

Prove that

$$\sum m(0, 3, 5, 6, 9, 10, 12, 15) = A \oplus B \oplus C \oplus D$$

Reduce the fol f using k-map technique and implement using the gates

$$i) f(p, q, r, s) = \sum m(0, 1, 4, 8, 9, 10) + d(2, 11)$$

$$ii) f(A, B, C, D) = \prod M(0, 2, 4, 10, 11, 14, 15)$$

$$(A \odot B) / (C \odot D) + (\overline{A \odot B}) \cdot (\overline{C \odot D})$$

$$= (A \odot B) \odot (C \odot D)$$

(15)

$$x \odot y = \overline{x \oplus y}$$

$$\overline{x \oplus y} = x \odot y$$

$$\begin{array}{r|l} 2 & 15 \\ 2 & 7-1 \checkmark \\ 2 & 3-1 \\ & 1-1 \end{array}$$

$$1111_{(2)} = 15_{(10)}$$

$$\begin{array}{cccccc} 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 16 & 8 & 4 & 2 & 1 & \\ 0 & 1 & 1 & 1 & 1 & 1 \end{array}$$

100% magnum

$$\Rightarrow \overline{A} \overline{B} (\overline{C} \overline{D} + CD) + \overline{A} B (\overline{C} D + C \overline{D}) + AB (\overline{C} \overline{D} + CD) + A \overline{B} (\overline{C} D + C \overline{D})$$

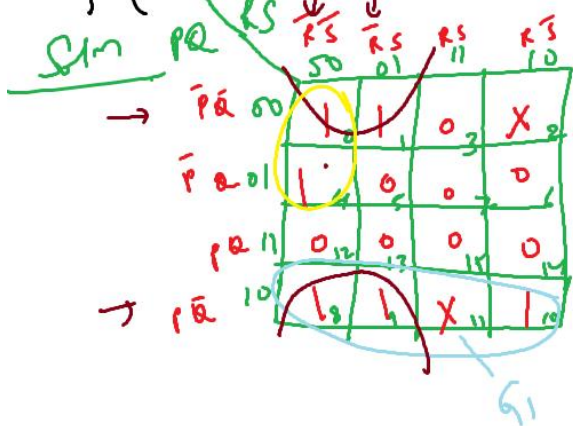
$$\overline{x} \overline{y} + xy = \overline{x \oplus y}$$

$$\overline{xy} + x\overline{y} = x \oplus y$$

$$\begin{aligned} &= (\overline{C} \overline{D} + CD) (\overline{A} \overline{B} + AB) + (\overline{C} D + C \overline{D}) (\overline{A} B + A \overline{B}) \\ &= (C \odot D) (A \odot B) + (C \oplus D) (A \oplus B) \text{ (Reorder)} \\ &= (A \odot B) (C \odot D) + (\overline{A \oplus B}) (\overline{C \oplus D}) \end{aligned}$$

~~K-map~~ & Realize/Implement using gates

$$f(p, a, r, s) = \sum m(\bar{0}, \bar{1}, \bar{4}, \bar{8}, \bar{9}, \bar{10}) + d(2, 11)$$



~~16~~ 8 4 2 1

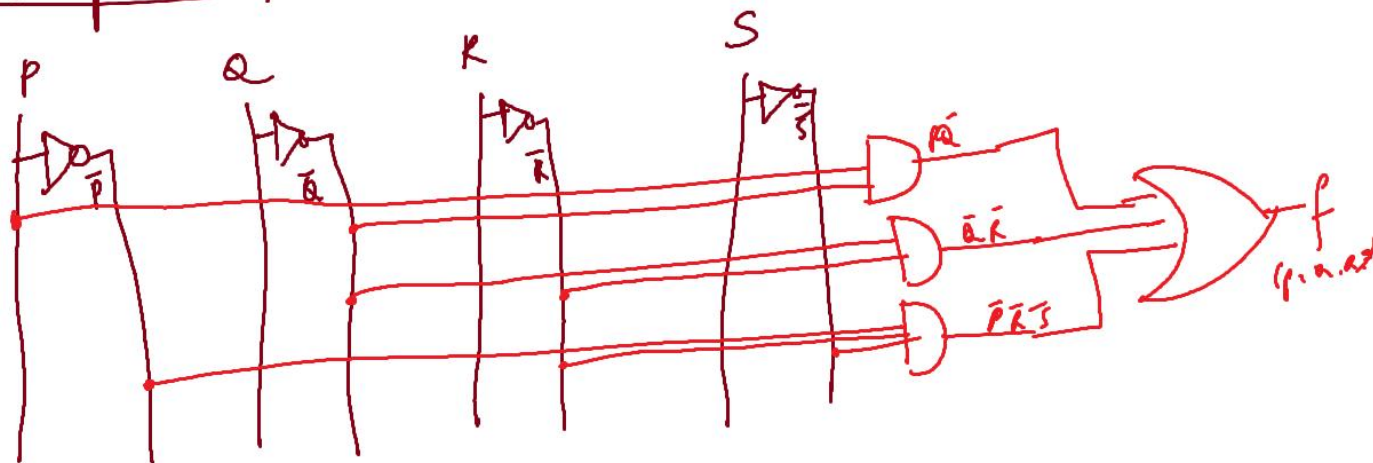
X $\rightarrow$ 6 1

Implement

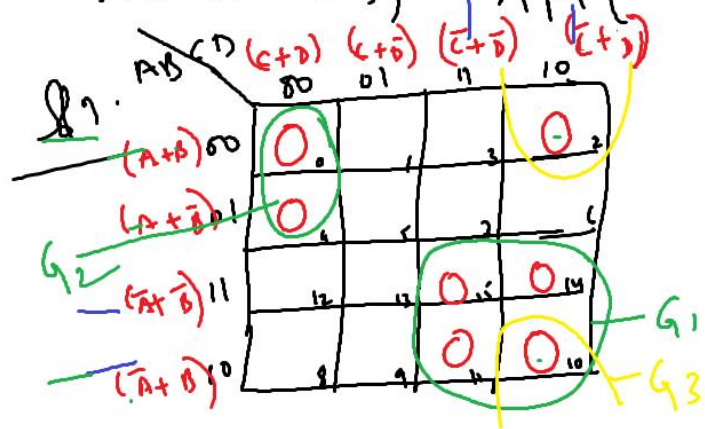
The simplified B.E = $Q_1 + Q_2 + Q_3$

$$\text{S.B.E} = \overline{P}Q + \overline{Q}R + \overline{P}R\overline{S}$$

Implementation / Realization



$$f(A, B, C, D) = \prod M(0, 2, 4, 10, 11, 14, 15)$$



Implement
using
gates

16/4

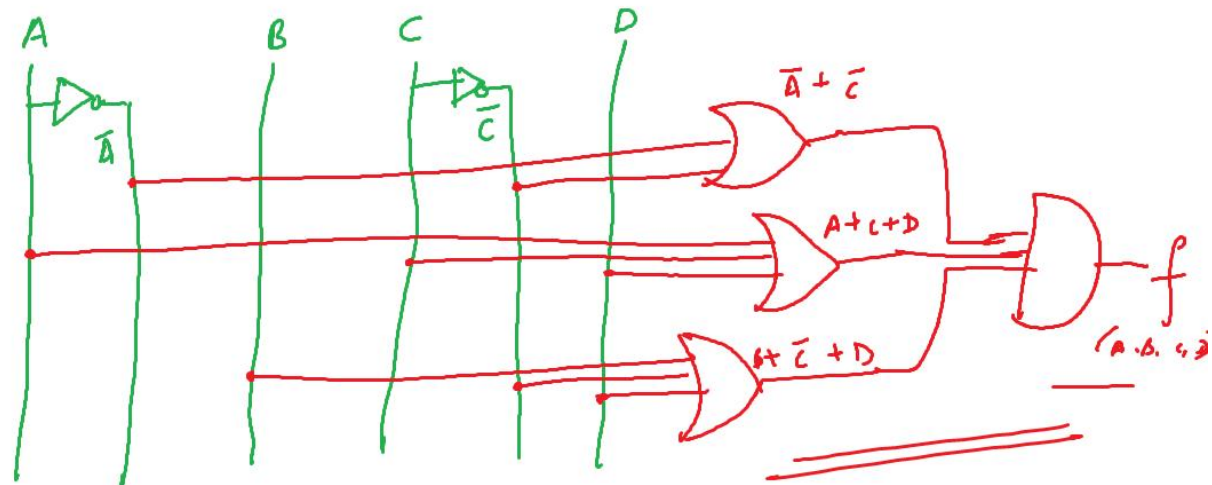
$$G_1 = \bar{A} + \bar{C}$$

$$G_2 = A + C + D$$

$$G_3 = B + \bar{C} + D$$

$$\text{Reduced B.E} = G_1 \cdot G_2 \cdot G_3$$

$$= (\bar{A} + \bar{C}) \cdot (A + C + D) \cdot (B + \bar{C} + D)$$



Quine-McCluskey Minimization technique (minterms) SOP method

Simplify the foll Boolean fn using Quine-McCluskey method $\Rightarrow 5$

$$f(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$$

- Soln:
- ① List all the minterms in binary form
 - ② Arrange the minterms according to no. of 1's
 - ③ Compare each binary no with every term in the adjacent next higher category and if they differ only by 1-posⁿ, then put a check mark and copy the term in the next column with hyphen (-) in the position that they differed.
 - ④ Apply the same process (as in step 3) until there is no eliminⁿ of literals.

$$f(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$$

Soln:

1's
2's

min	A	B	C	D	min	A	B	C	D	min	A	B	C	D
0	0	0	0	0	0	0	0	0	0	0, 2, 8, 10	0	0	0	0
2	0	0	1	0	2	0	0	1	0	2, 8, 2, 10	0	0	1	0
3	0	0	1	1	3	1	0	0	0	2, 3, 6, 7	0	0	0	0
6	0	1	1	0	6	0	0	1	1	2, 6, 3, 7	0	0	1	1
7	0	1	1	1	7	0	1	1	0		0	1	1	0
8	1	0	0	0	8	1	0	0	0		1	0	0	0
10	1	0	1	0	10	1	0	1	0		1	0	1	0
12	1	1	0	0	12	1	1	0	0		1	1	0	0
13	1	1	0	1	13	1	1	0	1		1	1	0	1

1st column

2nd column

3rd column

4th column

The prime implicants are

$\bar{B}\bar{D}$ (0, 2, 8, 10)

$\bar{A}C$ (2, 3, 6, 7)

$A\bar{C}\bar{D}$ (8, 10)

$AB\bar{C}$ (12, 13)

$A\bar{C}\bar{D}$ (8, 10)
 $AB\bar{C}$ (12, 13)
 $\bar{B}\bar{D}$ (0, 2, 8, 10)
 $\bar{A}C$ (2, 3, 6, 7)

Step 5: List all the prime implicants

Prime Implicants	Bin. Represent ⁿ
$A\bar{C}\bar{D}$ (8, 12)	- - 0 0
$AB\bar{C}$ (12, 13)	1 1 0 -
$\bar{B}\bar{D}$ (0, 2, 8, 10)	- 0 - 0
$\bar{A}C$ (2, 3, 6, 7)	0 - 1 -

Step 6: Select the min. no. of prime implicants which must cover ALL the minterms

Prime Implicants	m_0 cd=1	m_2 cd=2	m_3 cd=3	m_6 cd=4	m_7 cd=5	m_8 cd=6	m_{10} cd=7	m_{12} cd=8	m_{13} cd=9	
$A\bar{C}\bar{D}$ (8, 12)						⊙ ^x		⊙ ^x		
$\checkmark AB\bar{C}$ (12, 13)								⊙ ^x	⊙	
$\checkmark \bar{B}\bar{D}$ (0, 2, 8, 10)	⊙	⊙ ^x				⊙ ^x	⊙			
$\checkmark \bar{A}C$ (2, 3, 6, 7)		⊙ ^x	⊙	⊙	⊙					

Final Simplified Expressⁿ = $AB\bar{C} + \bar{B}\bar{D} + \bar{A}C$
 $(1\ 0\ -) + (-\ 0\ -0) + (0\ -\ 1\ -)$

Using Quine McCluskey tabulation method, obtain the set of prime implicants for the $f(a, b, c, d) = \Sigma(0, 1, 4, 5, 9, 10, 12, 14, 15) + d(2, 8, 13)$

Solution:

8 4 2 1 a b c d				8 4 2 1 a b c d				8 4 2 1 a b c d				8 4 2 1 a b c d				8 4 2 1 a b c d			
Min	Bin. Represent			Min	Bin. Represent			Min. Comp.	Bin. Represent			Min. Comp.	Bin. Represent			Rewrite this column			
m ₀	0	0	0	0	0	0	0	(0, 1)	0	0	0	0	0	0	0	(0, 1, 4, 5) =	0	0	0
m ₁	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	(0, 1, 8, 9) =	0	0	0
m ₄	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	(0, 2, 8, 10) =	0	0	0
m ₅	0	1	0	1	0	1	0	0	0	1	0	0	0	1	0	(0, 4, 8, 12) =	0	0	0
m ₉	1	0	0	1	1	0	0	0	0	1	0	0	0	1	0	(1, 5, 9, 13) =	0	0	0
m ₁₀	1	0	1	0	1	0	1	0	0	1	0	1	0	0	0	(8, 12, 9, 13) =	0	0	0
m ₁₂	1	1	0	0	1	1	0	0	0	1	1	0	0	0	0	(8, 12, 10, 14) =	0	0	0
m ₁₄	1	1	1	0	1	1	1	0	0	1	1	1	0	0	0	(4, 12, 5, 13) =	0	0	0
m ₁₅	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	(12, 13, 14, 15) =	0	0	0
m ₂	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0				
m ₈	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0				
m ₁₃	1	1	0	1	1	1	0	1	0	1	1	0	1	0	0				

1st step 2nd step

Rewrite this solution

$(0, 1, 4, 5) = 0$	0	0	0	1
$(0, 1, 8, 9) = 0$	0	0	0	1
$(0, 2, 8, 10) = 0$	0	0	1	0
$(0, 4, 8, 12) = 0$	0	0	1	0
$(1, 5, 9, 13) = 0$	1	0	0	1
$(8, 12, 9, 13) = 1$	1	0	0	1
$(8, 12, 10, 14) = 1$	1	0	0	1
$(1, 12, 5, 13) = 0$	1	0	0	1
$(12, 13, 14, 15) = 1$	1	1	0	0

$\bar{b}\bar{d}$ (points to first two rows)
 $a\bar{d}$ (points to rows 5-8)
 ab (points to last row)

Step 6: Prime Implicants

min. comp	Bin	Exp
$\bar{b}\bar{d} = 1$	0	0
$a\bar{d} = 1$	1	0
$ab = 1$	1	1
$\bar{c} = 1$	0	0

Minterms Comparison Binary Representation

$(0, 1, 4, 5, 8, 12, 9, 13)$	- - 0 1 -
$(0, 1, 8, 9, 4, 12, 5, 13)$	- - 0 1 -
$(0, 4, 8, 12, 1, 5, 9, 13)$	- - 0 1 -

Step 5

$(0, 1, 4, 5, 8, 12, 9, 13)$

a	b	c	d
-	-	0	1

$\downarrow \bar{c}$

Step 7: Select min. prime - ALL minterms

Prime Implicants	m ₀	m ₁	m ₄	m ₅	m ₉	m ₁₀	m ₁₂	m ₁₄	m ₁₅	dm ₂	dm ₈	dm ₁₃
$\bar{b}\bar{d} = (0, 2, 8, 10)$	0					0				0	0	
$a\bar{d} = (8, 12, 10, 14)$						0	0	0		0	0	
$ab = (12, 13, 14, 15)$							0	0	0			0
$\bar{c} = (0, 1, 4, 5, 8, 12, 9, 13)$	0	0	0	0	0		0		0	0	0	

$f(A, B, C, D) = \bar{c} + ab + \bar{b}\bar{d}$

+ final simplified Boolean Expression using Quine Mc-Cluskey.

