# **BJT AC Analysis**

#### **AMPLIFICATION IN THE AC DOMAIN**

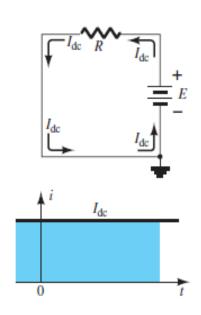
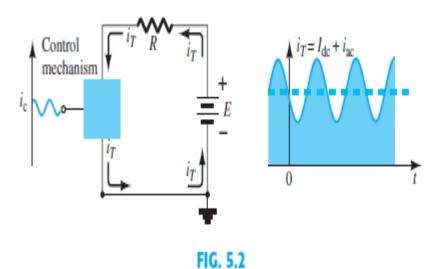


FIG. 5.1 Steady current established by a dc supply.



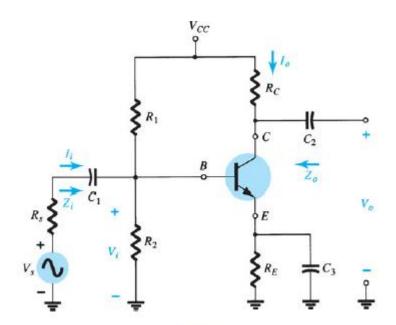
Effect of a control element on the steady-state flow of the electrical system of Fig. 5.1.

$$i_{ac(p-p)} \gg i_{c(p-p)}$$

The superposition theorem is applicable for the analysis and design of the dc and ac components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

# **BJT TRANSISTOR MODELING**

A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.



Transistor circuit under examination in this introductory discussion and Boylstead

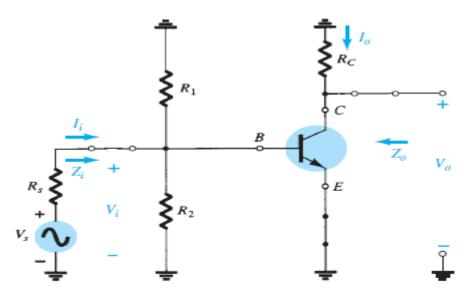


FIG. 5.4

The network of Fig. 5.3 following removal of the dc supply and insertion of the short-circuit equivalent for the capacitors.

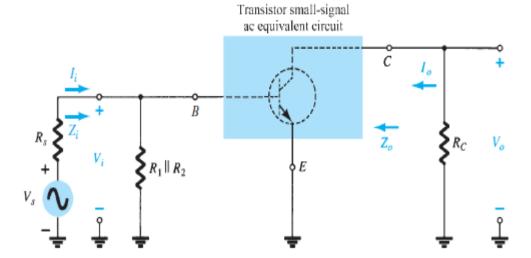


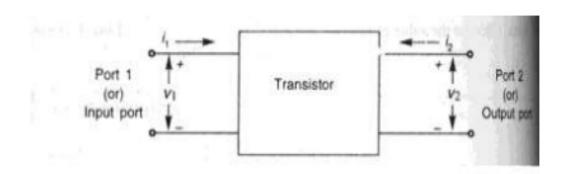
FIG. 5.7

Circuit of Fig. 5.4 redrawn for small signal ice analysis ircuits Nashelsky and Boylstead

- 1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
- 2. Replacing all capacitors by a short-circuit equivalent
- 3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
- 4. Redrawing the network in a more convenient and logical form

#### Two port devices & Network Parameters:-

A transistor can be treated as a two part network. The terminal behavior of any two part network can be specified by the terminal voltages V<sub>1</sub> & V<sub>2</sub> at parts 1 & 2 respectively and current i<sub>1</sub> and i<sub>2</sub>, entering parts 1 & 2, respectively, as shown in figure.



## Two port network

Of these four variables V<sub>1</sub>, V<sub>2</sub>, i<sub>1</sub> and i<sub>2</sub>, two can be selected as independent variables and the remaining two can be expressed in terms of these independent variables. This leads to various two part parameters out of which the following three are more important.

