## BJT AC Analysis

## AMPLIFICATION IN THE AC DOMAIN



FIG. 5.1
Steady current established by a dc supply.


FIG. 5.2
Effect of a control element on the steady-state flow of the electrical system of Fig. 5.1.

$$
i_{\operatorname{ac}(p-p)} \gg i_{c(p-p)}
$$

The superposition theorem is applicable for the analysis and design of the dc and ac components of a BJT network, permitting the separation of the analysis of the dc and ac responses of the system.

## BJT TRANSISTOR MODELING

A model is a combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device under specific operating conditions.


FIG. 5.3
Transistor circuit under Exanichaxion in this indrorcuuits Nashelsky
and Boylstead


FIG. 5.4
The network of Fig. 5.3 following removal of the dc supply and insertion of the short-circuit equivalent for the capacitors.


FIG. 5.7


1. Setting all dc sources to zero and replacing them by a short-circuit equivalent
2. Replacing all capacitors by a short-circuit equivalent
3. Removing all elements bypassed by the short-circuit equivalents introduced by steps 1 and 2
4. Redrawing the network in a more convenient and logical form

## Two port devices \& Network Parameters:-

A transistor can be treated as a two part network. The terminal behavior of any two part network can be specified by the terminal voltages $\mathrm{V}_{1} \& \mathrm{~V}_{2}$ at parts 1 \& 2 respectively and current $\mathrm{i}_{1}$ and $\mathrm{i}_{2}$, entering parts $1 \& 2$, respectively, as shown in figure.


## Two port network

Of these four variables $\mathrm{V}_{1}, \mathrm{~V}_{2}$, i1 and 12 , two can be selected as independent variables and the remaining two can be expressed in terms of these independent variables. This leads to various two part parameters out of which the following three are more important.

## Hybrid parameters (or) h - parameters:-

$\rightarrow$ If the input current $\mathrm{i}_{1}$ and output Voltage $\mathrm{V}_{2}$ are takes as independent variables, the input voltage $\mathrm{V}_{1}$ and output current $\mathrm{i}_{2}$ can be written as

$$
\begin{aligned}
& V_{1}=h_{11} i_{1}+h_{12} V_{2} \\
& i_{2}=h_{21} i_{1}+h_{22} V_{2}
\end{aligned}
$$

The four hybrid parameters $h_{11}, h_{12}, h_{21}$ and $h_{22}$ are defined as follows.

$$
\begin{aligned}
\mathrm{h}_{11} & =\left[\mathrm{V}_{1} / \mathrm{i}_{1}\right] \text { with } \mathrm{V}_{2}=0 \\
& =\text { Input Impedance with output part short circuited. } \\
\mathrm{h}_{22} & =\left[\mathrm{i}_{2} / \mathrm{V}_{2}\right] \text { with } \mathrm{i}_{1}=0
\end{aligned}
$$

$=$ Output admittance with input part open circuited.

$$
\mathrm{h}_{12}=\left[\mathrm{V}_{1} / \mathrm{V}_{2}\right] \text { with } \mathrm{i}_{1}=0
$$

$=$ reverse voltage transfer ratio with input part open circuited.
$\mathrm{h}_{21}=\left[\mathrm{i}_{2} / \mathrm{i}\right]$ with $\mathrm{V}_{2}=0$
$=$ Forward current gain with output part short circuited.

## The dimensions of $h$ - parameters are as follows:

$h_{11}-\Omega$
h22-mhos
h12, h21-dimension less.
$\rightarrow$ as the dimensions are not alike, (ie) they are hybrid in nature, and these parameters are called as hybrid parameters.
$\mathrm{I}=11=$ input ; $0=22=$ output ;
$\mathrm{F}=21=$ forward transfer ; $\mathrm{r}=12=$ Reverse transfer.

## Notations used in transistor circuits:-

$\mathrm{h}_{\mathrm{ie}}=\mathrm{h}_{1 l_{\mathrm{e}}}=$ Short circuit input impedance
$h_{0_{\mathrm{e}}}=\mathrm{h}_{22_{\mathrm{e}}}=$ Open circuit output admittance
$h_{\text {re }}=h_{12 \mathrm{e}}=$ Open circuit reverse voltage transfer ratio
$h_{f e}=h_{2 l e}=$ Short circuit forward current Gain.