Obtain all the prime implicants of the function

$$f(v, w, x, y, z) = \sum m(4, 5, 9, 11, 12, 14, 15, 27, 30) + d_c(1, 17, 25, 26, 31)$$

Sol:

Bin. Represent ⁿ						Bin. Represent ⁿ						Bin. Represent ⁿ						Bin. Represent ⁿ						
Min	v	w	x	y	z	Min	v	w	x	y	z	Min. Group ^s	v	w	x	y	z	Min. Group ^s	v	w	x	y	z	
m₁	0	0	1	0	0	m₁	0	0	0	0	0	1	(1, 5)	0	0	0	0	1	(1, 9, 17, 25)	-	-	0	0	1
m₅	0	0	1	0	1	m₅	0	0	1	0	0	0	(1, 9)	0	0	1	0	0	(1, 17, 9, 25)	-	-	0	0	1
m₉	0	1	0	0	1	m₉	0	0	1	0	1	0	(1, 17)	0	0	1	0	0	(9, 11, 25, 27)	-	1	0	-	1
m₁₁	0	1	0	1	1	m₁₁	0	1	0	0	1	0	(4, 5)	0	0	1	0	-	(1, 25, 11, 27)	-	1	0	-	1
m₁₂	0	1	1	0	0	m₁₂	0	1	1	0	0	0	(4, 12)	0	-	1	0	0	(11, 15, 27, 31)	-	1	-	1	1
m₁₄	0	1	1	1	0	m₁₄	1	0	0	0	0	1	(1, 1)	0	1	0	-	1	(1, 22, 15, 31)	-	1	-	1	1
m₁₅	0	1	1	1	1	m₁₅	0	1	0	1	1	1	(9, 25)	0	1	0	1	1	(14, 15, 30, 31)	-	1	1	1	-
m₂₇	1	1	0	1	1	m₂₇	0	1	1	1	1	0	(12, 14)	0	1	1	-	0	(14, 30, 15, 31)	-	1	1	1	-
m₃₀	1	1	1	1	0	m₃₀	1	1	0	0	1	1	(4, 25)	1	0	0	0	1	(26, 27, 30, 31)	1	1	-	1	-
m₃₁	1	1	1	1	1	m₃₁	1	1	0	1	0	1	(1, 15)	0	1	0	1	1	(26, 30, 15, 31)	1	1	-	1	-
m₁₃	1	0	0	0	1	m₁₃	0	1	1	1	1	1	(1, 27)	0	1	1	1	1						
m₂₅	1	1	0	0	1	m₂₅	1	1	0	1	1	1	(14, 15)	0	1	1	1	0						
m₂₆	1	1	0	1	0	m₂₆	1	1	1	1	0	1	(11, 30)	0	1	1	1	0						
m₃₁	1	1	1	1	1	m₃₁	1	1	1	1	1	1	(9, 27)	1	1	0	0	1						

Step (1) Step - 2 Step - 3 - Comparison

9 terms → prime implicants

Prime Implicants	Binary Representation
$(1, 5) = \bar{v} \bar{w} \bar{y} z$	0 0 0 1
$(4, 5) = \bar{v} \bar{w} x \bar{y}$	0 0 1 0
$(4, 12) = \bar{v} x \bar{y} \bar{z}$	0 1 1 0
$(12, 14) = \bar{v} w x \bar{z}$	0 1 1 0
$(1, 9, 17, 25) = \bar{x} \bar{y} z$	0 0 1 1
$(9, 11, 25, 27) = w \bar{x} z$	1 0 0 1
$(11, 15, 27, 31) = w y z$	1 1 1 1
$(14, 15, 30, 31) = w x y$	1 1 1 1
$(26, 27, 30, 31) = v w y$	1 1 1 1

$$\Sigma m(4, 5, 9, 11, 12, 15, 27, 30) + d_c(1, 17, 25, 26, 31)$$

$$\text{Final Expression} = \bar{x} \bar{y} z + v w y = (\bar{x} \bar{y} z) + (v w y)$$

Last step:

Prime Implicants	m ₄	m ₅	m ₉	m ₁₁	m ₁₂	m ₁₄	m ₁₅	m ₂₇	m ₃₀	d _{m1}	d _{m17}	d _{m25}	d _{m26}	d _{m31}
$(1, 5) = \bar{v} \bar{w} \bar{y} z$	0	0								0				
$(4, 5) = \bar{v} \bar{w} x \bar{y}$	0	0												
$(4, 12) = \bar{v} x \bar{y} \bar{z}$	0				0									
$(12, 14) = \bar{v} w x \bar{z}$					0	0								
$(1, 9, 17, 25) = \bar{x} \bar{y} z$			0							0	0	0		
$(9, 11, 25, 27) = w \bar{x} z$			0	0				0				0		
$(11, 15, 27, 31) = w y z$				0			0	0						0
$(14, 15, 30, 31) = w x y$						0	0		0					0
$(26, 27, 30, 31) = v w y$								0	0				0	0

Simplify using Quine McCluskey Tabulation algorithm,
 $V = f(a, b, c, d) = \sum (2, 3, 4, 5, 13, 15) + \sum d(8, 9, 10, 11)$

Solution

Min	a	b	c	d	Min	a	b	c	d	Min. Comp	Bin. Represent
m ₂	0	0	1	0	m ₂	0	0	1	0	(2, 3)	0 0 1 0
m ₃	0	0	1	1	m ₃	0	0	1	1	(2, 10)	0 0 1 1
m ₄	0	1	0	0	m ₄	0	1	0	0	(4, 5)	0 1 0 0
m ₅	0	1	0	1	m ₅	0	1	0	1	(4, 9)	0 1 0 1
m ₆	1	1	0	1	m ₆	1	1	0	1	(8, 10)	1 1 0 1
m ₁₃	1	1	1	1	m ₁₃	1	1	1	1	(9, 11)	1 1 1 1
m ₈	1	0	0	0	m ₈	1	0	0	0	(5, 13)	1 0 0 1
m ₉	1	0	0	1	m ₉	1	0	0	1	(5, 11)	1 0 0 1
m ₁₀	1	0	1	0	m ₁₀	1	0	1	0	(6, 13)	1 0 1 0
m ₁₁	1	0	1	1	m ₁₁	1	0	1	1	(6, 10)	1 0 1 1
										(11, 15)	1 1 1 1
										(13, 15)	1 1 1 1

Step (1) Step (2)

Final Expr = $\bar{a}\bar{b}\bar{c} + \bar{b}c + a\bar{b} + ad$ Step (3) ~~comp~~

Min. Comp	a	b	c	d
(2, 3, 10, 11)	-	0	1	-
(2, 10, 3, 11)	-	0	1	-
(8, 9, 10, 11)	1	0	-	-
(8, 10, 9, 11)	1	0	-	-
(9, 13, 11, 15)	1	-	-	1
(9, 11, 13, 15)	1	-	-	1

Step (4) - comp

Min. Comp	a	b	c	d
(2, 3, 10, 11)	-	0	1	-
(8, 9, 10, 11)	1	0	-	-
(9, 13, 11, 15)	1	-	-	1

Step (5) - comp

Prime Implicants

- (4, 5) = $\bar{a}\bar{b}\bar{c}$
- (5, 13) = $\bar{b}\bar{c}d$
- (2, 3, 10, 11) = $\bar{b}c$
- (8, 9, 10, 11) = $a\bar{b}$
- (9, 13, 11, 15) = ad

	1	2	3	4	5	6	7	8	9	10	11
	m ₂	m ₃	m ₄	m ₅	m ₁₃	m ₁₅	d _{m8}	d ₉	d ₁₀	d ₁₁	
(4, 5)			0	0							
(5, 13)				0	0						
(2, 3, 10, 11)	0	0							0	0	
(8, 9, 10, 11)							0	0	0	0	
(9, 13, 11, 15)						0	0		0	0	

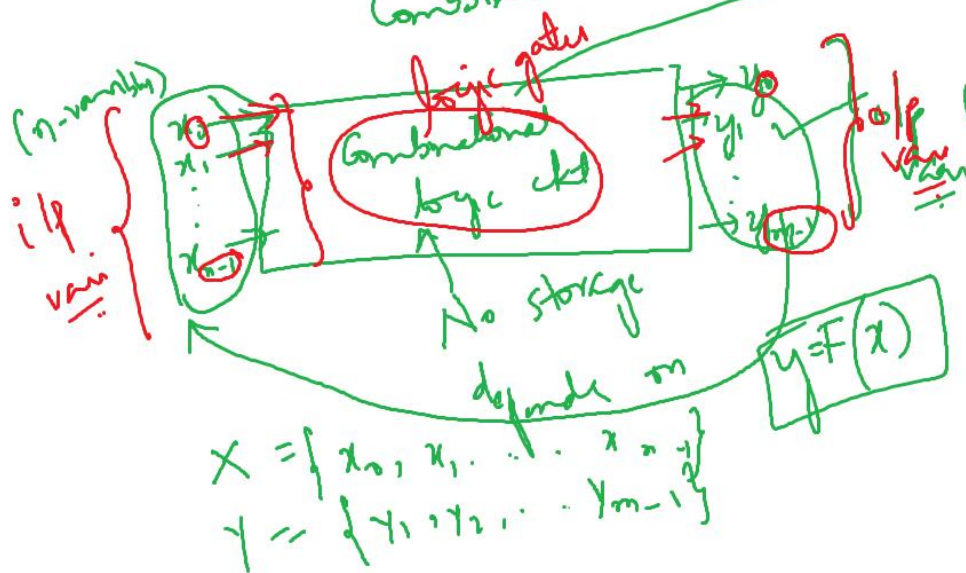
Unit - 2

Data processing circuits

Collection of logic gates

Combinational

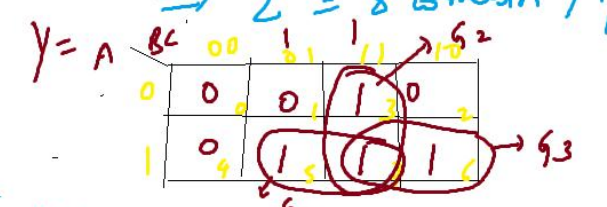
Sequential



Ex: A combinational logic circuit with 3 ip vars that will produce a majority when > one ip vars are = 1.
 Derive the T.T. & implement the ckt.

Soln: 3 ip variables $\Rightarrow 2^3 = 8$ combinations
Design of C.L.C.

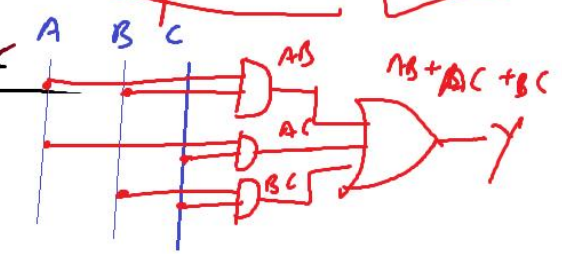
	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1



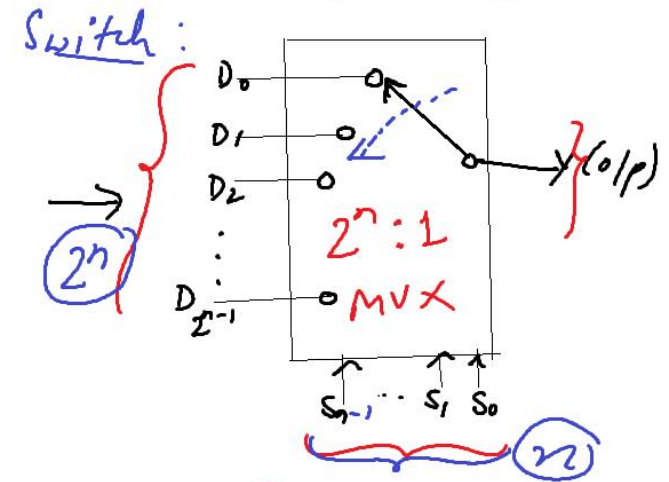
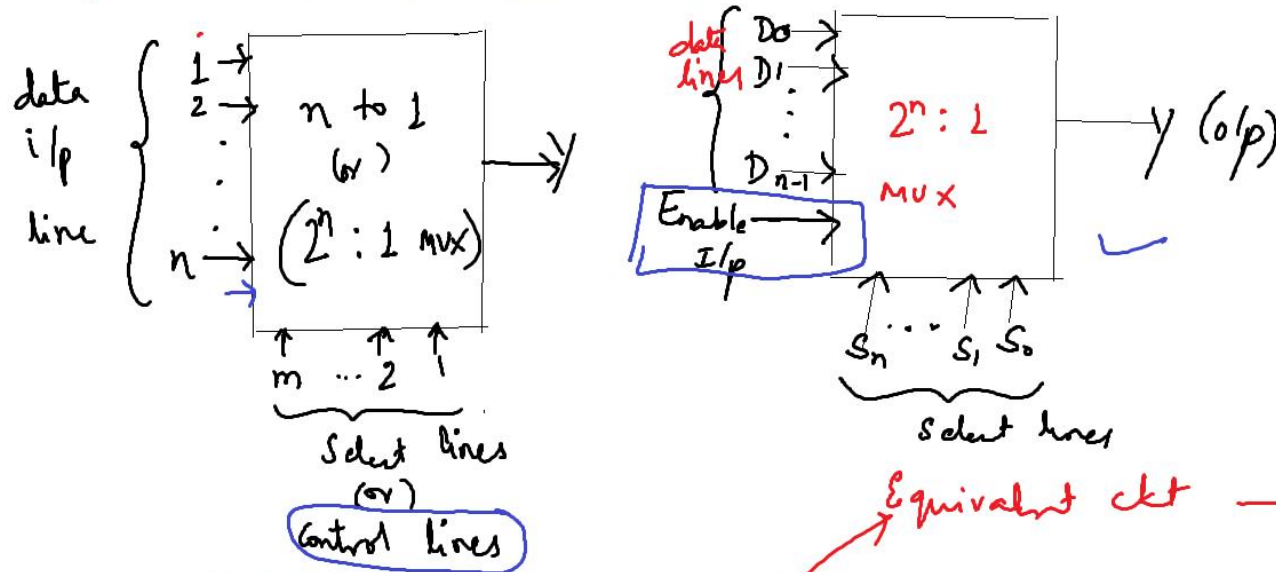
$$Y = AC + BC + AB$$

$$\overline{Y} = \overline{AB} + \overline{AC} + \overline{BC}$$

Implement:



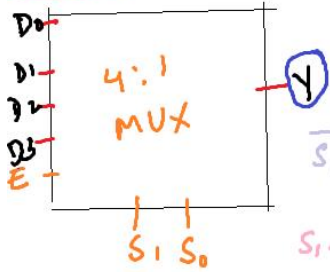
Multiplexers: (MUX) $\rightarrow$ Select single data line from several data-i/p lines
 $\rightarrow$ data selector - many to 1 -



$\rightarrow$ Equivalent ckt $\rightarrow$ Equivalent ckt.

Block diagram of $2^n:1$ MUX

4:1 MUX:

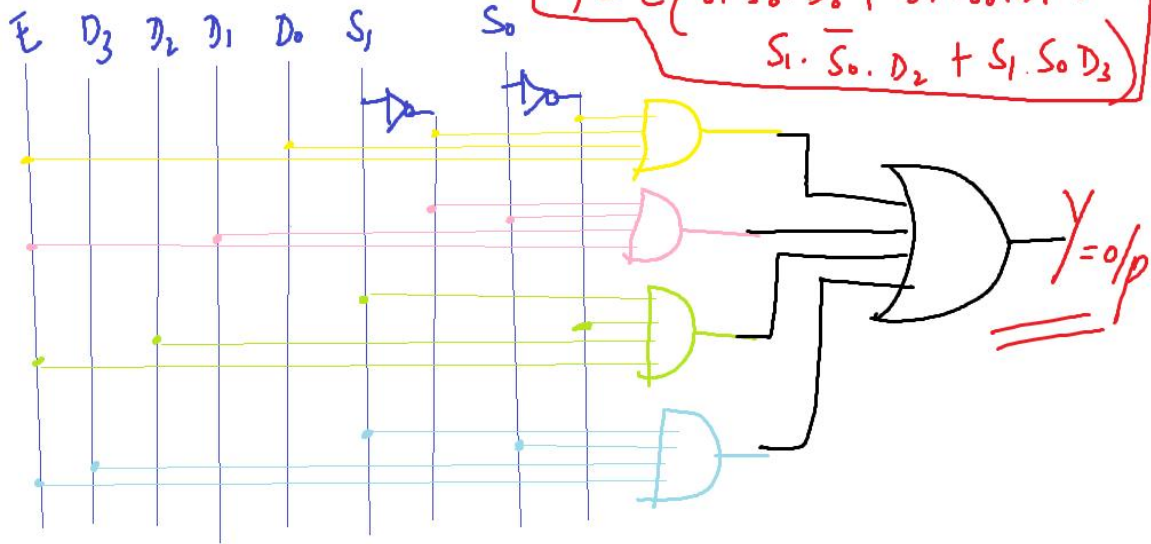


Function table/Truth table

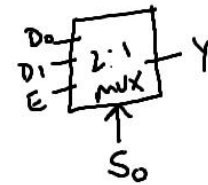
S ₁	S ₀	Y	E
0	0	D ₀	1
0	1	D ₁	1
1	0	D ₂	1
1	1	D ₃	1

$\bar{S}_1 \cdot \bar{S}_0 \cdot D_0$ (E)
 $\bar{S}_1 \cdot S_0 \cdot D_1$ (E)
 $S_1 \cdot \bar{S}_0 \cdot D_2$ (E)
 $S_1 \cdot S_0 \cdot D_3$ (E)

$$Y = E(\bar{S}_1 \cdot \bar{S}_0 \cdot D_0 + \bar{S}_1 \cdot S_0 \cdot D_1 + S_1 \cdot \bar{S}_0 \cdot D_2 + S_1 \cdot S_0 \cdot D_3)$$



2:1 MUX:

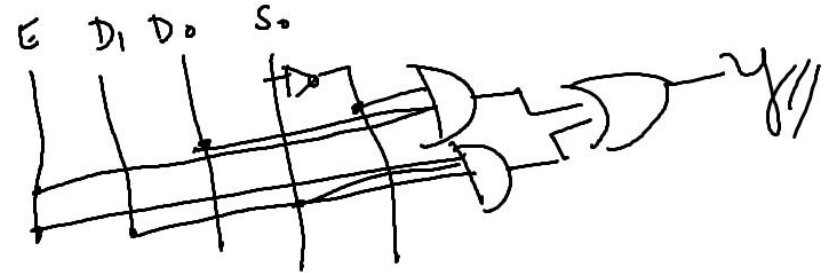


select line = 1
 data lines = 2

Fⁿ table/Truth table

E	S ₀	Y
1	0	D ₀
1	1	D ₁

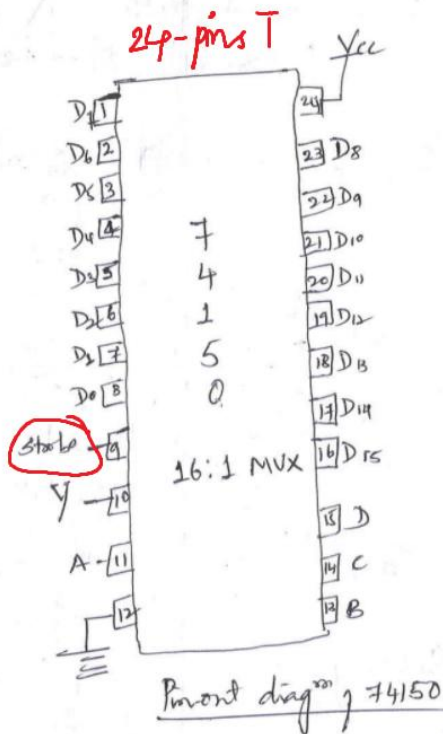
$$Y = E\bar{S}_0 \cdot D_0 + E S_0 \cdot D_1 = E(\bar{S}_0 \cdot D_0 + S_0 \cdot D_1)$$



Strobe/ Enable	A	B	C	D	Y
$\bar{L}$	L	L	L	L	$\bar{D}_0$
$\bar{L}$	L	L	L	H	$\bar{D}_1$
$\bar{L}$	L	L	H	L	$\bar{D}_2$
.	.	.	.	.	.
.	.	.	.	.	.
$\bar{L}$	H	H	H	H	$\bar{D}_3$
H	X	X	X	X	H

74150 Truth Table

can't be determined



Bubbles on Signal line:

$y \Rightarrow$ is the o/p that is the complement of the selected data-bit.
 $\bar{D}_0 = y$

$y \Rightarrow$ bubble on i/p pin, means the signal is active low.
Low-state

$1 = 1$
 $0 = 74150$

E	S ₁	S ₀	Y
0	0	0	$\bar{D}_0$
0	0	1	$\bar{D}_1$
0	1	0	$\bar{D}_2$
0	1	1	$\bar{D}_3$
1	X	X	X

Realization of 4 variables using 4:1 MUX
 Soln: 4-variables $\rightarrow 2^4 \rightarrow 16:1$ MUX