# <u>Hybrid parameters (or) h - parameters</u>:-

 $\rightarrow$  If the input current i<sub>1</sub> and output Voltage V<sub>2</sub> are takes as independent variables, the input voltage V<sub>1</sub> and output current i<sub>2</sub> can be written as

$$V_1 = h_{11} i_1 + h_{12} V_2$$
  
 $i_2 = h_{21} i_1 + h_{22} V_2$ 

The four hybrid parameters h<sub>11</sub>, h<sub>12</sub>, h<sub>21</sub> and h<sub>22</sub> are defined as follows.

$$h_{11} = [V_1 / i_1]$$
 with  $V_2 = 0$ 

= Input Impedance with output part short circuited.

$$h_{22} = [i_2 / V_2]$$
 with  $i_1 = 0$ 

= Output admittance with input part open circuited.

$$h_{12} = [V_1 / V_2]$$
 with  $i_1 = 0$ 

= reverse voltage transfer ratio with input part open circuited.

$$h_{21} = [i_2 / i_1]$$
 with  $V_2 = 0$ 

= Forward current gain with output part short circuited.

# The dimensions of h - parameters are as follows:

$$h_{11}$$
 -  $\Omega$   
 $h_{22}$  - mhos  
 $h_{12}$ ,  $h_{21}$  - dimension less.

→ as the dimensions are not alike, (ie) they are hybrid in nature, and these parameters are called as hybrid parameters.

$$I = 11 = input$$
;  $0 = 22 = output$ ;  $F = 21 = forward transfer$ ;  $r = 12 = Reverse transfer$ .

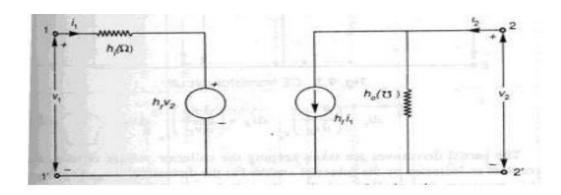
# Notations used in transistor circuits:-

 $h_{ie} = h_{11e} = Short circuit input impedance$ 

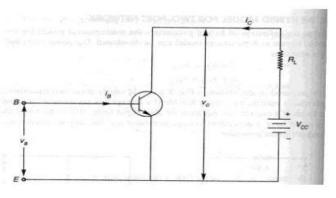
 $h_{0e} = h_{22e} = Open$  circuit output admittance

 $h_{re} = h_{12e} = Open$  circuit reverse voltage transfer ratio

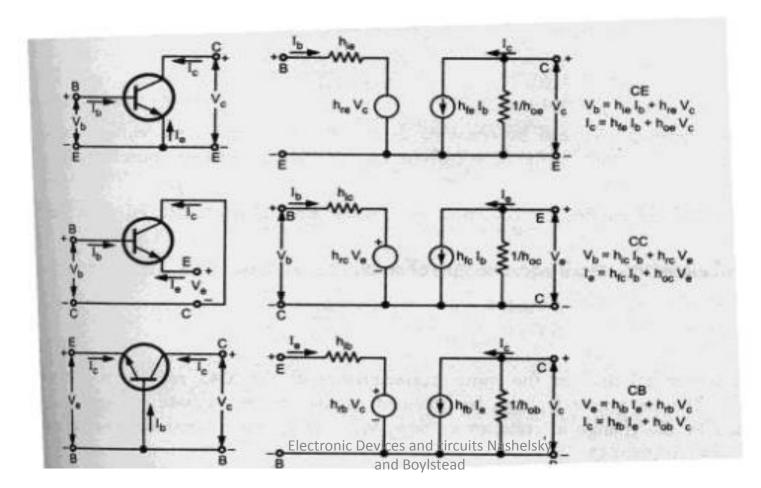
 $h_{fe} = h_{21e} = Short circuit forward current Gain.$ 



#### The Hybrid Model for Two-port Network



CE Transistor Circuit



# THE $r_e$ TRANSISTOR MODEL

# **Common-Emitter Configuration**

The equivalent circuit for the common-emitter configuration will be constructed using the device characteristics and a number of approximations. Starting with the input side, we find the applied voltage  $V_i$  is equal to the voltage  $V_{be}$  with the input current being the base current  $I_b$  as shown in Fig. 5.8.