The Hybrid Model for Two-port Network


CE Transistor Circuit


## THE $r_{e}$ TRANSISTOR MODEL

## Common-Emitter Configuration

The equivalent circuit for the common-emitter configuration will be constructed using the device characteristics and a number of approximations. Starting with the input side, we find the applied voltage $V_{i}$ is equal to the voltage $V_{b e}$ with the input current being the base current $l_{b}$ as shown in Figg 5.8 .
Recall from Chapter 3 that because the current through the forward-biased junction of the transistor is $I_{E}$, the characteristics for the input side appear as shown in Fig. 5.9 for various levels of $V_{B E}$. Taking the average value for the curves of Fig. 5.9 w will result in the single curve of Fig. 5.5 , which is simply that of a forward-biased diode.


FIG. 5.8
Finding the input equivalent circuit for a BIT transistor.


FIG. 5.10
Equivalent circuit for the input side of a BJT transistor.


FIG. 5.12

$$
r_{D}=26 \mathrm{mV} / I_{D}
$$

BJT equivalent circuit.


FIG. 5.13

$$
\begin{aligned}
Z_{i} & =\frac{V_{i}}{I_{b}}=\frac{V_{b e}}{I_{b}} \\
V_{b e} & =I_{e_{e} r_{e}}=\left(I_{c}+I_{b}\right) r_{e}=\left(\beta I_{b}+I_{b}\right) r_{e} \\
& =(\beta+1) l_{b_{e}} r_{e} \\
Z_{i} & =\frac{V_{b e}}{I_{b}}=\frac{(\beta+1) l_{b} r_{e}}{I_{b}}
\end{aligned}
$$

$$
Z_{i}=(\beta+1) r_{e} \cong \beta r_{e}
$$

Defining the level of $Z_{i}$.


FIG. 5.14
Improved BJT equivalent circuit.

## COMMON-EMITTER FIXED-BIAS <br> CONFIGURATION



FIG. 5.20
Common-emitter fixed-bias configuration.


FIG. 5.21
Network of Fig. 5.20 following the removal of the effects of $V_{C C}, C_{1}$, and $C_{2}$.

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FIG. 5.22
Substituting the $r_{e}$ model into the network of Fig. 5.21 .
$Z_{i}$ Figure 5.22 clearly shows that

$$
Z_{i}=R_{B} \| \beta r_{e} \quad \text { ohms }
$$

$$
Z_{i} \cong \beta r_{\ell}
$$

ohms
$Z_{0}$ Recall that the output impedance of any system is defined as the impedance $Z_{o}$ determined when $V_{i}=0$. For Fig. 5.22 , when $V_{i}=0, I_{i}=I_{b}=0$, resulting in an opencircuit equivalence for the current source. The result is the configuration of Fig. 5.23. We have

$$
\begin{equation*}
Z_{o}=R_{C} \| r_{o} \quad \text { ohms } \tag{5.7}
\end{equation*}
$$



$$
Z_{0}=R_{C} \| r_{0} \quad \text { ohms }
$$

FIG. 5.23
Determining $Z_{o}$ for the network of Fig. 5.22.

If $r_{o} \geq 10 R_{C}$, the approximation $R_{C} \| r_{o} \cong R_{C}$ is frequently applied, and

$$
Z_{o} \cong R_{C}
$$

$$
r_{o} \geq 10 R_{C}
$$

$A_{\mathbf{V}}$ The resistors $r_{o}$ and $R_{C}$ are in parallel, and

$$
V_{o}=-\beta I_{b}\left(R_{C} \| r_{o}\right)
$$

but

$$
I_{b}=\frac{V_{i}}{\beta r_{e}}
$$

so that

$$
V_{o}=-\beta\left(\frac{V_{i}}{\beta r_{e}}\right)\left(R_{C} \| r_{o}\right)
$$

and

$$
A_{v}=\frac{V_{o}}{V_{i}}=-\frac{\left(R_{C} \| r_{o}\right)}{r_{e}}
$$

If $r_{o} \geq 10 R_{C}$, so that the effect of $r_{o}$ can be ignored,


[^0]

## For the network of Fig. :

a. Determine $r_{e}$.
b. Find $Z_{i}$ (with $r_{o}=\infty \Omega$ ).
c. Calculate $\mathrm{Z}_{0}\left(\right.$ with $\left.r_{o}=\infty \Omega\right)$.
d. Determine $A_{v}$ (with $r_{o}=\infty \Omega$ ).
e. Repeat parts (c) and (d) including $r_{o}=50 \mathrm{k} \Omega$ in all calculations and compare results.