2 variables $\rightarrow 2^2 \rightarrow 4:1$ MUX

wx yz

	00	01	11	10	
00	1	1	0	0	$\rightarrow I_0$
01	0	1	1	1	$\rightarrow I_1$
11	0	1	0	1	$\rightarrow I_3$
10	0	1	0	0	$\rightarrow I_2$

Common - constant literals
 selected line
 varying $\rightarrow$ data line

Submaps: for I_0, I_1, I_2 and I_3

w=0, x=0

	00	01	11	10
00	1	1	0	0

$I_0 = \bar{y}$ ✓

w=0, x=1

	00	01	11	10
00	0	1	1	1

$I_1 = y + z$ ✓

w=1, x=0

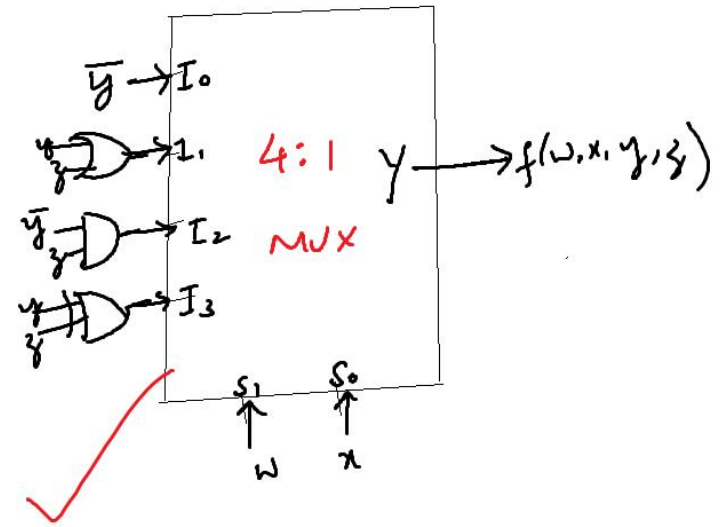
	00	01	11	10
00	0	1	0	0

$I_2 = \bar{y}z$ ✓

w=1, x=1

	00	01	11	10
00	0	1	0	1

$I_3 = \bar{y}z + y\bar{z} = y \oplus z$
 $I_3 = y \oplus z$ ✓



w, x y, z

	00	01	11	10
00	1	1	0	0
01	0	1	1	1
11	0	1	0	0
10	0	1	1	0
	I_0	I_1	I_2	I_3

$y=0$

1
1
1
1

$$x(\bar{w}\bar{x}) + \bar{w}x + wx + w\bar{x}$$

$$(w \oplus x) + (\bar{w} \oplus x)$$

$$1 + 0 = 1$$

$\bar{I}_1 = 1$

$y=1$

0
1
0
1

$$\bar{w}x + w\bar{x}$$

$I_3 = w \oplus x$

Submaps for I_0, I_1, I_2, I_3

$I_0 = yz=00$ select lines

$w, x \rightarrow$ data lines

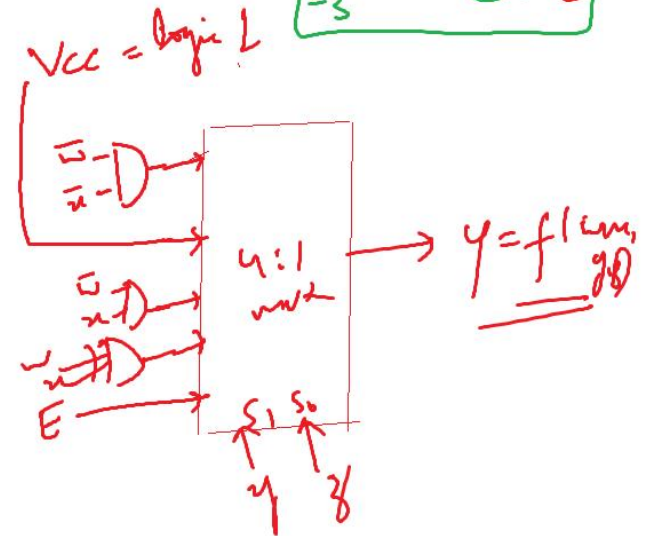
$y, z \rightarrow$ select lines

$I_0 = \bar{w} \cdot \bar{x}$

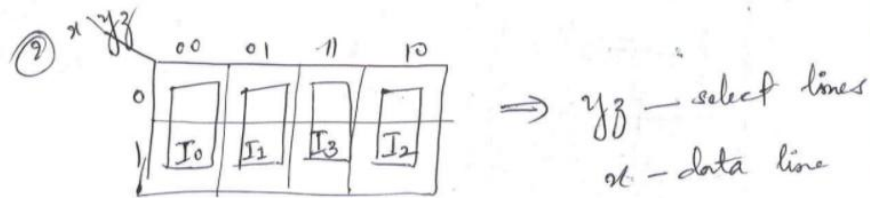
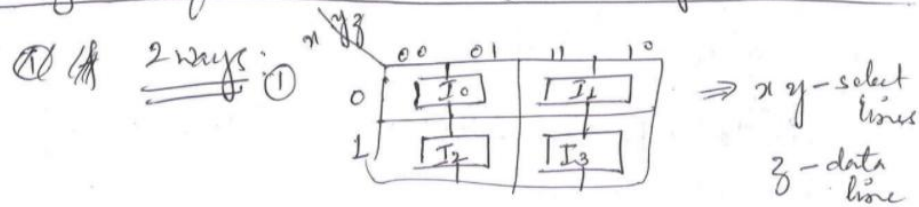
$y=1$

0
1
0
0

$\bar{I}_2 = \bar{w}x$

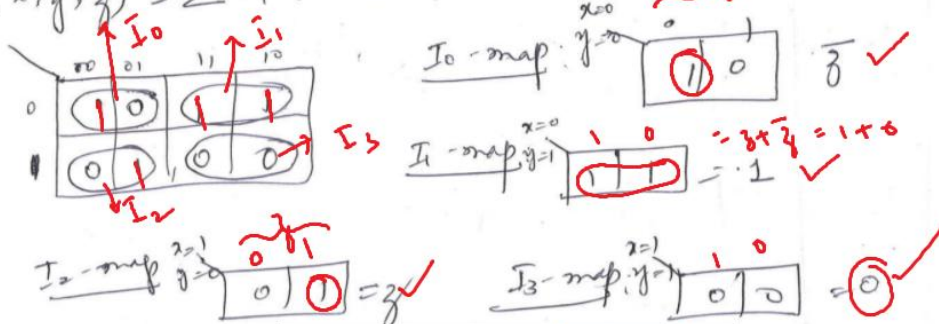


Realization of 3 vari boolean f using 4-to-1 mux

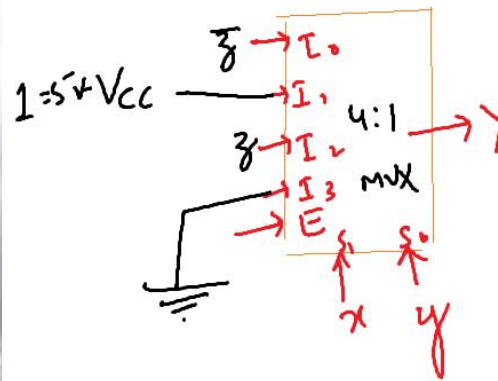


Eg. First method:

$f(x, y, z) = \sum m(0, 2, 3, 5)$ Submaps



$I_0 = \bar{z}$ $I_1 = 1$ $I_2 = z$ $I_3 = 0$

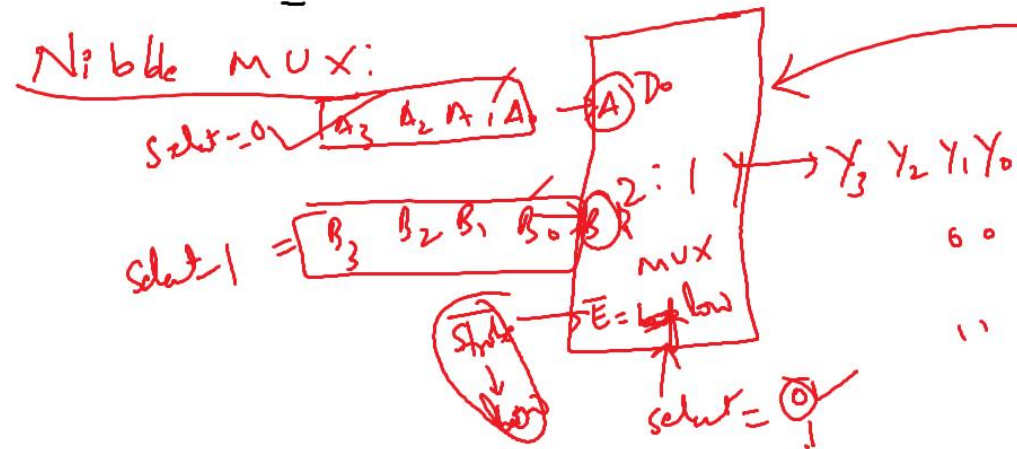
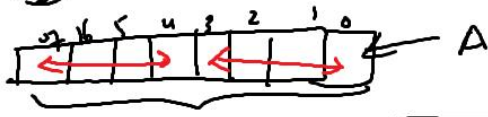


1 bit = 0 or 1

1 byte = 8-bits

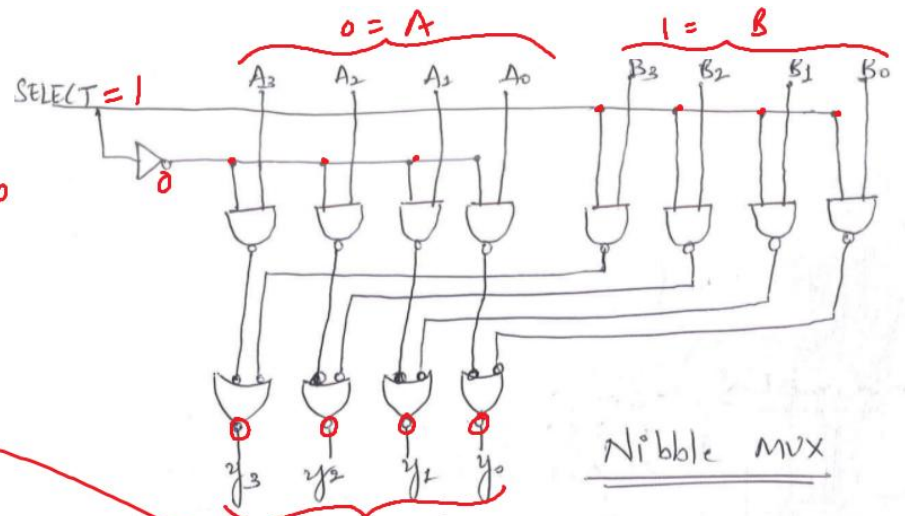
1 word = 2-bytes = A B

1 nibble = 1 byte = 8-bits = 4 bits

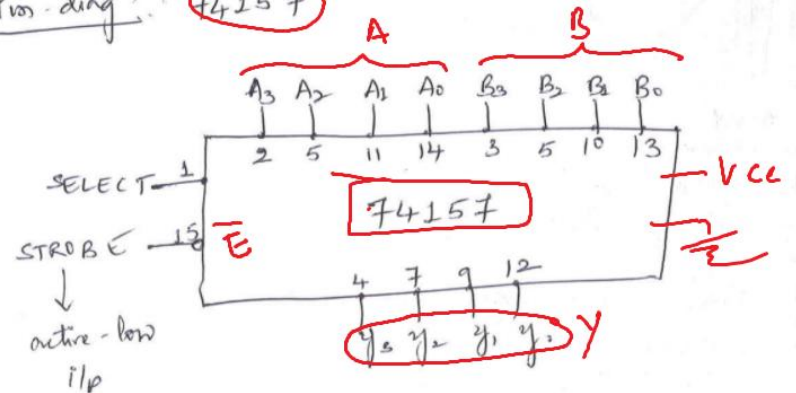


$A < 0$

A	B
0	0
0	1
1	0
1	1

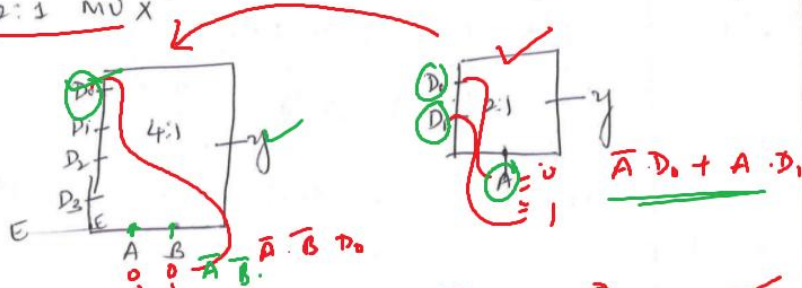


Pin-ding: 74157



1) Show how 4:1 MUX can be obtained using only 2:1 MUX

Soln:



Logic eqⁿ for 2:1 MUX : $y = \bar{A} D_0 + A D_1$ (1)

Logic eqⁿ for 4:1 MUX : $y = \bar{A} \bar{B} D_0 + \bar{A} B D_1 + A \bar{B} D_2 + A B D_3$

$$y = \bar{A} (\bar{B} D_0 + B D_1) + A (\bar{B} D_2 + B D_3)$$

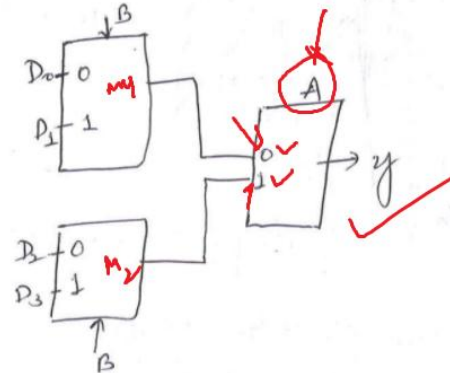
$\Rightarrow$ 4 s.
cho
 $\Rightarrow$ 2-to
A the

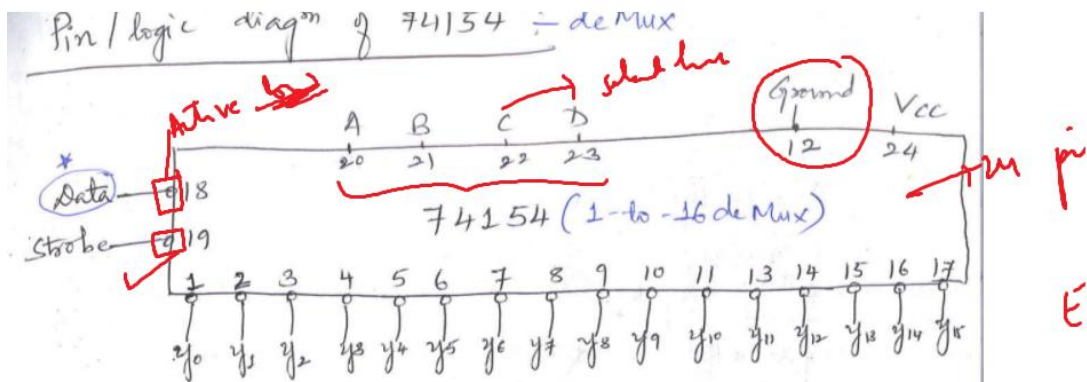
The logic eqⁿ of 4:1 MUX can be re-written as

$$y = \bar{A} (\bar{B} D_0 + B D_1) + A (\bar{B} D_2 + B D_3) \rightarrow (2)$$

Compare eqⁿ (1) & (2).

We need two 2:1 MUX to realize the two bracketed terms where B serves as select i/p. The o/p of these 2 MUXs can be sent to a 3rd MUX as data i/p's where A serves as select i/p & we get 4:1 MUX.





D	^{strobe} E	A	B	C	D	Y ₀	Y ₁	Y ₁₅
0	0	0	0	0	0	1	1	1
0	0	0	0	1	1	0	1	1
1		X	X	X	X	1	1	1
	1	X	X	X	X	1	1	1

Worked Example: Show how two 1-to-16 MUX can be connected to get 1-to-32 deMUX.

Solⁿ: 1-to-32 deMUX has 5 select @ variables A B C D E.

Four of them B C D E — fed to two 1-to-16 deMUX.

Fifth A — used to select one of these two deMUX through strobe-i/p.

If $A = 0$, the top 74154 is chosen.

∴ B C D E directs data to one of the 15 o/p's of that IC.