Recall from Chapter 3 that because the current through the forward-biased junction of the transistor is  $I_E$ , the characteristics for the input side appear as shown in Fig. 5.9a for various levels of  $V_{BE}$ . Taking the average value for the curves of Fig. 5.9a will result in the single curve of Fig. 5.9b, which is simply that of a forward-biased diode.

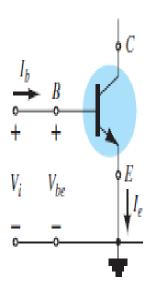


FIG. 5.8

Finding the input equivalent circuit for a BJT transistor.

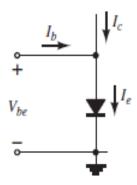
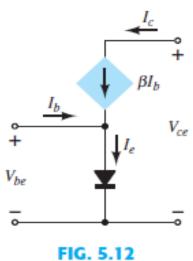


FIG. 5.10

Equivalent circuit for the input side of a BJT transistor.



BJT equivalent circuit.

$$r_D = 26 \text{ mV/}I_D$$
.

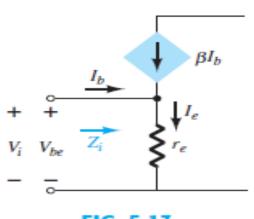


FIG. 5.13
Defining the level of Z<sub>i</sub>.

$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

$$V_{be} = I_e r_e = (I_c + I_b) r_e = (\beta I_b + I_b) r_e$$

$$= (\beta + 1) I_b r_e$$

$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta + 1) I_b r_e}{I_b}$$

 $Z_i = (\beta + 1)r_e \cong \beta r_e$ 

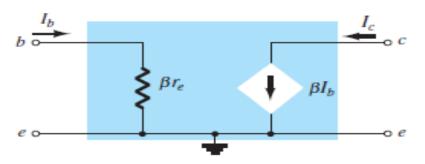
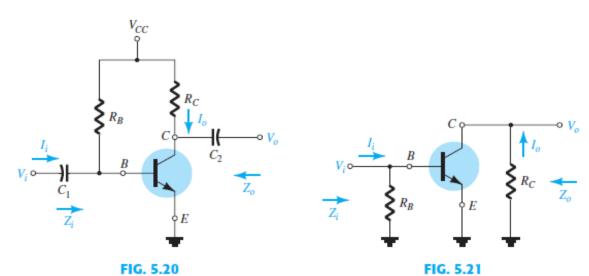


FIG. 5.14
Improved BJT equivalent circuit.

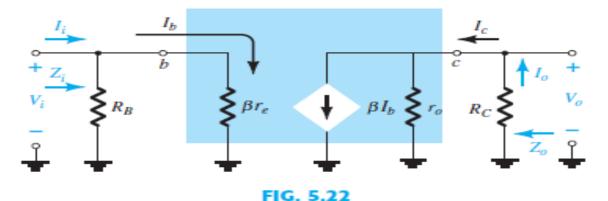
# COMMON-EMITTER FIXED-BIAS CONFIGURATION



Common-emitter fixed-bias configuration.

Network of Fig. 5.20 following the removal of the effects of  $V_{CC}$ ,  $C_1$ , and  $C_2$ .

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Substituting the  $r_e$  model into the network of Fig. 5.21.

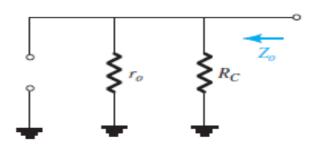
Z<sub>i</sub> Figure 5.22 clearly shows that

$$Z_i = R_B \| \beta r_e \|$$
 ohms

$$Z_i \cong \beta r_\ell$$
 ohms  $R_B \ge 10\beta r_\epsilon$ 

**Z<sub>0</sub>** Recall that the output impedance of any system is defined as the impedance  $Z_0$  determined when  $V_i = 0$ . For Fig. 5.22, when  $V_i = 0$ ,  $I_i = I_b = 0$ , resulting in an open-circuit equivalence for the current source. The result is the configuration of Fig. 5.23. We have

$$Z_o = R_C \| r_o$$
 ohms
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 $Z_o = R_C \| r_o \|$  ohms

FIG. 5.23

Determining  $Z_o$  for the network of Fig. 5.22.