a. DC analysis:

$$
\begin{aligned}
& I_{B}=\frac{V_{C C}-V_{B E}}{R_{B}}=\frac{12 \mathrm{~V}-0.7 \mathrm{~V}}{470 \mathrm{k} \Omega}=24.04 \mu \mathrm{~A} \\
& I_{E}=(\beta+1) I_{B}=(101)(24.04 \mu \mathrm{~A})=2.428 \mathrm{~mA} \\
& r_{e}=\frac{26 \mathrm{mV}}{I_{E}}=\frac{26 \mathrm{mV}}{2.428 \mathrm{~mA}}=10.71 \Omega
\end{aligned}
$$

b. $\beta r_{e}=(100)(10.71 \Omega)=1.071 \mathrm{k} \Omega$

$$
Z_{i}=R_{B}\left\|\beta r_{e}=470 \mathrm{k} \Omega\right\| 1.071 \mathrm{k} \Omega=1.07 \mathrm{k} \Omega
$$

c. $Z_{o}=R_{C}=3 \mathrm{k} \Omega$
d. $A_{v}=-\frac{R_{C}}{r_{e}}=-\frac{3 \mathrm{k} \Omega}{10.71 \Omega}=-280.11$
e. $Z_{o}=r_{o}\left\|R_{C}=50 \mathrm{k} \Omega\right\| 3 \mathrm{k} \Omega=\mathbf{2 . 8 3} \mathbf{k} \boldsymbol{\Omega}$ vs. $3 \mathrm{k} \Omega$

$$
A_{v}=-\frac{r_{o} \| R_{C}}{r_{e}}=\frac{2.83 \mathrm{k} \Omega}{10.71 \Omega}=-264.24 \mathrm{vs} .-280.11
$$

## VOLTAGE-DIVIDER BIAS



FIG. 5.26
Voltage-divider bias configuration.


FIG. 5.27
Substituting the $r_{e}$ equivalent circuit into the ac equivalent network of Fig. 5.26.

$$
R^{\prime}=R_{1} \| R_{2}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}
$$

$Z_{i}$ From Fig. 5.27

$$
Z_{i}=R^{\prime} \| \beta r_{e}
$$

$Z_{0}$ From Fig. 5.27 with $V_{i}$ set to 0 V , resulting in $I_{b}=0 \mu \mathrm{~A}$ and $\beta I_{b}=0 \mathrm{~mA}$,

$$
\begin{equation*}
Z_{o}=R_{C} \| r_{o} \tag{5.13}
\end{equation*}
$$

If $r_{o} \geq 10 R_{C}$,

$$
\begin{equation*}
Z_{o} \cong R_{C} \quad r_{0} \geq 10 R_{C} \tag{5.14}
\end{equation*}
$$

$A_{\boldsymbol{V}}$ Because $R_{C}$ and $r_{o}$ are in parallel,

$$
V_{o}=-\left(\beta I_{b}\right)\left(R_{C} \| r_{o}\right)
$$

and

$$
I_{b}=\frac{V_{i}}{\beta r_{e}}
$$

so that

$$
V_{o}=-\beta\left(\frac{V_{i}}{\beta r_{e}}\right)\left(R_{C} \| r_{o}\right)
$$

and

$$
\begin{equation*}
A_{v}=\frac{V_{o}}{V_{i}}=\frac{-R_{C} \| r_{o}}{r_{e}} \tag{5.15}
\end{equation*}
$$

which you will note is an exact duplicate of the equation obtained for the fixed-bias configuration.

For $r_{o} \geq 10 R_{C}$,

$$
\begin{equation*}
A_{v}=\frac{V_{o}}{V_{i}} \cong-\frac{R_{C}}{r_{e}} \tag{5.16}
\end{equation*}
$$

Phase Relationship The negative sign of Eq. (5.15) reveals a $180^{\circ}$ phase shift between $V_{o}$ and $V_{i}$.

## For the network of Fig. :

a. $r_{e}$.
b. $Z_{i}$.
c. $Z_{o}\left(r_{o}=\infty \Omega\right)$.
d. $A_{v}\left(r_{o}=\infty \Omega\right)$.
e. The parameters of parts (b) through (d) if $r_{o}=50 \mathrm{k} \Omega$ and compare results.