If $A = 1$, the top 74154 is chosen,

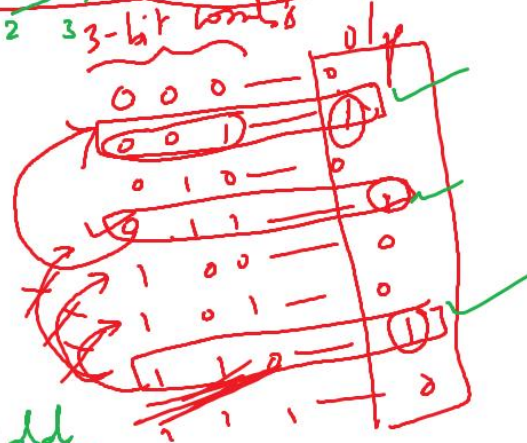
Combinational ckts: Adders & Subtractors

MUX, deMUX

forward i/p numbers

0 1 0 ← 1

0 1 1 ← 1



Half Adder: 2-bit Adder

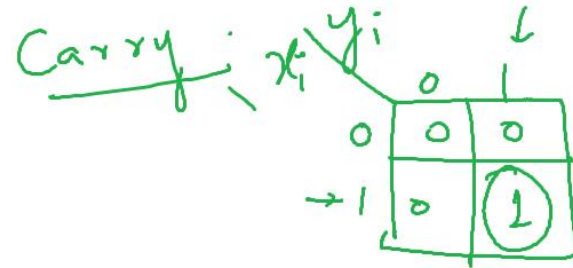
2 binary i/p's: Augend and addend bits

2 binary o/p's: Sum and carry

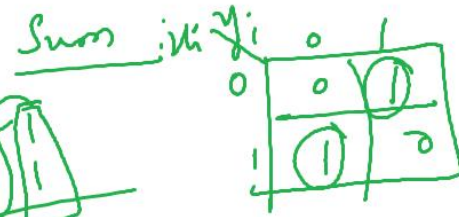
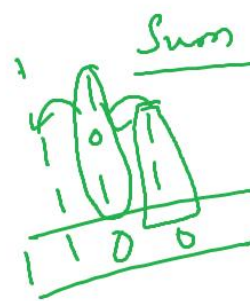
x_i	y_i	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

3
11

0 0 1 1
0 0 1 1
0 1 1 1
1 0 1 1
2 10



$$\therefore C = x_i \cdot y_i$$



$$\overline{x_i} y_i + x_i \overline{y_i} \therefore \text{Sum} = x_i \oplus y_i$$

Full Adder: $\rightarrow$ 3 i/p's - x_i, y_i, C_i
Half Adder: 2 i/p's - Sum
 Carry bit C_{i+1}

$$\begin{array}{r} 0 \\ 0 \\ \hline 00 \end{array}$$

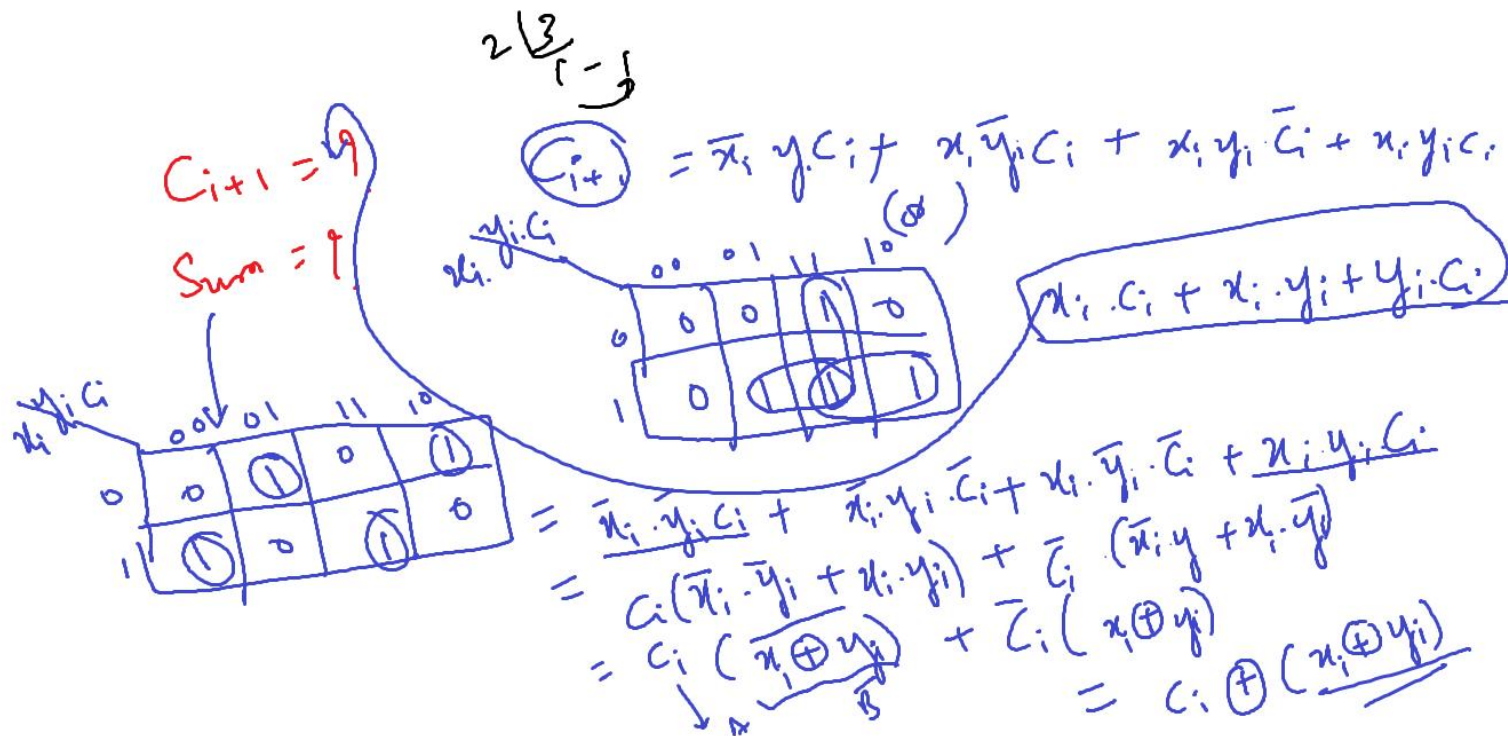
$$\begin{array}{r} 0 \\ 0 \\ \hline 01 \end{array}$$

$$\begin{array}{r} 0 \\ 1 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ \hline 11 \end{array}$$

x_i	y_i	C_i	C_{i+1}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Logic Diagram:

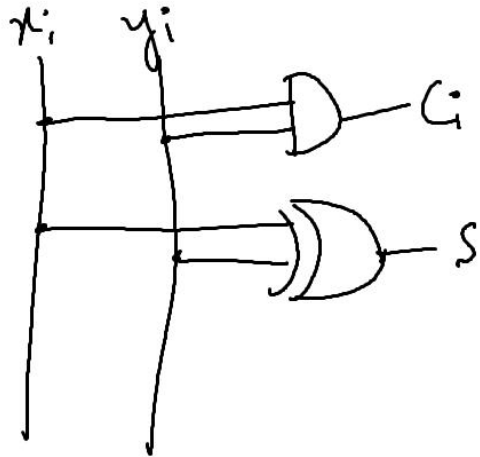


Logic diagram for half adder and full adder

$$C_i = x_i \cdot y_i$$

$$S = x_i \oplus y_i$$

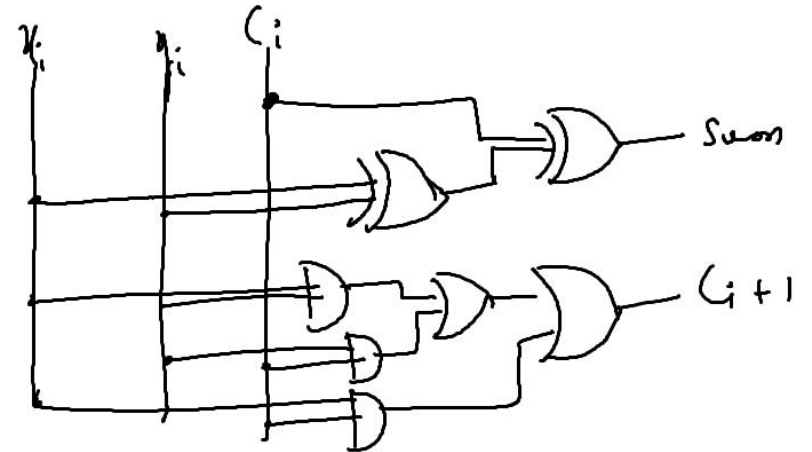
inputs:



$$Sum = C_i \oplus (x_i \oplus y_i)$$

$$C_{i+1} = x_i y_i + y_i C_i + x_i C_i$$

inputs:



Subtractor C.G. $\begin{cases} \text{HS} - 2\text{-bit} - 2\text{ bit} \\ \text{FS} - 3\text{-bit} - 2\text{ bit} \end{cases}$

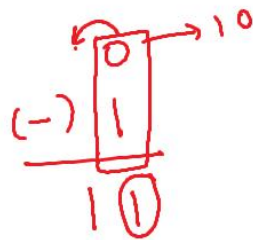
Binary Adder & Binary Subtractor

Binary half Subtractor: $(x_i - y_i)$

2 binary i/p: Minuend $- x_i$
Subtrahend $- y_i$

2 binary o/p: Difference $- d$
borrow $- b$

x_i	y_i	d	b
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0



$$\begin{array}{r} 1 \\ (-) 0 \\ \hline 01 \end{array}$$

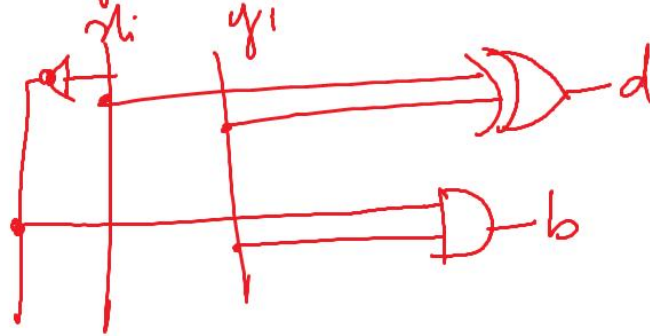
$$\begin{array}{r} 0 \\ (-) 0 \\ \hline 00 \end{array}$$

K-map: Diff

$$\begin{array}{c|c|c} & y_i & \\ \hline x_i & 0 & 1 \\ \hline 0 & 0 & 1 \\ \hline 1 & 1 & 0 \end{array} = \bar{x}_i \cdot y_i + x_i \cdot \bar{y}_i = x_i \oplus y_i$$

Borrow: $\begin{array}{c|c|c} & y_i & \\ \hline x_i & 0 & 1 \\ \hline 0 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array} \therefore b = \bar{x}_i \cdot y_i$

Logic diagram:



Binary Full Subtractor: 3 i/p's — 3 binary i/p's — minuee — x_i
 2 o/p's — 2 binary o/p's. — Subtrahend — y_i
 borrow-in from previous

borrow-in

x_i	y_i	b_i	d_i	b_{i+1}
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

borrow-out

$$\begin{array}{r} 0 \\ 0 \\ 0 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 0 \\ \boxed{10} \\ \hline 11 \end{array}$$

$$\begin{array}{r} 00 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 00 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 11 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 1 \\ 0 \\ 0 \\ \hline 01 \end{array}$$

$$\begin{array}{r} 10 \\ \hline 11 \\ \hline 01 \end{array}$$

d_i

$x_i \cdot y_i \cdot b_i$

x_i	y_i	b_i	$x_i \cdot y_i \cdot b_i$
0	0	0	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

b_{i+1}

$x_i \cdot y_i \cdot b_i$

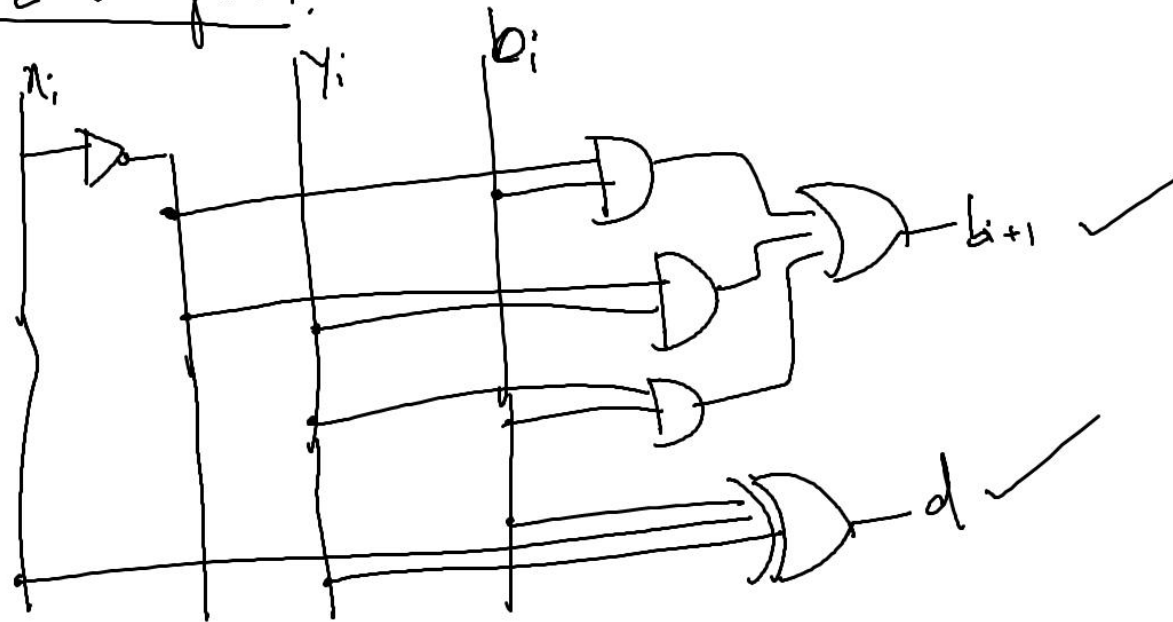
x_i	y_i	b_i	$x_i \cdot y_i \cdot b_i$
0	0	0	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

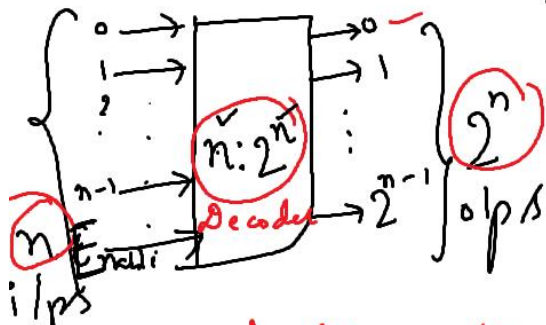
$$b_{i+1} = \bar{x}_i \bar{y}_i b_i + \bar{x}_i y_i b_i + x_i \bar{y}_i b_i + x_i y_i b_i$$

$$\begin{aligned} d_i &= \bar{x}_i \bar{y}_i b_i + \bar{x}_i y_i \bar{b}_i + x_i \bar{y}_i \bar{b}_i + x_i y_i b_i \\ &= b_i (\bar{x}_i \bar{y}_i + x_i y_i) + \bar{b}_i (\bar{x}_i y_i + x_i \bar{y}_i) \\ &= b_i (\bar{x} \oplus y) + \bar{b}_i (x \oplus y) \end{aligned}$$

$$d_i = b_i \oplus (x_i \oplus y_i)$$

Logic Diagram:



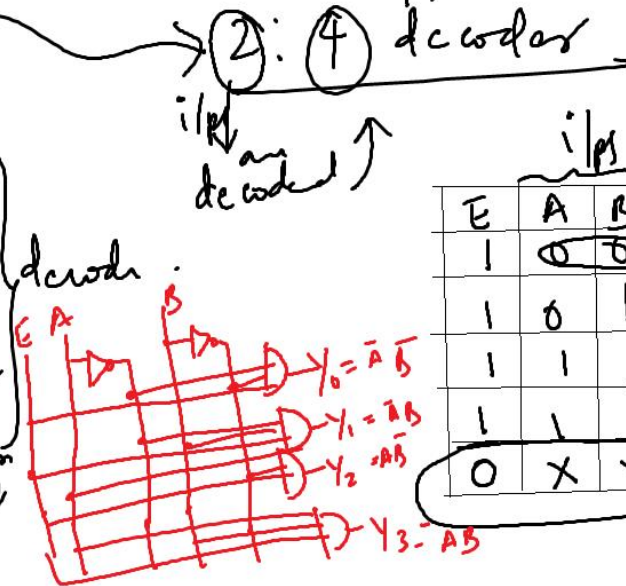


Decoder: Combinational L.C. which has multiple i/p's and multiple o/p's (logicckt) which converts the coded i/p's into coded o/p's where i/p - o/p's codes are different.

General structure of the decoder

Binary decoder: $n - 2^n \Rightarrow$
 n -bit binary i/p code

2 to 4
 3 to 8
 4 to 16
 5 to 32
 $\vdots$
 n to 2^n



i/p			o/p				
E	A	B	Y ₃	Y ₂	Y ₁	Y ₀	
1	0	0	0	0	0	1	= $\bar{A}\bar{B}$
1	0	1	0	0	1	0	= $\bar{A}B$
1	1	0	0	1	0	0	= $A\bar{B}$
1	1	1	1	0	0	0	= AB
0	X	X	0	0	0	0	

n: 2^n decoder: 3: 8 Decoder: 74138.
 $3: 2^3$
 $3: 8$
 3 bin. i/p's
 8 individual active **LOW** $\checkmark$
 $\bar{E} = 0$, active **low** enable

$f_1(A, B, C) = \sum m(0, 4, 6)$ **D**

$f_2(A, B, C) = \sum m(0, 5)$ **D**

$f_3(A, B, C) = \sum m(1, 2, 3, 7)$ **D**

Since @ the decoder o/p, we'll get all the 8 minterms, we use them as shown to get the boolean fns.

$m = \overline{M}$ (or) $M = \overline{m}$

$\sum m(0, 4, 6)$

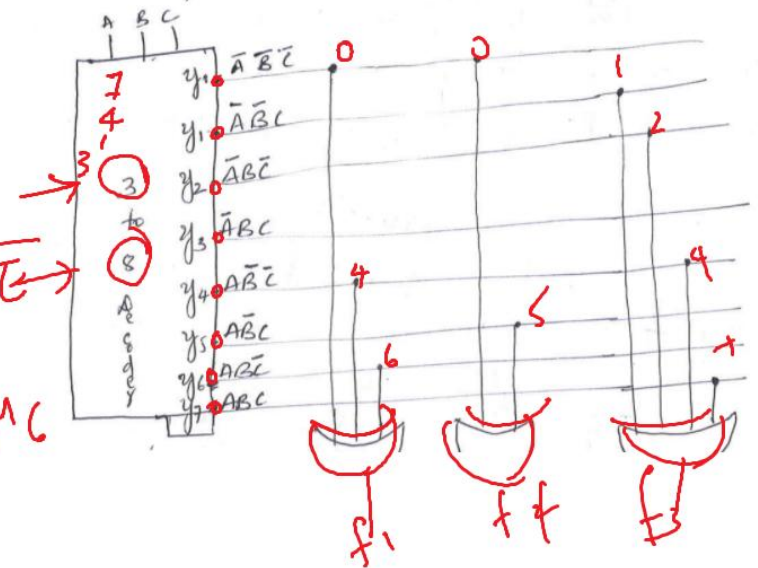
ABC
 $\overline{m}_0 = \overline{A} \cdot \overline{B} \cdot \overline{C}$
 0 0 0

$M_0 = \overline{A \cdot B \cdot C}$
 $= \overline{A} + \overline{B} + \overline{C}$

$A \cdot B \cdot C \Rightarrow A \cdot \overline{B} \cdot \overline{C}$
 $\Rightarrow \overline{A} + \overline{B} + \overline{C} = (\overline{A} + B + C) = M_4$

C.V = 1
 U.V = 0 $\bar{E}$

$M_0 \vee M_4 \vee M_6$



BCD to Decimal decoder

Binary coded Decimal

(nibble - 4 bits)

Decimal no 429 $\Rightarrow$ BCD: $\begin{array}{ccc} 4 & 2 & 9 \\ \hline 0100 & 0010 & 1001 \end{array}$

BCD: $\begin{array}{cccc} 1000 & 0101 & 0010 & 0111 \\ \hline 8 & 5 & 2 & 7 \end{array} \Rightarrow \text{Decimal} = 8527_{(10)}$

Convert the decimal no 563 into BCD.

