{matrix}$ do not have a solution unless $a+c=2b$.

7. Prove that the equations $\begin{matrix} 5x+3y+2z=12 \\ 2x+4y+5z=2 \\ 39x+43y+45z=c \end{matrix}$ are incompatible unless $c=74$; and in that case the equations are satisfied by $x=2+t, y=2-3t, z=-2+2t$, where t is any arbitrary quantity.

8. For what values of k the equations $\begin{matrix} x+y+z=1 \\ 2x+y+4z=k \\ 4x+y+10z=k^2 \end{matrix}$ have a solution and solve them completely in each case. **Ans**: when $k=1,\ x=-3z, y=2z+1$, when $k=2$, $x=1-3z, y=2z.$

9. Test for consistency $\begin{matrix} -2x+y+z=a \\ x-2y+z=b \\ x+y-2z=c \end{matrix}$ where a,b and c are constants.

Ans: if $a+b+c \neq 0$ inconsistent, if $a+b+c=0$, then infinitely many solution.

III. Solution of linear system of equations by Gauss elimination method:

1. Solve the following system of equations by Gauss elimination method

a) $\begin{matrix} 2x+y+z=10 \\ 3x+2y+3z=18 \\ x+4y+9z=16 \end{matrix}$.
Ans: $x=7, y=-9, z=5$

b) $\begin{matrix} 2x+2y+z=12 \\ 3x+2y+2z=8 \\ 5x+10y-8z=10 \end{matrix}$.
Ans: $x=\frac{-51}{4}, y=\frac{115}{8}, z=\frac{35}{4}$

c) $\begin{matrix} 2x_1+4x_2+x_3=3 \\ 3x_1+2x_2-2x_3=-2 \\ x_1-x_2+x_3=6 \end{matrix}$.
Ans: $x_1=2,\ x_2=-1,\ x_3=3.$

d) $\begin{matrix} 10x+2y+z=9 \\ 2x+20y-2z=-44 \\ -2x+3y+10z=22 \end{matrix}$.
Ans: $x=1, y=-2, z=3$

e) $\begin{matrix} 2x_1+x_2+4x_3=12 \\ 8x_1-3x_2+2x_3=20 \\ 4x_1+11x_2-x_3=33 \end{matrix}$.
Ans: $x_1=3,\ x_2=2,\ x_3=1.$

f) $\begin{matrix} x_1+4x_2-x_3=-5 \\ x_1+x_2-6x_3=-12 \\ 3x_1-x_2-x_3=4 \end{matrix}$.
Ans: $x_1=\frac{117}{71},\ x_2=-\frac{81}{71},\ x_3=\frac{148}{71}.$

g) $2x_1 + x_2 + 3x_3 = 1$
$4x_1 + 4x_2 + 7x_3 = 1$
$2x_1 + 5x_2 + 9x_3 = 3$.
Ans: $x_1 = -1/2,\ x_2 = -1,\ x_3 = 1.$

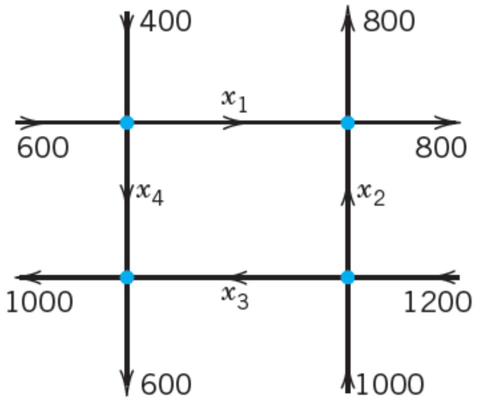
h) $2x_1 - 7x_2 + 4x_3 = 9$
$x_1 + 9x_2 - 6x_3 = 1$
$-3x_1 + 8x_2 + 5x_3 = 6$.
Ans: $x_1 = 4, x_2 = 1, x_3 = 2$

i) $2x_1 + x_2 + x_3 = 10$
$3x_1 + 2x_2 + 3x_3 = 18$
$x_1 + 4x_2 + 9x_3 = 16$.
Ans: $x_1 = 7,\ x_2 = -9,\ x_3 = 5.$

j) $5x_1 + x_2 + x_3 + x_4 = 4$
$x_1 + 7x_2 + x_3 + x_4 = 12$
$x_1 + x_2 + 6x_3 + x_4 = -5$
$x_1 + x_2 + x_3 + 4x_4 = -6$.
Ans: $x_1 =$, $x_2 =$,
$x_3 =$, $x_4 =$.