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a. DC: Testing $\beta R_{E}>10 R_{2}$,

$$
\begin{aligned}
(90)(1.5 \mathrm{k} \Omega) & >10(8.2 \mathrm{k} \Omega) \\
135 \mathrm{k} \Omega & >82 \mathrm{k} \Omega(\text { satisfied })
\end{aligned}
$$

Using the approximate approach, we obtain

$$
\begin{aligned}
V_{B} & =\frac{R_{2}}{R_{1}+R_{2}} V_{C C}=\frac{(8.2 \mathrm{k} \Omega)(22 \mathrm{~V})}{56 \mathrm{k} \Omega+8.2 \mathrm{k} \Omega}=2.81 \mathrm{~V} \\
V_{E} & =V_{B}-V_{B E}=2.81 \mathrm{~V}-0.7 \mathrm{~V}=2.11 \mathrm{~V} \\
I_{E} & =\frac{V_{E}}{R_{E}}=\frac{2.11 \mathrm{~V}}{1.5 \mathrm{k} \Omega}=1.41 \mathrm{~mA} \\
r_{e} & =\frac{26 \mathrm{mV}}{I_{E}}=\frac{26 \mathrm{mV}}{1.41 \mathrm{~mA}}=\mathbf{1 8 . 4 4 \Omega}
\end{aligned}
$$

b. $R^{\prime}=R_{1}\left\|R_{2}=(56 \mathrm{k} \Omega)\right\|(8.2 \mathrm{k} \Omega)=7.15 \mathrm{k} \Omega$

$$
\begin{aligned}
Z_{i} & =R^{\prime}\left\|\beta r_{e}=7.15 \mathrm{k} \Omega\right\|(90)(18.44 \Omega)=7.15 \mathrm{k} \Omega \| 1.66 \mathrm{k} \Omega \\
& =1.35 \mathrm{k} \Omega
\end{aligned}
$$

c. $Z_{o}=R_{C}=6.8 \mathrm{k} \Omega$
d. $A_{v}=-\frac{R_{C}}{r_{e}}=-\frac{6.8 \mathrm{k} \Omega}{18.44 \Omega}=-368.76$
e. $Z_{i}=1.35 \mathrm{k} \Omega$
$Z_{o}=R_{C}\left\|r_{o}=6.8 \mathrm{k} \Omega\right\| 50 \mathrm{k} \Omega=\mathbf{5 . 9 8} \mathbf{k} \boldsymbol{\Omega}$ vs. $6.8 \mathrm{k} \Omega$
$A_{v}=-\frac{R_{C} \| r_{o}}{r_{e}}=-\frac{5.98 \mathrm{k} \Omega}{18.44 \Omega}=-324.3 \mathrm{vs} .-368.76$
There was a measurable difference in the results for $Z_{o}$ and $A_{v}$, because the condition $r_{o} \geq 10 R_{C}$ was not satisfied.

## CE EMITTER-BIAS CONFIGURATION

## Bypassed

If $R_{E}$ of Fig. 5.29 is bypassed by an emitter capacitor $C_{E}$, the complete $r_{e}$ equivalent model can be substituted, resulting in the same equivalent network as Fig. 5.22. Equations (5.5) to (5.10) are therefore applicable.


For the network of Fig. :
a. $r_{e}=$
b. $Z_{i}$.
c. $Z_{o}$.
d. $A_{p}$.

a. DC:

$$
\begin{aligned}
I_{B} & =\frac{V_{C C}-V_{B E}}{R_{B}+(\beta+1) R_{E}}=\frac{20 \mathrm{~V}-0.7 \mathrm{~V}}{470 \mathrm{k} \Omega+(121) 0.56 \mathrm{k} \Omega}=35.89 \mu \mathrm{~A} \\
I_{E} & =(\beta+1) I_{B}=(121)(35.89 \mu \mathrm{~A})=4.34 \mathrm{~mA} \\
\text { and } \quad r_{E} & =\frac{26 \mathrm{mV}}{I_{E}}=\frac{26 \mathrm{mV}}{4.34 \mathrm{~mA}}=5.99 \Omega
\end{aligned}
$$

b. $R_{E}$ is "shorted out" by $C_{E}$ for the ac analysis. Therefore,

$$
\begin{aligned}
Z_{i} & =R_{B}\left\|Z_{b}=R_{B}\right\| \beta r_{e}=470 \mathrm{k} \Omega \|(120)(5.99 \Omega) \\
& =470 \mathrm{k} \Omega \| 718.8 \Omega \cong 717.70 \Omega
\end{aligned}
$$

c. $Z_{o}=R_{C}=2.2 \mathrm{k} \Omega$
d. $A_{v}=-\frac{R_{C}}{r_{\ell}}$

$$
=-\frac{2.2 \mathrm{k} \Omega}{5.99 \Omega}=-367.28(\text { a significant increase })
$$


[^0]:    