$\begin{array}{ccc} 0101 & 0110 & 0011 \\ \hline \end{array}$ (each digit - 4 bits)

1) $\begin{array}{c} \text{BCD} \\ 010101110101 = 9575_{(10)} \end{array}$
msb $\leftarrow$ $\leftarrow$ LSB
Decimal

Group 4-bits together

BCD to Decimal Decoder (7445)

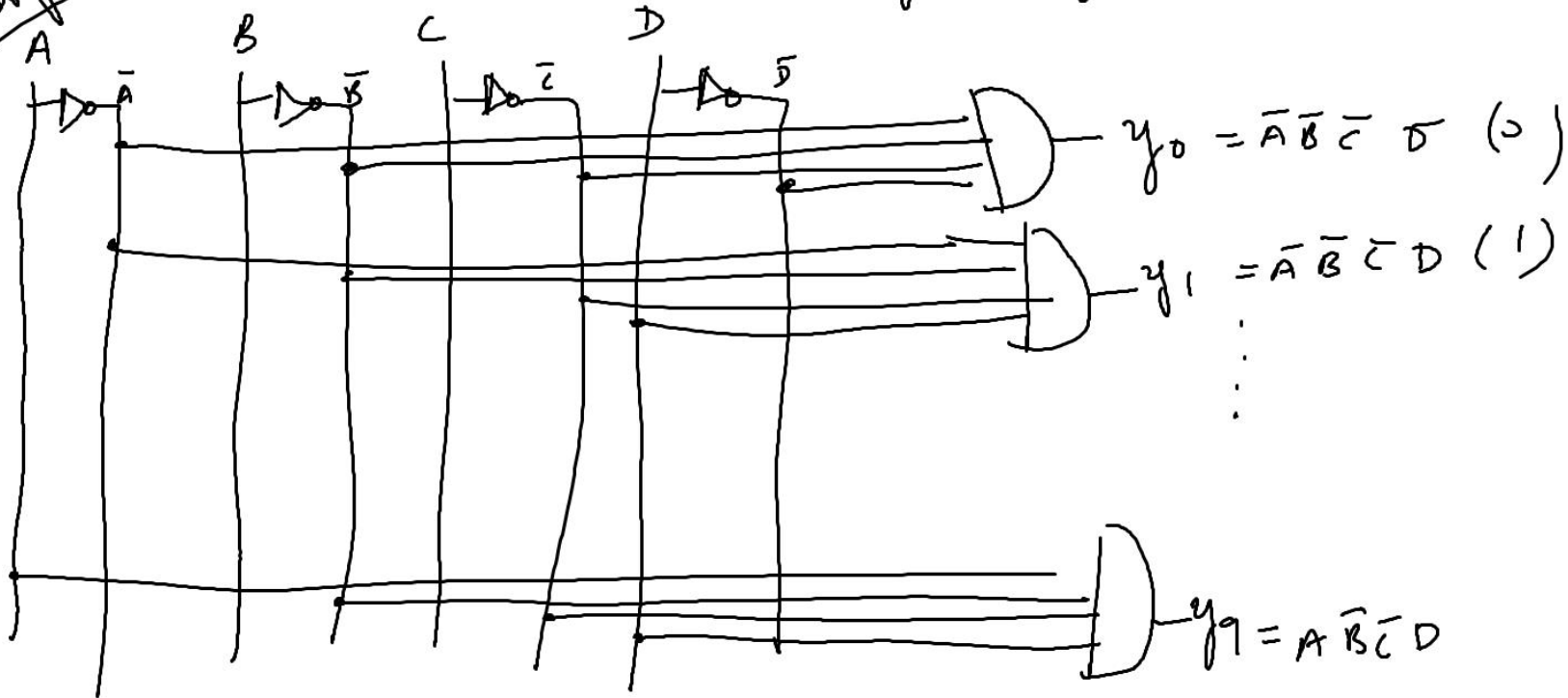
$$2^4 = 16 \rightarrow 0 \dots 9$$

0000 — 1001 (0 through 9)

($y_0 \dots y_9$)

10 ... 15 — can NOT exist in BCD form.

Logic Diagram

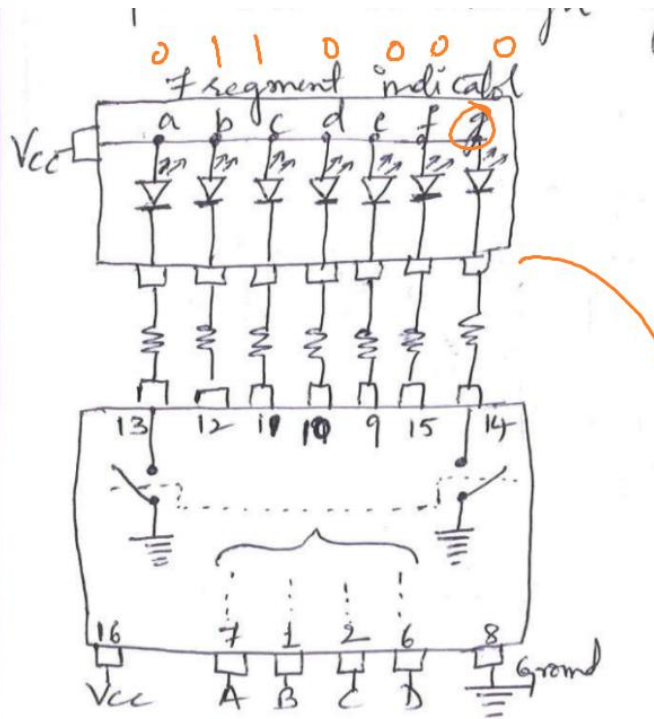


1061

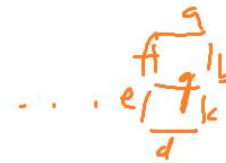
Decimal NO	BCD Inputs				Outputs (10 of lines)										MSB
	A	B	C	D	y ₀	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇	y ₈	y ₉	
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1
2	0	0	1	0	1	1	0	1	1	1	1	1	1	1	1
3	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1
4	0	1	0	0	1	1	1	1	0	1	1	1	1	1	1
5	0	1	0	1	1	1	1	1	1	0	1	1	1	1	1
6	0	1	1	0	1	1	1	1	1	1	0	1	1	1	1
7	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1
8	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1
9	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1
10	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0
11	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0
12	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0
13	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0
14	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
15	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0

These combinations forces output lines to HIGH state

7445 → generates active LOW outputs
(0000 — 1001)



1b
1c



1 0 0 1 1 1 1

0 0 0 0
0 0 0 1
0 0 1 0
.
1 0 0 0
1 0 0 1

Decimal no in BCD

Logic design using decoders: ($n : 2^n$)



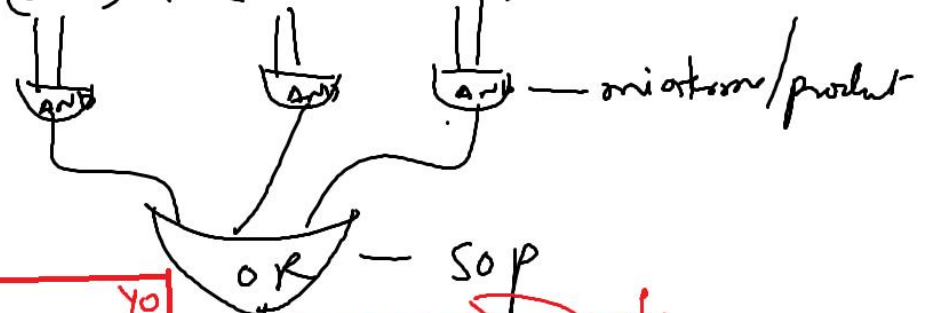
74138 LOW d/p's

$SOP = m_1 + m_2 + m_4 + m_5$

Boolean fn — Sum of minterms — SOP

minterms — product terms

$(ab) + (bc) + (ca)$

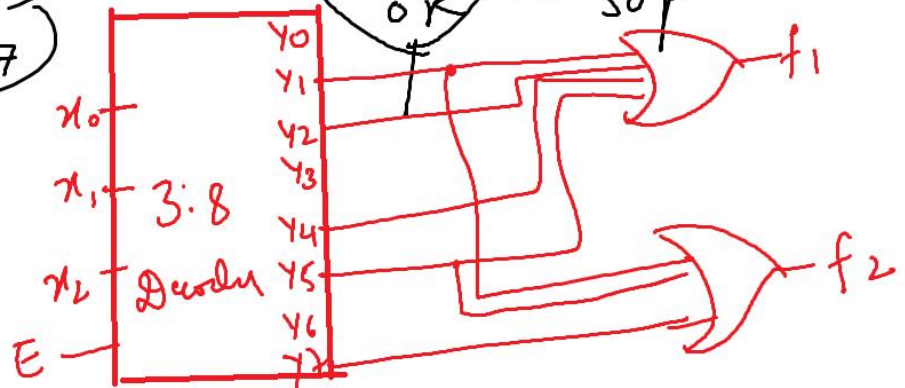


Ex: (1) $f_1(x_2, x_1, x_0) = \sum m(1, 2, 4, 5)$

$f_2(x_2, x_1, x_0) = \sum m(1, 5, 7)$

Realize decoder circuit.

(Default: Active HIGH d/p)



Realize the decoder circuit for given 2 f's

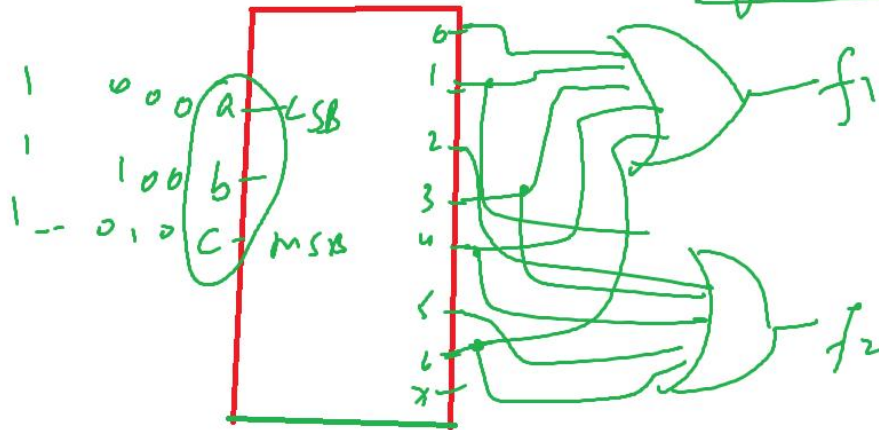
$$f_1(c, b, a) = \sum m(0, 1, 3, 4, 5, 6)$$

$$f_2(c, b, a) = \sum m(1, 2, 3, 4, 6)$$

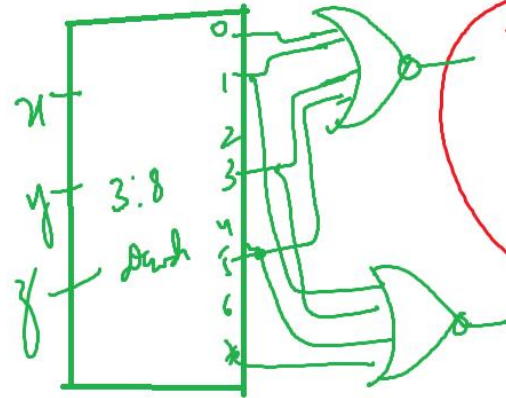
Default o/p: HIGH

$$f_1(z, y, x) = \prod M(0, 1, 3, 5)$$

$$f_2(z, y, x) = \prod M(1, 3, 6, 7)$$



$$m = \bar{M}$$



$$m = (m_1 + m_2 + m_3) \rightarrow$$

$$(m_1 + m_2 + m_3) \rightarrow$$

$$(\bar{m}_1) \cdot (\bar{m}_2) \cdot (\bar{m}_3)$$

$$M_1 \cdot M_2 \cdot M_3 \rightarrow$$

max max max } pos

Active HIGH o/p $\left\{ \begin{array}{l} \text{minterms (SOP)} - \text{OR} \\ \text{maxterms (POS)} - \text{NOR} \end{array} \right.$

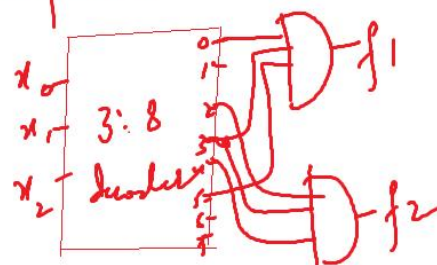
Active LOW o/p (74138) $\left\{ \begin{array}{l} \text{minterms (SOP)} - \text{NAND} \\ \text{maxterms (POS)} - \text{AND} \end{array} \right.$

$n: 2^n \rightarrow \text{NAND gates}$

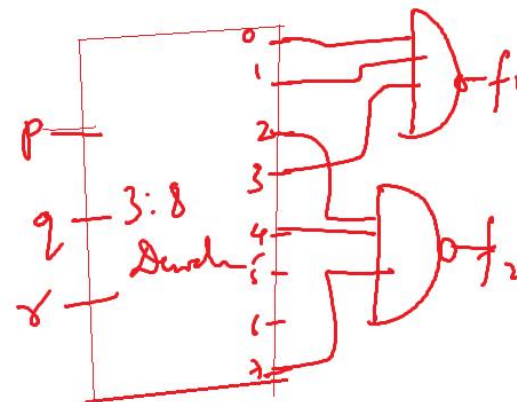
Realize decoder (Low o/p) f1 gives 2 fns:

$$1) f_1(x_2, x_1, x_0) = \sum m(0, 3, 5)$$

$$f_2(x_2, x_1, x_0) = \sum m(2, 3, 4)$$

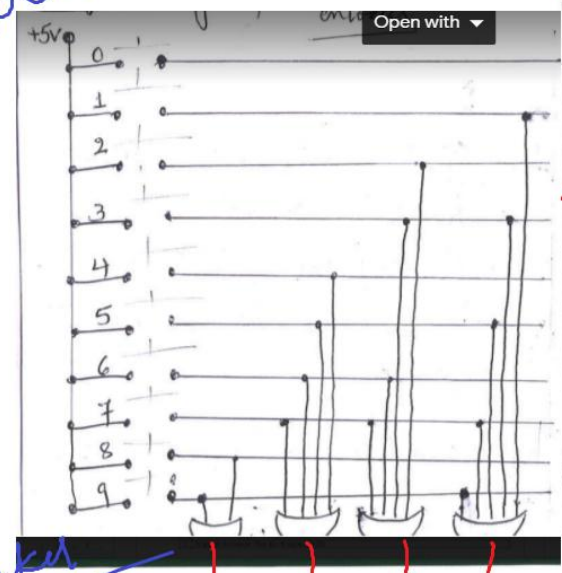
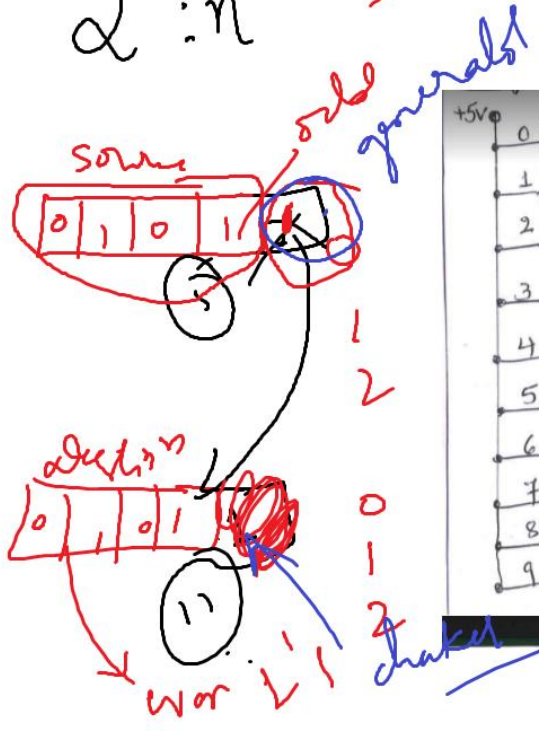


2) Realize active low o/p decoder f1 gives 2 fns:
 $f_1(x, y, p) = \sum m(0, 1, 3)$
 $f_2(x, y, p) = \sum m(2, 4, 7)$



Encoders $2^n : n$

Decoder $n : 2^n$



Decimal value	10 - Inputs										Outputs 4			
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	A	B	C	D	
0	1	1	1	1	1	1	1	1	1	1	1	1	1	
1	0	1	1	1	1	1	1	1	1	1	1	1	0	
2	0	0	1	1	1	1	1	1	1	1	1	0	0	
3	0	0	0	1	1	1	1	1	1	1	1	0	0	
4	0	0	0	0	1	1	1	1	1	1	0	1	1	
5	0	0	0	0	0	1	1	1	1	0	1	0	0	
6	0	0	0	0	0	0	1	1	1	0	0	1	1	
7	0	0	0	0	0	0	0	1	1	0	0	0	1	
8	0	0	0	0	0	0	0	0	1	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	0	0	

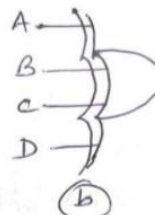
Four Inputs:



$A \oplus B$



$(A \oplus B) \oplus (C \oplus D)$



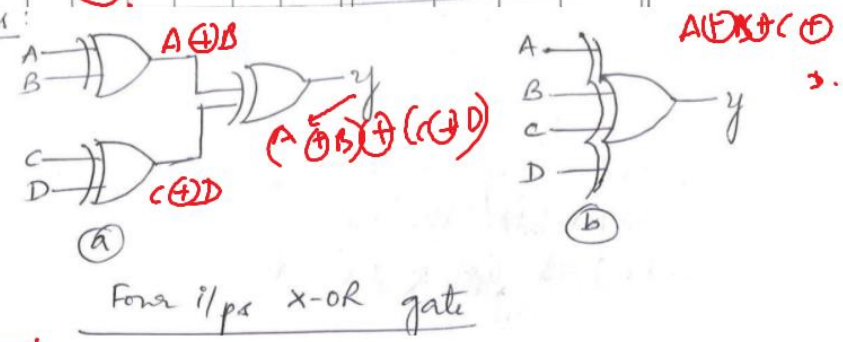
$A \oplus B$

(a) (b)

high 0/1

low 0/1

0011 1

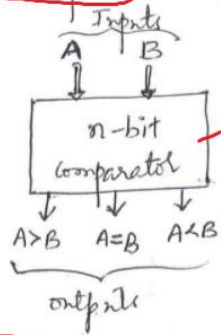


Magnitude Compar

Open with

Defⁿ: A comparator is a special combinational circuit designed primarily to compare the relative magnitude of 2-binary nos.