If  $r_o \ge 10R_C$ , the approximation  $R_C || r_o \cong R_C$  is frequently applied, and

$$Z_0 \cong R_C$$

$$r_0 \ge 10R_C$$

 $A_v$  The resistors  $r_o$  and  $R_C$  are in parallel, and

$$V_o = -\beta I_b(R_C || r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

but

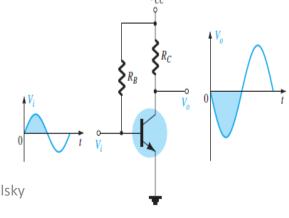
so that

 $V_o = -\beta \left(\frac{V_i}{\beta r_e}\right) (R_C || r_o)$ 

and

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C \| r_o)}{r_e}$$

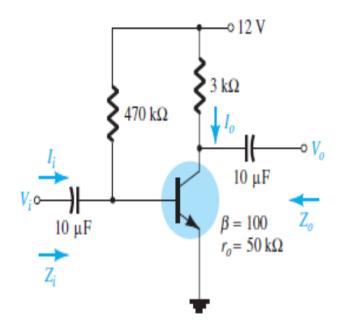
If  $r_o \ge 10R_C$ , so that the effect of  $r_o$  can be ignored,



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### For the network of Fig. :

- a. Determine  $r_e$ .
- b. Find  $Z_i$  (with  $r_o = \infty \Omega$ ).
- c. Calculate  $Z_o$  (with  $r_o = \infty \Omega$ ).
- d. Determine  $A_v$  (with  $r_o = \infty \Omega$ ).
- e. Repeat parts (c) and (d) including  $r_o = 50 \, \mathrm{k}\Omega$  in all calculations and compare results.



a. DC analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 \text{ V} - 0.7 \text{ V}}{470 \text{ k}\Omega} = 24.04 \,\mu\text{A}$$
 $I_E = (\beta + 1)I_B = (101)(24.04 \,\mu\text{A}) = 2.428 \,\text{mA}$ 
 $r_e = \frac{26 \,\text{mV}}{I_E} = \frac{26 \,\text{mV}}{2.428 \,\text{mA}} = 10.71 \,\Omega$ 

- b.  $\beta r_e = (100)(10.71 \ \Omega) = 1.071 \ k\Omega$  $Z_i = R_B \|\beta r_e = 470 \ k\Omega \|1.071 \ k\Omega = 1.07 \ k\Omega$
- c.  $Z_o = R_C = 3 \text{ k}\Omega$

d. 
$$A_v = -\frac{R_C}{r_e} = -\frac{3 \text{ k}\Omega}{10.71 \Omega} = -280.11$$

e. 
$$Z_o = r_o \| R_C = 50 \text{ k}\Omega \| 3 \text{ k}\Omega = 2.83 \text{ k}\Omega \text{ vs. } 3 \text{ k}\Omega$$
  
 $A_v = -\frac{r_o \| R_C}{r_e} = \frac{2.83 \text{ k}\Omega}{10.71 \Omega} = -264.24 \text{ vs. } -280.11$ 

# **VOLTAGE-DIVIDER BIAS**

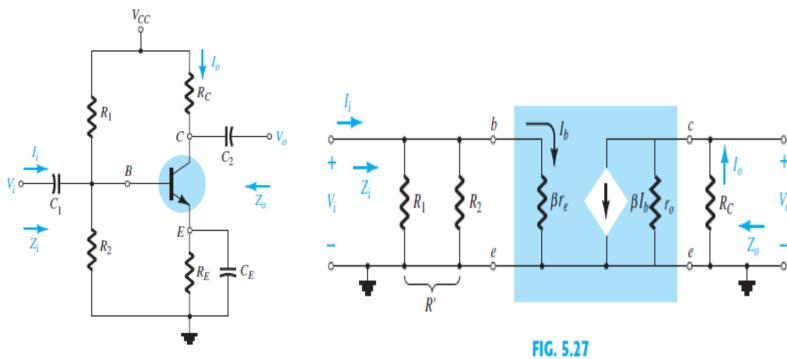


FIG. 5.26 Voltage-divider bias configuration.