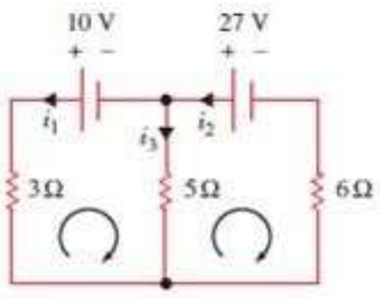
k) $2x_1 + 2x_2 + x_3 + 2x_4 = 7$
$-x_1 + 2x_2 + x_4 = -2$
$-3x_1 + x_2 + 2x_3 + x_4 = -3$
$-x_1 + 2x_4 = 0$
Ans: $\frac{80}{37}, -\frac{17}{37}, \frac{53}{37}, \frac{40}{37}$

2. Balance the chemical equation $x_1C_3H_8 + x_2O_2 \rightarrow x_3CO_2 + x_4H_2O$ finding x_1, x_2, x_3, x_4 using Gauss elimination method
3. Find the traffic flow in the net of one-way streets directions shown in the figure



4. Using Kirchhoff's voltage and current law, find the system of equations in current from the following circuit and then find the current using Gauss elimination method:1)

i)



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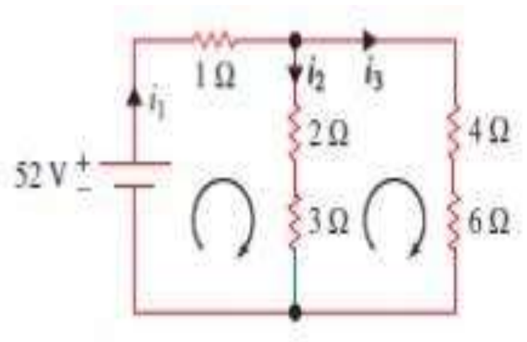

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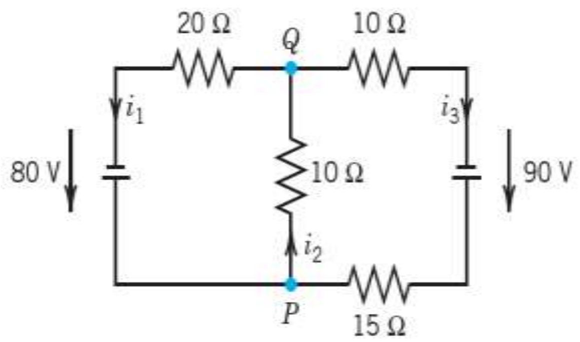

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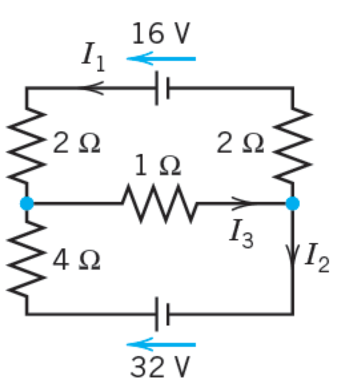

IV. **Gauss-Seidel Iteration method**

The solution converges if the system is diagonally dominant.

Suppose $AX = B$ is diagonally dominant with $A = (a_{ij})$, $X = (x_i)$ and $B = (b_i)$.

Then the iterative formula is

$$x_1^{n+1} = \frac{1}{a_{11}}\left(b_1 - a_{12}x_2^n - a_{13}x_3^n \cdots\cdots\right)$$
$$x_2^{n+1} = \frac{1}{a_{22}}\left(b_2 - a_{21}x_1^{n+1} - a_{23}x_3^n \cdots\cdots\right)$$
$$x_3^{n+1} = \frac{1}{a_{33}}\left(b_3 - a_{31}x_1^{n+1} - a_{32}x_2^{n+1} \cdots\cdots\right)$$
$$\vdots$$

a. $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.
Ans: $x = 1,\ y = -1,\ z = 1$

b. $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$.
Ans: $x = 0.996,\ y = 2,\ z = 3$

c. $2x + y + 6z = 9$, $8x + 3y + 2z = 13$, $x + 5y + z = 7$.
Ans: $x = 1,\ y = 1,\ z = 1$