Block diagⁿ:



It receives two n -bit nos A and B as inputs and the outputs are $A > B$, $A = B$ and $A < B$. Depending upon the relative magnitude of the 2 nos, one of the o/p's will be high.

① Design 1-bit comparator using basic gates.

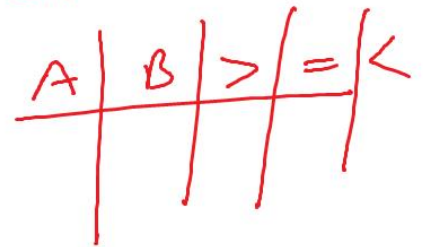
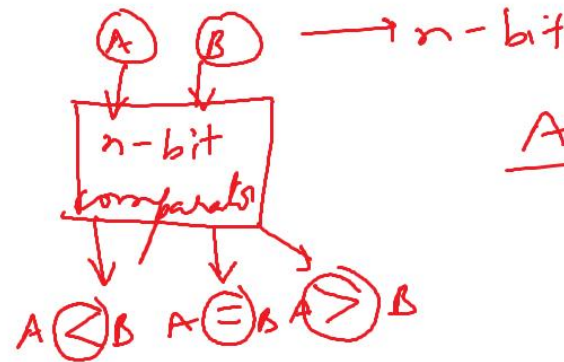
Solⁿ: Consider two 1-bit no A and B .

Inputs		Outputs		
A	B	$Y_{A>B}$	$Y_{A=B}$	$Y_{A<B}$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

$Y_{A>B}$:- k-map

A \ B	0	1
0	0	0
1	1	0

$$Y_{A>B} = A\bar{B}$$



A	B	$Y_{A<B}$	$Y_{A=B}$	$Y_{A>B}$
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

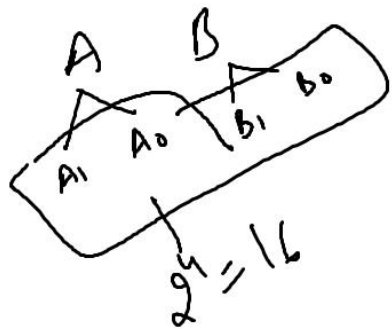
$$A\bar{B}$$

$$\bar{A}\bar{B} + AB$$

$$A \oplus B$$

$$A\bar{B}$$

2-bit comparator:



Inputs				Outputs		
A ₁	A ₀	B ₁	B ₀	A < B	A > B	A = B
0	0	0	0	0	0	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	1	0
0	1	0	1	0	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	0	0	1
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	0	0	1

$$A = B$$

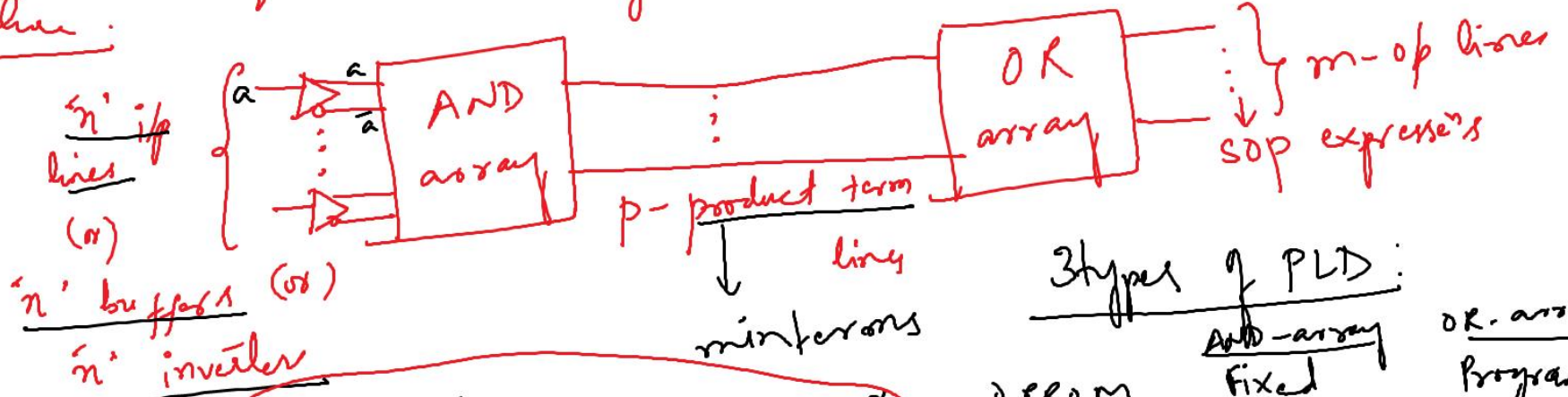
$$(A_0 \odot B_0) \cdot (A_1 \odot B_1)$$

Programmable array logic / logic array: (PAL/PLA)

Programmable logic devices (PLDs): ICs
 ARRAY of AND & OR gates

LOGIC Devices
 Combinational Sequen
NOT programming

General structure:



Buffer/inverter/not gate:

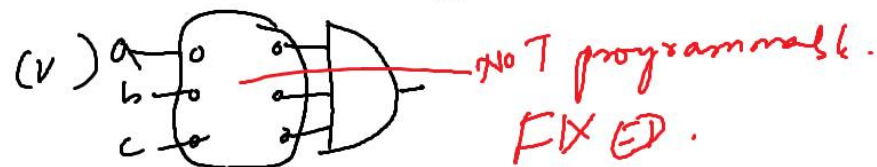
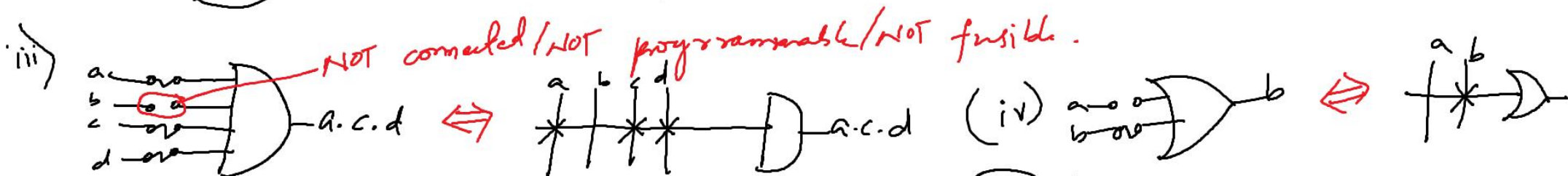
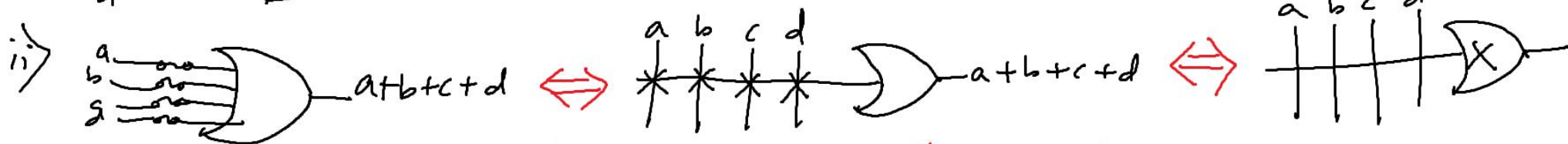
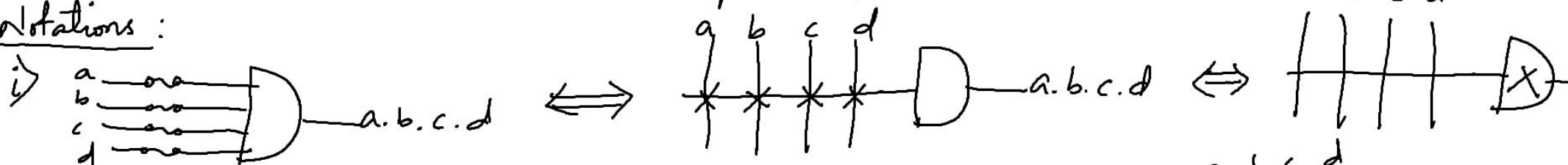


3 types of PLD:

	<u>AND-array</u>	<u>OR-array</u>
1) PROM	Fixed	Programmable
2) PLA	Programmable	Programmable
3) PAL	Programmable	Fixed

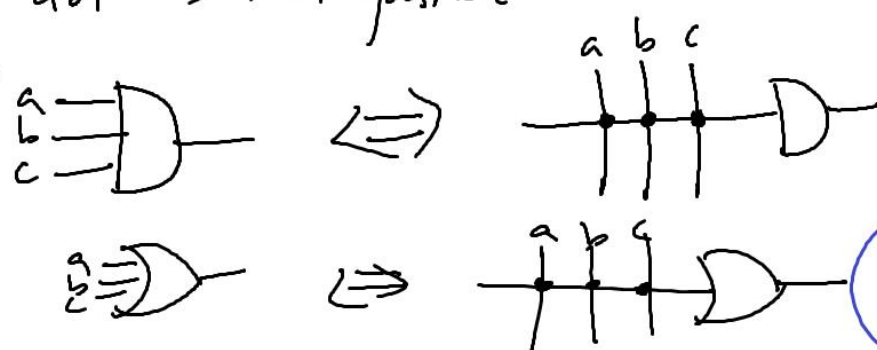
In PLDs, connections to each gate can be modified
 i.e., connect each i/p to fuse (X - fusible/programmable)

Notations:



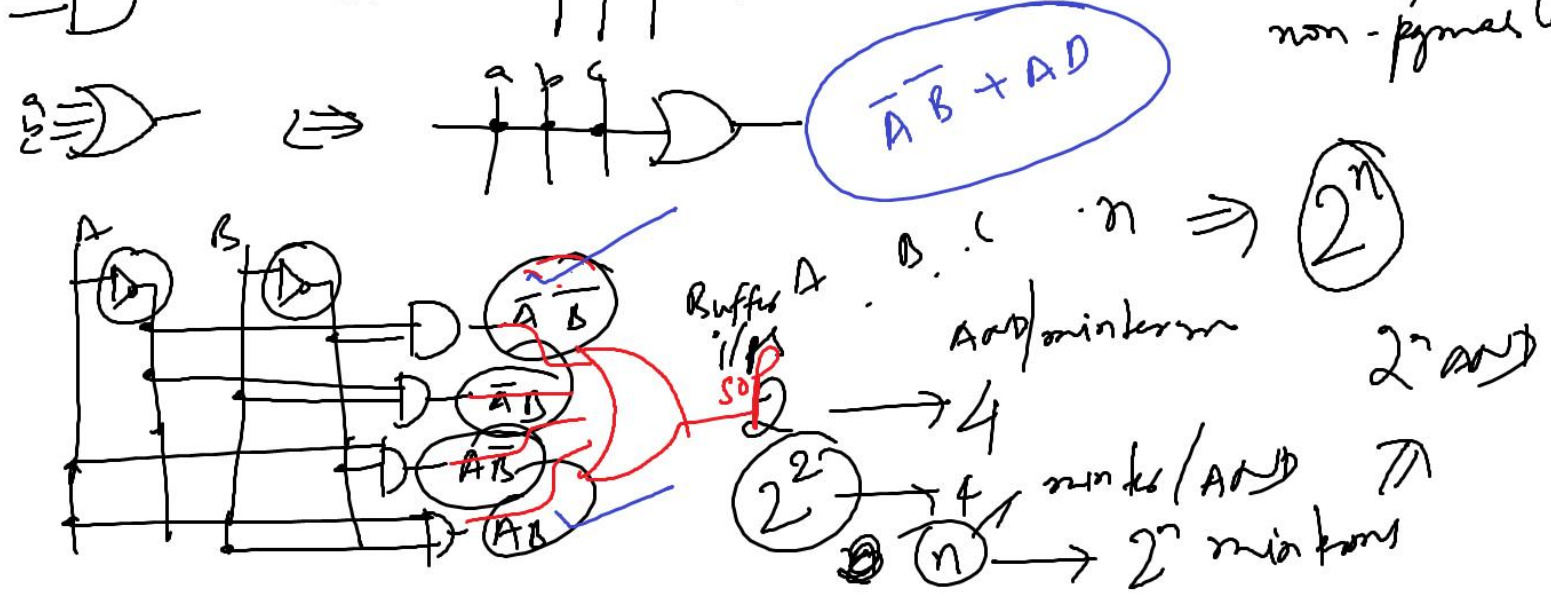
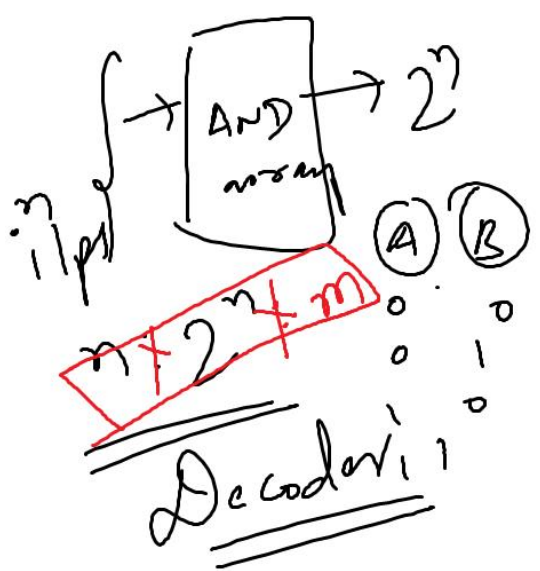
- Imp 1:
- 1) cross @ intersection $\Rightarrow$ fusible link which is intact
 - 2) lack of cross $\Rightarrow$ fuse/connection is blown (or) No connection exist
 - 3) Junction dot $\Rightarrow$ Not fusible

Non/not fusible inputs:



X - programmable

- non fusible
- non-programmable



Buffer A

Inputs $n \Rightarrow 2^n$

Add/minterm

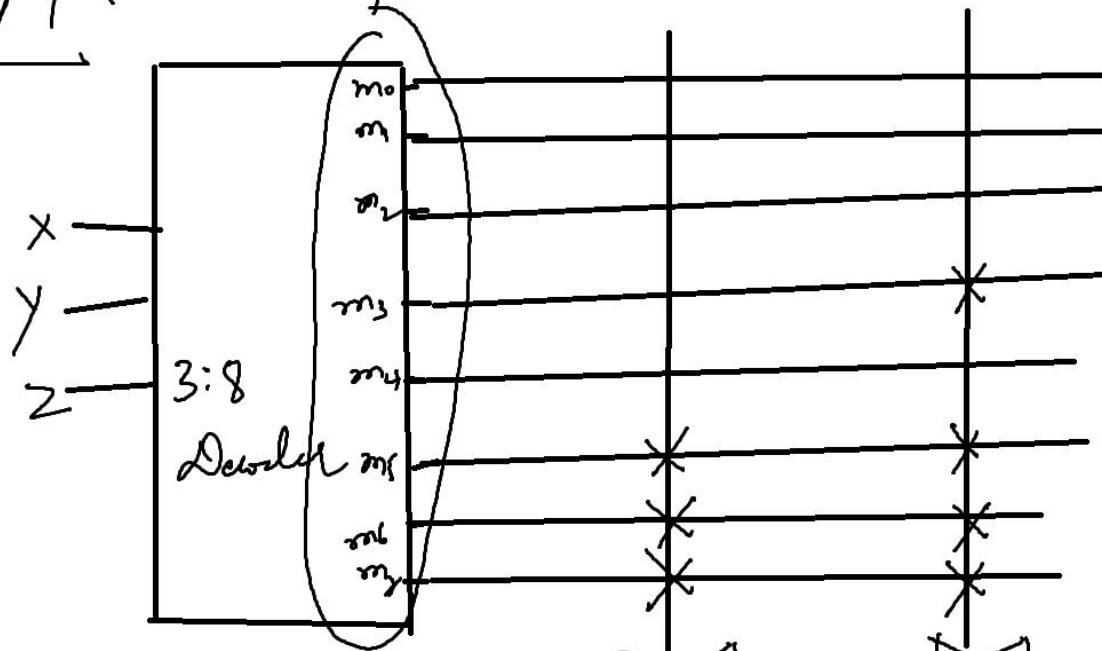
2^n AND

minis/AND $\Rightarrow$

minis

Reading PROM: $A \xrightarrow{\text{AND array fixed}} A = \{5, 6, 7\} \rightarrow B = \Sigma(3, 5, 6, 7)$

8m.



$3 \times 8 \times 2$
PROM

$A = \Sigma m(5, 6, 7)$ $B = \Sigma m(3, 5, 6, 7)$

Let us implement the following **Boolean functions** using PAL.

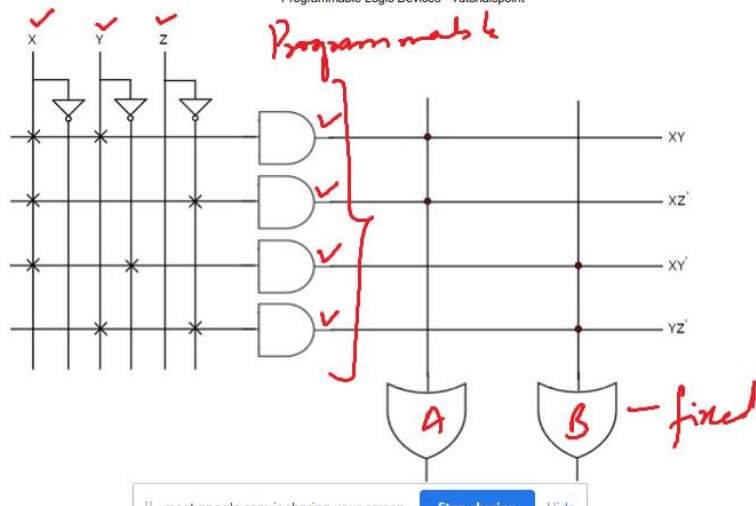
$$A = XY + XZ' = \text{sop}$$

$$B = XY' + YZ' = \text{sop}$$

The given two functions are in sum of products form. There are two product terms present in each Boolean function. So, we require four programmable AND gates & two fixed OR gates for producing those two functions. The corresponding **PAL** is shown in the following figure.