Substituting the  $r_e$  equivalent circuit into the ac equivalent network of Fig. 5.26.

$$R' = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

**Z**<sub>i</sub> From Fig. 5.27

$$Z_i = R' \|\beta r_e$$

**Z<sub>0</sub>** From Fig. 5.27 with  $V_i$  set to 0 V, resulting in  $I_b = 0 \,\mu\text{A}$  and  $\beta I_b = 0 \,\text{mA}$ ,

$$Z_o = R_C \| r_o \tag{5.13}$$

If  $r_o \geq 10R_C$ ,

$$Z_o \cong R_C \qquad (5.14)$$

 $A_V$  Because  $R_C$  and  $r_o$  are in parallel,

$$V_o = -(\beta I_b)(R_C \| r_o)$$

$$I_b = \frac{V_i}{\beta r_e}$$

so that

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right) (R_C || r_o)$$

and

$$A_{v} = \frac{V_{o}}{V_{i}} = \frac{-R_{C} \| r_{o}}{r_{e}}$$
 (5.15)

which you will note is an exact duplicate of the equation obtained for the fixed-bias configuration.

For  $r_o \geq 10R_C$ ,

$$A_{v} = \frac{V_{o}}{V_{i}} \cong -\frac{R_{C}}{r_{e}}$$

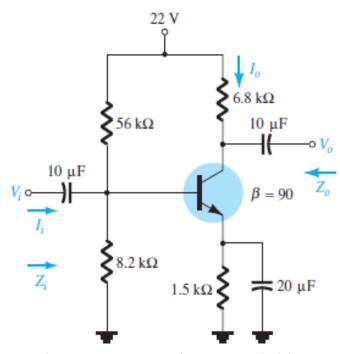
$$r_{o} \geq 10R_{C}$$

$$(5.16)$$

**Phase Relationship** The negative sign of Eq. (5.15) reveals a 180° phase shift between  $V_o$  and  $V_i$ .

### For the network of Fig. :

- a.  $r_e$ .
- b.  $Z_i$ .
- c.  $Z_o(r_o = \infty \Omega)$ . d.  $A_v(r_o = \infty \Omega)$ .
- e. The parameters of parts (b) through (d) if  $r_o = 50 \, \mathrm{k}\Omega$  and compare results.



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a. DC: Testing  $\beta R_E > 10R_2$ ,

$$(90)(1.5 \text{ k}\Omega) > 10(8.2 \text{ k}\Omega)$$
  
 $135 \text{ k}\Omega > 82 \text{ k}\Omega \text{ (satisfied)}$ 

Using the approximate approach, we obtain

$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{(8.2 \text{ k}\Omega)(22 \text{ V})}{56 \text{ k}\Omega + 8.2 \text{ k}\Omega} = 2.81 \text{ V}$$

$$V_E = V_B - V_{BE} = 2.81 \text{ V} - 0.7 \text{ V} = 2.11 \text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11 \text{ V}}{1.5 \text{ k}\Omega} = 1.41 \text{ mA}$$

$$r_e = \frac{26 \text{ mV}}{I_E} = \frac{26 \text{ mV}}{1.41 \text{ mA}} = 18.44 \Omega$$

b. 
$$R' = R_1 \| R_2 = (56 \text{ k}\Omega) \| (8.2 \text{ k}\Omega) = 7.15 \text{ k}\Omega$$
  