d. $28x + 4y - z = 32$, $x + 3y + 10z = 24$, $2x + 17y + 4z = 35$.
Ans: $x = 0.998,\ y = 1.723,\ z = 2.024$

e. $10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$.
Ans: $x = 1,\ y = -2,\ z = 3$

f. $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$, $3x + 8y + 29z = 71$.
Ans: $x = 1.06, y = 1.37, z = 1.96$

g. $54x + y + z = 110$, $2x + 15y + 6z = 72$, $-x + 6y + 27z = 85$.
Ans: $x = 1.926,\ y = 3.573,\ z = 2.425$

h. $5x_1 - x_2 = 9$, $-x_1 + 5x_2 - x_3 = 4$, $-x_2 + 5x_3 = -6$.
Ans: $x_1 = 1.99,\ x_2 = 0.99,\ x_3 = -1$

i. $8x_1 + x_2 - x_3 = 8$, $2x_1 + x_2 + 9x_3 = 12$, $x_1 - 7x_2 + 2x_3 = -4$.
Ans: $x_1 = 1\ x_2 = 1,\ x_3 = 1$

j. $4x_1 + 2x_2 + x_3 = 11$, $-x_1 + 2x_2 = 3$, $2x_1 + x_2 + 4x_3 = 16$.
Ans: $x_1 = 1,\ x_2 = 2, x_3 = 3$

k. Start with $(2, 2, -1)$ and solve
$5x_1 - x_2 + x_3 = 10$
$2x_1 + 4x_2 = 12$
$x_1 + x_2 + 5x_3 = -1$
$Ans: x_1 = 2.5555, x_2 = 1.7222,\ x_3 = -1.0555$

l. $10x_1 + x_2 + x_3 = 12$, $2x_1 + 10x_2 + x_3 = 13$, $2x_1 + 2x_2 + 10x_3 = 14$.
Ans: $x_1 = x_2 = x_3 = 1$

m. $27x_1 + 6x_2 - x_3 = 85$, $6x_1 + 15x_2 + 2x_3 = 72$, $x_1 + x_2 + 54x_3 = 110$.
Ans: $x_1 = 2.4255,\ x_2 = 3.573,\ x_3 = 1.926$

n. $x_1 - 8x_2 + 3x_3 = -4$, $2x_1 + x_2 + 9x_3 = 12$, $8x_1 + 2x_2 - 2x_3 = 8$.
$Ans: x_1 = x_2 = x_3 = 1$

o. $\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$.
Ans: $x = 1,\ y = 2,\ z = 3, u = 4$

V. Characteristic values (Eigen values) and characteristic vectors (Eigen vectors)

If A is a square matrix, then λ is said to be an eigen value of the matrix if there exists a non-zero vector X such that $AX = \lambda X$. X is called the eigen vector corresponding to the eigen value λ .

$\lambda X = \lambda IX \Rightarrow (A - \lambda I)X = 0$. We seek non-trivial solution of $(A - \lambda I)X = 0$.

X is non-trivial if $\rho(A - \lambda I) < n \Rightarrow |A - \lambda I| = 0$.

If A is matrix of size 3×3 then $-\lambda^3 + Tr(A)\lambda^2 - \sum M_{ii}^{2\times 2}\lambda + |A| = 0$.

a. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Ans: $\lambda = -2, 3, 6; x_1 = [-k, 0, k],$.
$x_2 = [k, -k, k],\ x_3 = [k, 2k, k]$

b. $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

Ans: $\lambda = 2, 3, 5\ \ x_1 = k_1[1, -1, 0]$
$x_2 = k_2[1, 0, 0],\ x_3 = k_3[3, 2, 1]$

c. $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Ans: $\lambda = 0, 3, 15\ \ x_1 = [1, 2, 2],$
$x_2 = [2, 1, -2], x_3 = [2, -2, 1]$

d. $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

Ans: $\lambda = 1, 2, 3\ \ x_1 = [1, 0, -1]$
$x_2 = [0, 1, 0], x_3 = [1, 0, 1]$

e. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Ans: $\lambda = 5, -3, -3\ \ x_1 = k[1, 2, -1]$.
$x_2 = [3k_1 - 2k_2, k_1, k_2]$

f. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Ans: $\lambda = 8, 2, 2;\ x_1 = [2, -1, 1],$.
$x_2 = [1, 0, -2], x_3 = [1, 2, 0]$

g. $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$

Ans: $\lambda = 1, -1,\ 4;\ x_1 = [2, -1, 1],$.
$x_2 = [0, 1, 1], x_3 = [-1, -1, 1]$

h. $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Ans: $\lambda = 1, 2, 2;\ x_1 = k_1[1, 1, -1],$.
$x_2 = k_2[2, 1, 0], x_3 = k_3[2, 1, 0]$

i. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Ans: $\lambda = -2, 3, 6; x_1 = [-1, 0, 1],$.
$x_2 = [1, -1, 1], x_3 = [1, 2, 1]$

j. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Ans: $\lambda = 5, 1, 1;\ x_1 = [1, 1, 1],$.
$x_2 = [1, 0, -1], x_3 = [2, -1, 0]$

k. $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

Ans: $\lambda = 2, 2, 3$.
$x_1 = x_2 = [5, 2, -3], x_3 = [1, 1, -2]$

l. $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

Ans: $\lambda = 2, 2, -2$.
$x_1 = x_2 = [0, 1, 1]\ \ x_3 = [-4, -1, 1]$

m. $\begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$

Ans: $\lambda = -5, 2, 2$.

$x_1 = [3, 2, 4],\ x_2 = x_3 = [1, 3, -1]$

VI. Determine the largest (dominant) Eigen value and the corresponding Eigen vector of the following matrices using Rayleigh's Power Method

1. $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

   $Ans: \lambda = 3.41$

2. $A = \begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}$.

   $Ans: \lambda = 4.418$

3. $\begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$

   Ans: $\lambda = -5$

4. $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$.

   $Ans: \lambda = 6$

5. $\begin{bmatrix} -8 & 6 & -2 \\ 6 & -7 & 4 \\ -2 & 4 & -3 \end{bmatrix}$

   $\text{Ans}: \lambda = -15$

6. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

   $Ans: \lambda = 8$

7. $\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

   $Ans: \lambda = 4$

8. $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$.

   $Ans: \lambda = 25.182$

9. $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$

   $Ans: \lambda = 11$

SELF STUDY:

VII. Solution of system of equations by Gauss Jacobi iterative Method

a. $2x + y + 6z = 9$
$8x + 3y + 2z = 13$
$x + 5y + z = 7$.
Ans: $x = 1,\ y = 1,\ z = 1$

b. $28x + 4y - z = 32$
$x + 3y + 10z = 24$
$2x + 17y + 4z = 35$
Ans: $x = 0.998,\ y = 1.723,\ z = 2.024$
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c. $10x + 2y + z = 9$
$2x + 20y - 2z = -44$
$-2x + 3y + 10z = 22$.
Ans: $x = 1,\ y = -2,\ z = 3$

d. $83x + 11y - 4z = 95$
$7x + 52y + 13z = 104$
$3x + 8y + 29z = 71$.
Ans: $x = 1.06, y = 1.37, z = 1.96$

e. $$\begin{aligned}54x+y+z&=110\\2x+15y+6z&=72\\-x+6y+27z&=85\end{aligned}$$
Ans: $x=1.926,\ y=3.573,\ z=2.425$

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f. $$\begin{aligned}5x_1-x_2&=9\\-x_1+5x_2-x_3&=4\\-x_2+5x_3&=-6\end{aligned}$$
Ans: $x_1=1.99,\ x_2=0.99,\ x_3=-1$

.

g. $$\begin{aligned}8x_1+x_2-x_3&=8\\2x_1+x_2+9x_3&=12\\x_1-7x_2+2x_3&=-4\end{aligned}$$.
Ans: $x_1=1\ x_2=1,\ x_3=1$

$$\begin{aligned}4x_1+2x_2+x_3&=11\\-x_1+2x_2&=3\\2x_1+x_2+4x_3&=16\end{aligned}$$
Ans: $x_1=1,\ x_2=2, x_3=3$

VIII. Using Cayley-Hamilton Theorem find the inverse of the matrix

1. $\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$.

2. $\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$.

3. $\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$.

4. $\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$.

5. $\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix}$.

6. $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$

7. $\begin{bmatrix}2&1&-1\\1&1&-2\\-1&-2&1\end{bmatrix}$