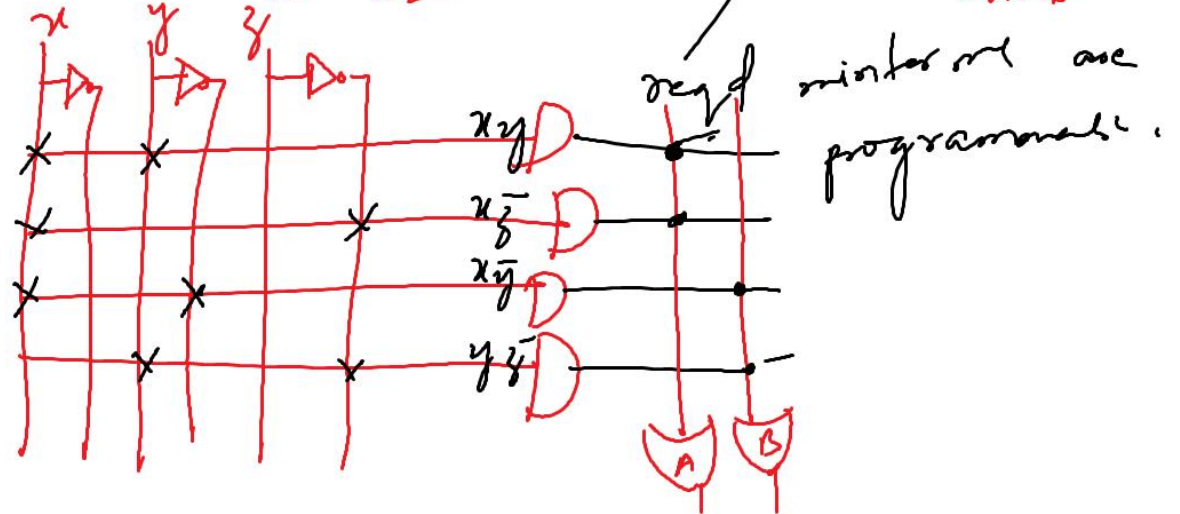
11/10/2020

Programmable Logic Devices - Tutorialspoint



PAL : $A = \textcircled{xy} + \textcircled{x\bar{z}}$ $\rightarrow 2$ minterms
 $B = \textcircled{x\bar{y}} + \textcircled{yz'}$ $\rightarrow 2$ minterms

xyz — 3 variable $\rightarrow 2^3 = 8$ combinations
 For 3-variables
 8 — minterms/product terms



Example

Let us implement the following **Boolean functions** using PLA.

$$A = XY + XZ'$$

$$B = XY' + YZ + XZ'$$

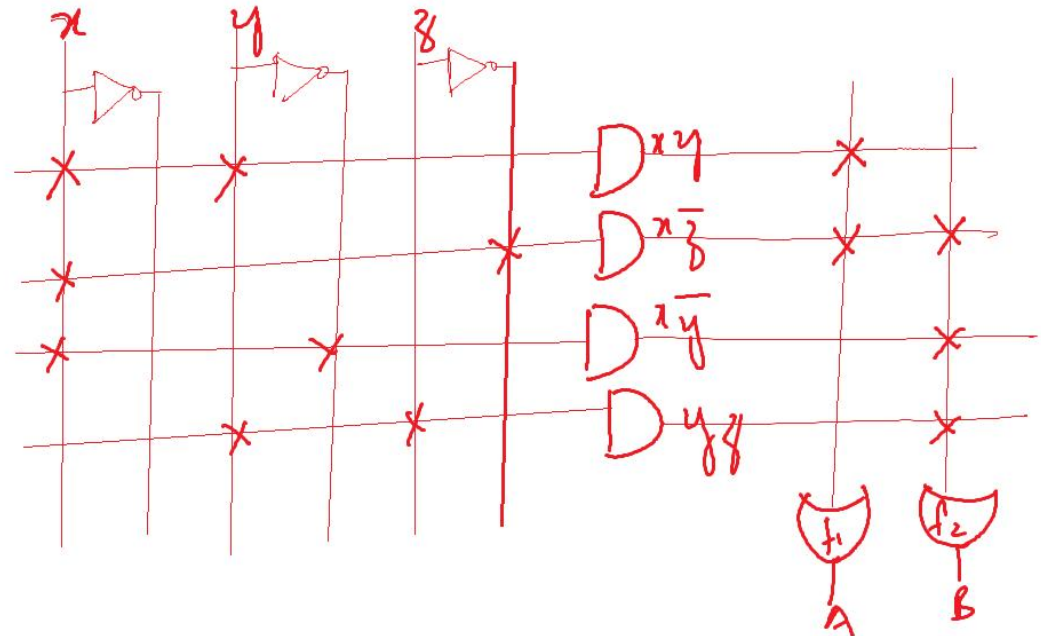
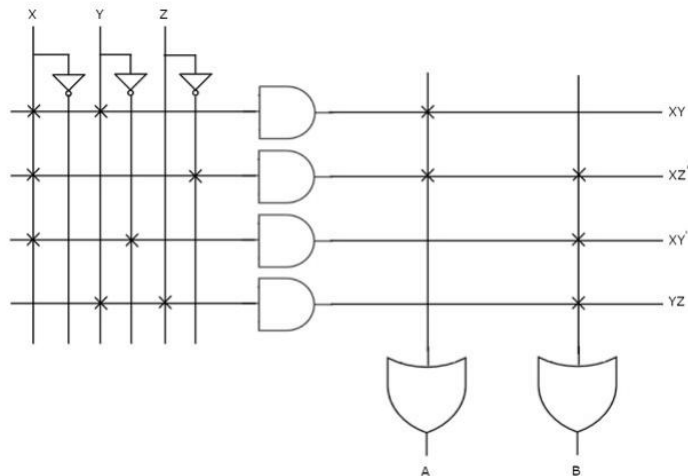
PLA: Both AND arrays and OR-arrays are programmable.
3 variables

$$A = x\check{y} + x\bar{z}$$

$$B = x\bar{y} + yz + x\bar{z}$$

The given two functions are in sum of products form. The number of product terms present in the given Boolean functions A & B are two and three respectively. One product term, $Z'X$ is common in each function.

So, we require four programmable AND gates & two programmable OR gates for producing those two functions. The corresponding **PLA** is shown in the following figure.



1) Realize the PROM for given 2 fns

$$f_1(A, B, C) = \sum m(0, 1, 2, 5, 7)$$

$$f_2(A, B, C) = \sum m(0, 1, 2, 4, 6)$$

2) Realize the PLA for given 2 fns

(1) D

$$\text{Ex: } f_1(x, y, z) = \sum m(0, 1, 3, 4)$$

$$f_2(x, y, z) = \sum m(1, 2, 3, 4, 5)$$

3) Realize the PAL for given 2 fns

$$f_1(x, y, z) = \sum m(1, 2, 4, 5, 7)$$

$$f_2(x, y, z) = \sum m(0, 1, 3, 5, 7)$$

PLDs: PROM — ^{AND} fixed ^{OR} programmable $\rightarrow n \times 2^n \times m$ — PROM
 IC
 PLA — programmable programmable
 PAL — programmable fixed

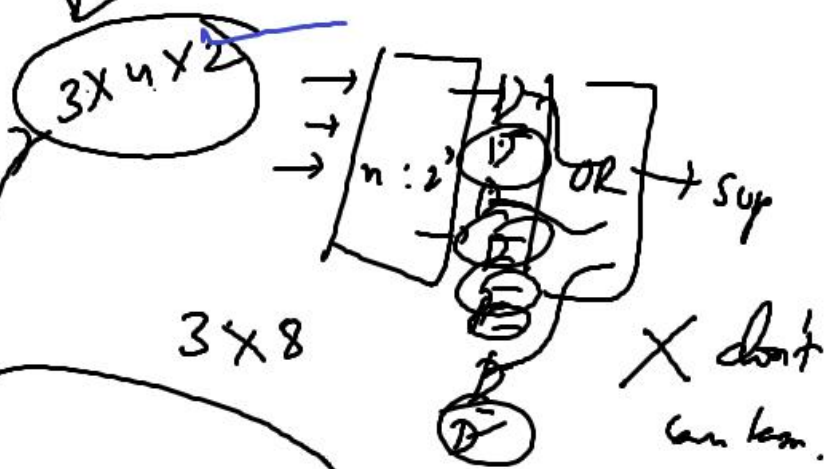
$3 \times 8 \times 2$ — PROM

$n \times p \times m$

$$xy + yz$$



$$PAL = n \times p \times m = 16 \times 48 \times 8$$



$$PROM = n \times 2^n \times m = 16 \times 2^{16} \times 8$$

X don't
can learn.

Realize PLA using 2 fns

1) $f_1(x, y, z) = \sum m(0, 1, 3, 4)$

$f_2(x, y, z) = \sum m(1, 2, 3, 4, 5)$

Soln

f_1 $\times$ y, z

	00	01	11	10
0	1	1	0	0
1	1	0	0	0

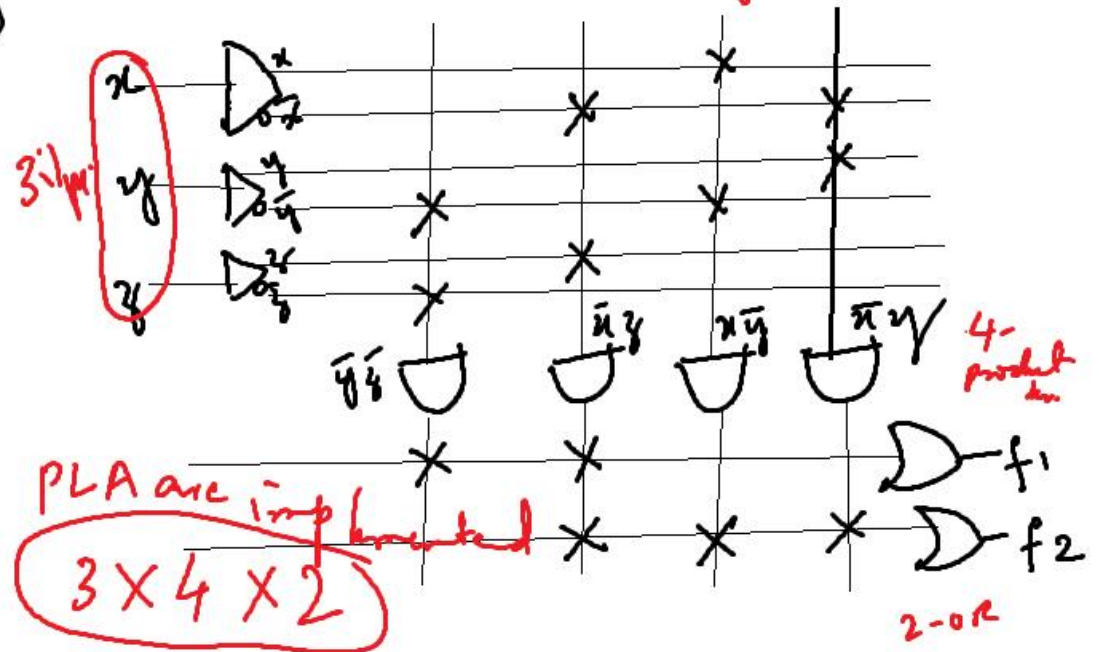
$\checkmark$
 $= \underline{\underline{\bar{y}\bar{z} + xz}}$

f_2 $\times$ y, z

	00	01	11	10
0	0	1	1	1
1	1	1	0	0

$= \underline{\underline{x\bar{y} + x\bar{z} + \bar{x}y}}$

— both AND & OR arrays are programmable



② Realize/Implement PLA (3x4x2) for the foll boolean fn.

$$f_1(a, b, c) = \sum m(0, 1, \textcircled{3}, \textcircled{5})$$

$$f_2(a, b, c) = \sum m(\textcircled{3}, \textcircled{5}, 7)$$

Solⁿ: f_1

	bc	00	01	11	10
a	0	1	1	1	0
1	0	1	1	0	0

$$= \bar{a}\bar{b} + \bar{a}c + \bar{b}c$$

No common product

f_2

	bc	00	01	11	10
a	0	0	0	1	0
1	0	1	1	0	0

$$= ac + bc$$

$$3 \times \textcircled{5} \times 2 \rightarrow 4$$

f_1

	bc	00	01	11	10
a	0	1	1	1	0
1	0	1	1	0	0

$$= \bar{a}\bar{b} + \bar{a}c + \textcircled{a\bar{b}c}$$

f_2

	bc	00	01	11	10
a	0	0	0	1	0
1	0	1	1	0	0

$$= bc + \textcircled{a\bar{b}c}$$

$$\therefore f_1 = \bar{a}\bar{b} + \bar{a}c + a\bar{b}c$$

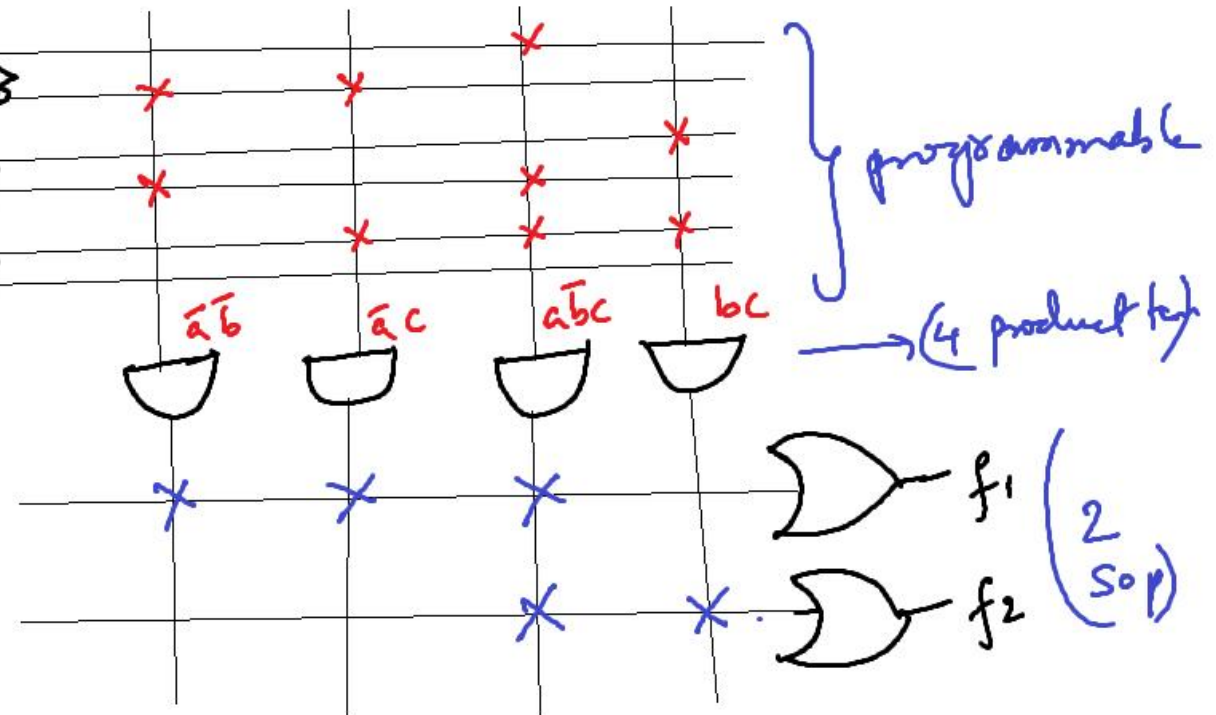
$$f_2 = bc + a\bar{b}c$$

$$f_1 = \bar{a}\bar{b} + \bar{a}c + a\bar{b}c$$

$$f_2 = bc + a\bar{b}c$$

3-buffer
i/p's

3x4x2 — PLA



PAL: Programmable Array Logic ($\text{AND}_{\text{array}}$ is programmable, OR_{array} is fixed)

Ex. $f_1(x, y, z) = \sum m(1, 2, 4, 5, 7)$

$f_2(x, y, z) = \sum m(0, 1, 3, 5, 7)$

Soln: f_1

	00	01	11	10
0	0	1	0	1
1	1	1	1	0

$$= \underline{x\bar{y}} + \underline{xz} + \underline{\bar{y}z} + \underline{\bar{x}y\bar{z}}$$