 $Z_i = R' \| \beta r_e = 7.15 \text{ k}\Omega \| (90)(18.44 \Omega) = 7.15 \text{ k}\Omega \| 1.66 \text{ k}\Omega$   
 $= 1.35 \text{ k}\Omega$ 

c. 
$$Z_o = R_C = 6.8 \text{ k}\Omega$$

d. 
$$A_v = -\frac{R_C}{r_e} = -\frac{6.8 \text{ k}\Omega}{18.44 \Omega} = -368.76$$

e. 
$$Z_i = 1.35 \text{ k}\Omega$$

$$Z_o = R_C \| r_o = 6.8 \,\mathrm{k}\Omega \| 50 \,\mathrm{k}\Omega = 5.98 \,\mathrm{k}\Omega \,\mathrm{vs.} \,6.8 \,\mathrm{k}\Omega$$

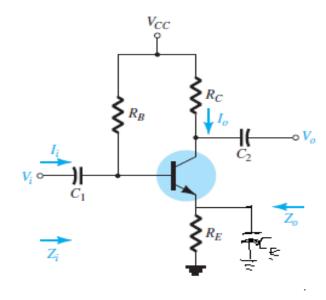
$$A_v = -\frac{R_C \| r_o}{r_e} = -\frac{5.98 \text{ k}\Omega}{18.44 \Omega} = -324.3 \text{ vs.} -368.76$$

There was a measurable difference in the results for  $Z_0$  and  $A_v$ , because the condition  $r_0 \ge 10R_C$  was *not* satisfied.

#### **CE EMITTER-BIAS CONFIGURATION**

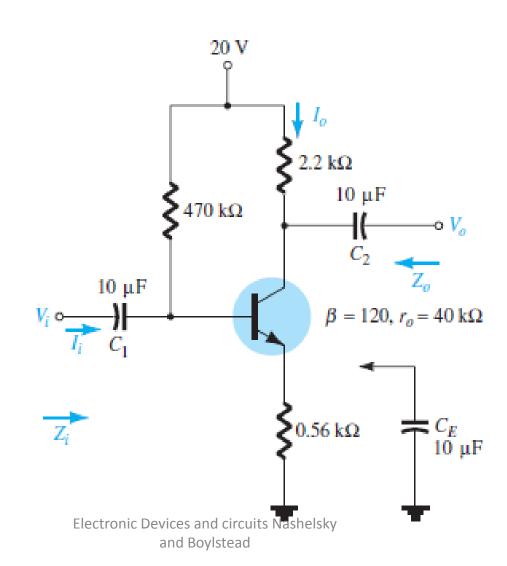
# **Bypassed**

If  $R_E$  of Fig. 5.29 is bypassed by an emitter capacitor  $C_E$ , the complete  $r_e$  equivalent model can be substituted, resulting in the same equivalent network as Fig. 5.22. Equations (5.5) to (5.10) are therefore applicable.



### For the network of Fig. :

- a.  $r_e$ .
- b.  $Z_i$ .
- c. Z<sub>o</sub>.
- d.  $A_{v}$ .



a. DC:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20 \text{ V} - 0.7 \text{ V}}{470 \text{ k}\Omega + (121)0.56 \text{ k}\Omega} = 35.89 \,\mu\text{A}$$

$$I_E = (\beta + 1)I_B = (121)(35.89 \,\mu\text{A}) = 4.34 \,\text{mA}$$
and 
$$r_e = \frac{26 \,\text{mV}}{I_E} = \frac{26 \,\text{mV}}{4.34 \,\text{mA}} = 5.99 \,\Omega$$

b.  $R_E$  is "shorted out" by  $C_E$  for the ac analysis. Therefore,

$$Z_i = R_B \| Z_b = R_B \| \beta r_e = 470 \text{ k}\Omega \| (120)(5.99 \Omega)$$
  
= 470 k\Omega \| 718.8 \Omega \approx 717.70 \Omega

c. 
$$Z_o = R_C = 2.2 \text{ k}\Omega$$

d. 
$$A_v = -\frac{R_C}{r_e}$$

$$= -\frac{2.2 \text{ k}\Omega}{5.99 \Omega} = -367.28 \text{ (a significant increase)}$$