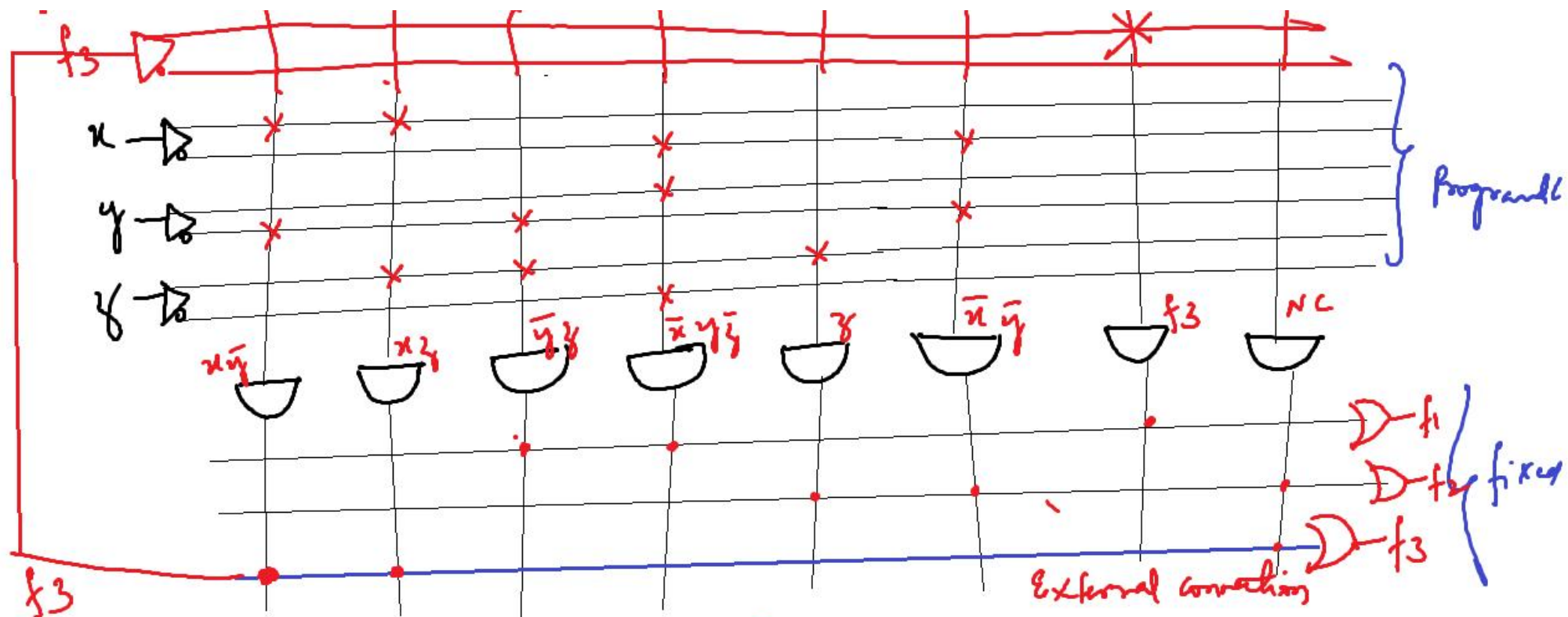
4 product terms

f_2

	00	01	11	10
0	1	1	1	0
1	0	1	1	0

$$= \underline{z} + \underline{\bar{x}\bar{y}}$$

2 pr



$$\bar{x}\bar{y} + xz + \bar{y}z + \bar{x}y\bar{z}$$

4 product terms
(max only 3 terms are realizable in PAL)

$$z + \bar{x}\bar{y}$$

$$f_1 = f_3 + \bar{y}z + \bar{x}y\bar{z}$$

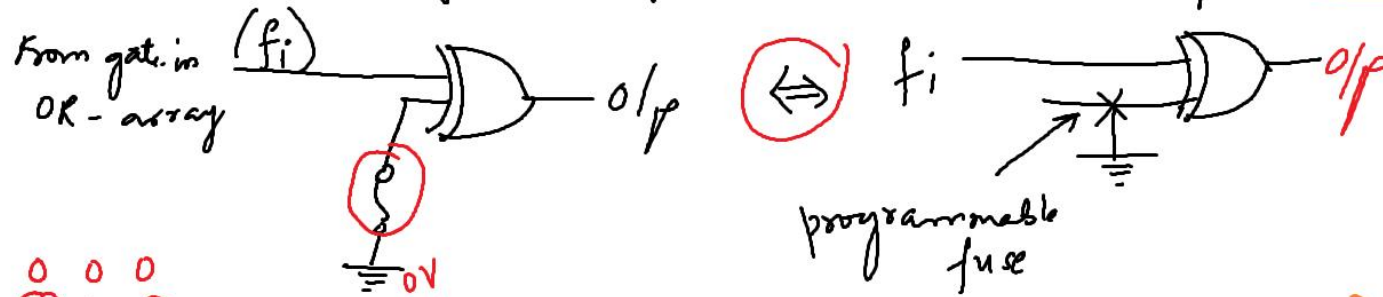
$$f_2 = z + \bar{x}\bar{y} + \text{NC}$$

$$f_3 = x\bar{y} + xz + \text{NC}$$

The informⁿ stored in the mem-elements @ any given time defines the present state of the Seq ckt. The present state and external i/p determine the outputs and the next state of the Seq ckt. Thus we can specify the Seq ckt by the TIME SEQUENCE of external i/p, internal states (present & next states), & o/p. Eg. Counters & Registers are common examples of Seq ckt.

Comparison: Combinational ckt	Sequential ckt
O/p of <u>vars</u> - all times depend on the combination of i/p <u>vars</u>	The o/p <u>vars</u> depend NOT only on the present i/p <u>vars</u> but also depend upon the past history of these.
Memory Unit - NOT reqd	Reqd to store the past history of i/p <u>vars</u> .
Speed - Faster, $\because$ delay b/w i/p & o/p is due to propagation delay of gates	Slower
Design - Easy to design	Comparatively harder to design

For greater flexibility, PLA provide true o/p (or) complemented o/p. USE EX-OR gates



Fuse intact = ground = logic 0
 $f_i \oplus 0 = f_i$ (same as i/p)

Fuse blown $\Rightarrow +V = \text{logic } 1$

$f_i \oplus 1 = \bar{f}_i$ (complemented o/p)

0	0	0
1	0	1
1	1	0

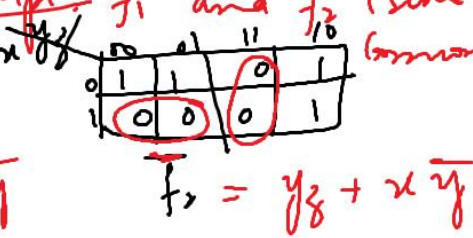
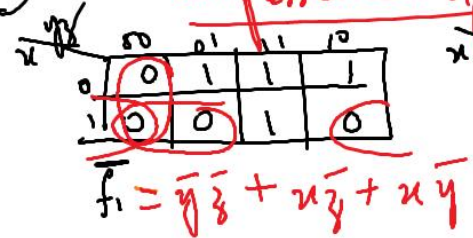
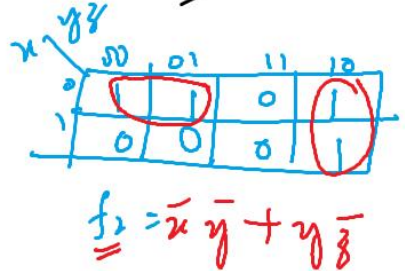
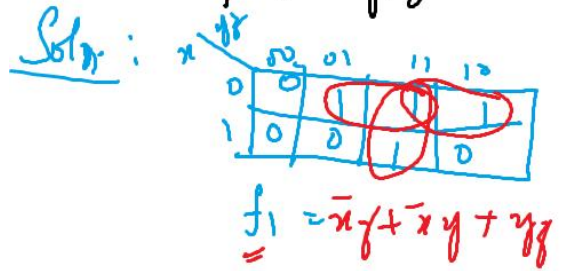
Implement PLA for the

$f_1(x, y, z) = \sum m(1, 2, 3, 7)$

$f_2(x, y, z) = \sum m(0, 1, 2, 6)$

$f_1 = \sum m(0, 4, 5, 6)$
 $f_2 = \sum m(3, 4, 5, 7)$

Complemented o/p: $\bar{f}_1$ and $\bar{f}_2$ (since NO common product terms)



$$f_1 = \bar{x}z + \bar{x}y + yz$$

$$f_2 = \bar{x}\bar{y} + y\bar{z}$$

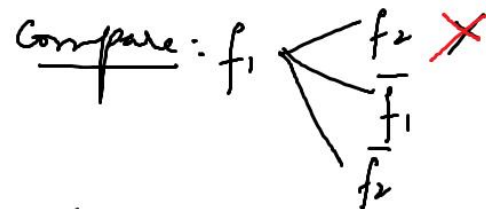
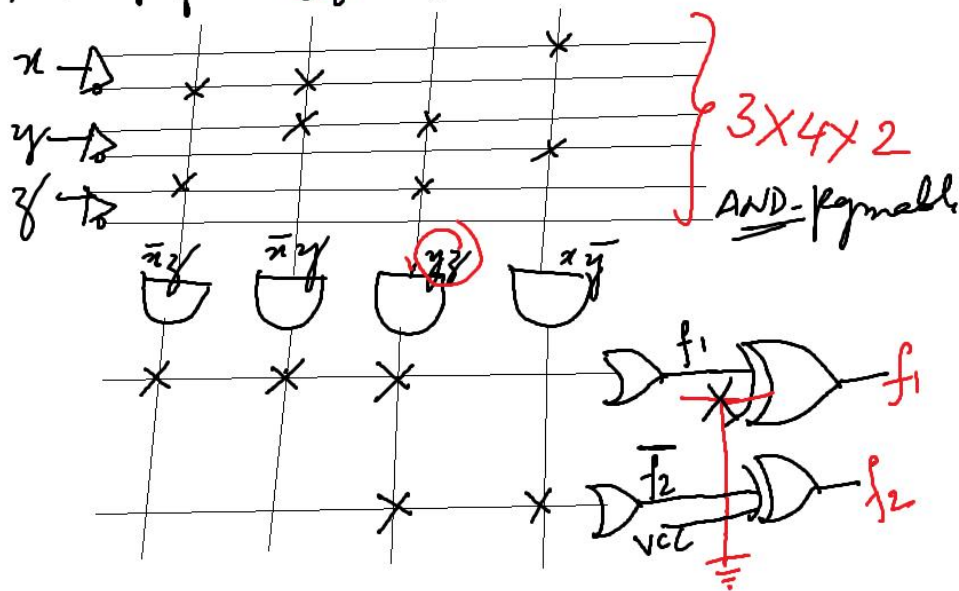
$$\bar{f}_1 = \bar{y}\bar{z} + x\bar{z} + x\bar{y}$$

$$\bar{f}_2 = yz + x\bar{y}$$

Case i: f_1 and $\bar{f}_2$ Ex-or gate:

$$f_1 \oplus \bar{f}_2(x, y, z) = \bar{x}z + \bar{x}y + yz$$

$$\bar{f}_2(x, y, z) = yz + x\bar{y}$$

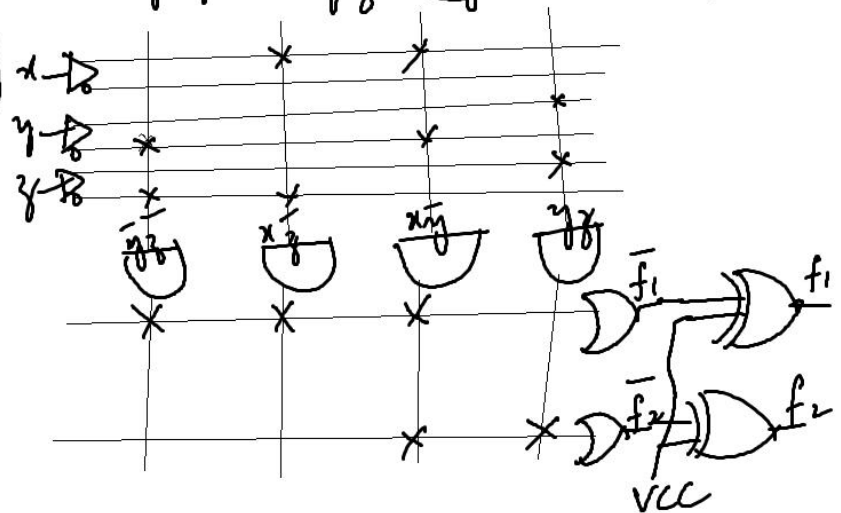


Case (ii): $\bar{f}_1$ and $\bar{f}_2$

$$\bar{f}_1(x, y, z) = \bar{y}\bar{z} + x\bar{z} + x\bar{y}$$

$$\bar{f}_2(x, y, z) = yz + x\bar{y}$$

total 4 product



$$f_i \oplus 0 = f_i \Rightarrow f_i$$

$$f_i \oplus 1 = \bar{f}_i \Rightarrow \bar{f}_i$$

$$\bar{f}_i \oplus 0 = \bar{f}_i$$

$$\bar{f}_i \oplus 1 = f_i$$

$$\bar{f}_i \oplus 1 = \bar{f}_i = f_i$$

+5V = logic 1

Common way of specifying the connections in PLA - PLA table

Case (i): f_1 and $\bar{f}_2$ Ex-or gate:

$$\begin{cases} f_1(x, y, z) = \bar{x}z + \bar{x}y + \bar{y}z \\ f_2(x, y, z) = yz + x\bar{y} \end{cases} \Rightarrow \text{Ex-or} \Rightarrow f_2$$

PLA table: case (i)
Product terms

4-rows

	Inputs			Outputs	
	x	y	z	f ₁	f ₂
$\bar{x}z$	0	-	1	1	-
$\bar{x}y$	0	1	-	1	-
yz	-	1	1	1	1
$x\bar{y}$	1	0	-	-	1

T/C
1 - true value
0 - complement

Inputs
0 → complemented form
1 → uncomplemented form
- → NO connection

Case (ii): $\bar{f}_1$ and $\bar{f}_2$

$$\begin{cases} \bar{f}_1(x, y, z) = \bar{y}\bar{z} + x\bar{z} + x\bar{y} \\ \bar{f}_2(x, y, z) = yz + x\bar{y} \end{cases}$$

f_1 - Ex-or f_2 - Ex-or

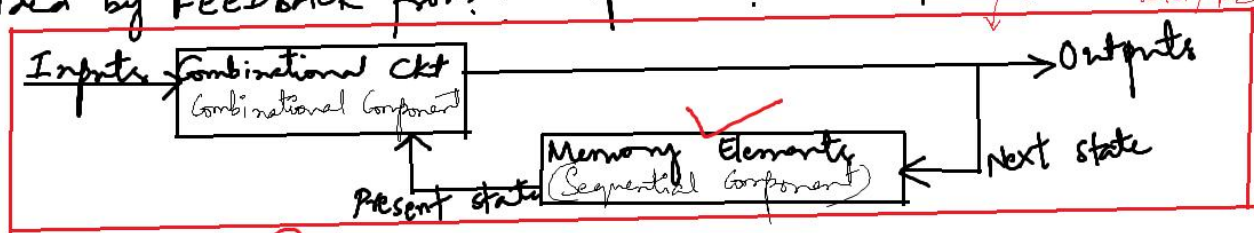
Product terms

	Inputs			Outputs	
	x	y	z	f ₁	f ₂
$\bar{y}\bar{z}$	-	0	0	1	-
$x\bar{z}$	1	-	0	1	-
$x\bar{y}$	1	0	-	1	1
yz	-	1	1	-	1

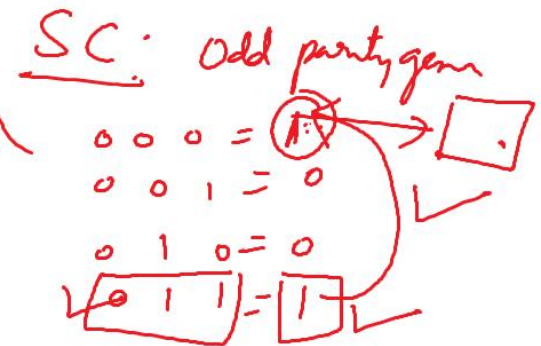
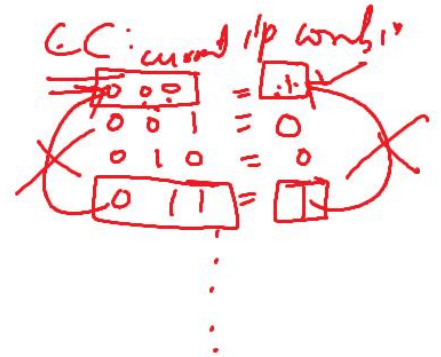
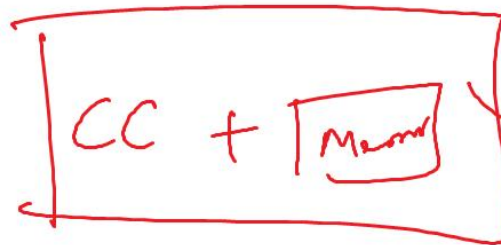
Outputs
1 → connection exists b/w AND-gate & OR-gate
- → NO connection exists

Unit -3 : Flip-flops

There are many applications in which digital o/p's are reqd to be generated in accordance with the sequence in which the input signals are received. This reqt can NOT be satisfied using a combinational logic system. These applications req o/p to be generated that are NOT only depend on the present i/p cond's but also depend up-on the past history of these i/p's. The past history is provided by FEEDBACK from the o/p back to the i/p.

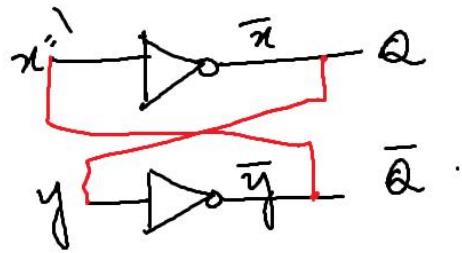


Sequential ckt



Block diagram of sequential ckt / FSM (Finite State M/c)

The basic bistable element:



When $x=0$: $Q = \bar{x} = 1$

$\therefore y=1$ $\therefore \bar{Q} = \bar{y} = 0$
 $\therefore Q = \bar{x} = y = 1$ and $\bar{Q} = \bar{y} = x = 0$
 $\boxed{Q=1}$ & $\boxed{\bar{Q}=0}$

When $x=1$: $Q = \bar{x} = 0$

$y=0$ & $\bar{Q} = \bar{y} = 1$
 $\therefore Q = \bar{x} = y = 0$ & $\bar{Q} = \bar{y} = x = 1$
 $\boxed{Q=0}$ & $\boxed{\bar{Q}=1}$

2 i/p's : x & y (two stable conditions or states) $\rightarrow$ Complemented o/p
2 o/p's : Q & $\bar{Q}$ $\rightarrow$ normal o/p
 2 cross-coupled NOT gates - o/p of 1st NOT gate serves as i/p to 2nd NOT gate & o/p of 2nd NOT gate serves as i/p to 1st NOT gate $\rightarrow$ feedback

When $Q=0$, then $\bar{Q}=1$ & $Q=1$ & $\bar{Q}=0$

$\therefore Q$ & $\bar{Q}$ are complementary.

Positive logic : o/p $Q=1 \rightarrow 1\text{-state (SET)}$
 o/p $Q=0 \rightarrow 0\text{-state (RESET)}$

Equilibrium condn: 2 o/p signals are about half way b/w logic-0 & logic-1 - o/p is NOT valid - metastable state.

Amount of time a device stays in met state is unpredictable due to ckt noise.
 $\therefore$ Must be avoided

$$\left. \begin{array}{l} Q=0 \\ \bar{Q}=1 \end{array} \right\} \text{valid} \quad \left. \begin{array}{l} Q=1 \\ \bar{Q}=0 \end{array} \right\} \text{valid}$$

Latches: storage device
 $\hookrightarrow$ o/p immediately responds to change in i/p-lines.

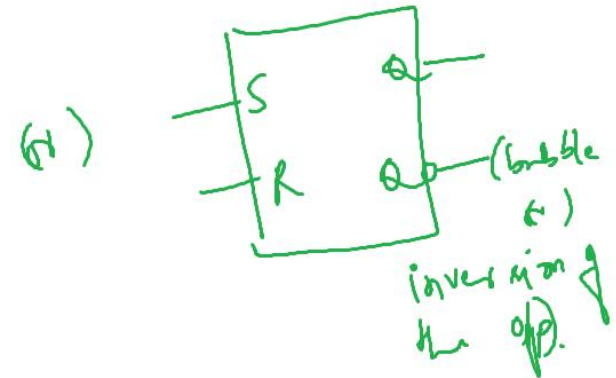
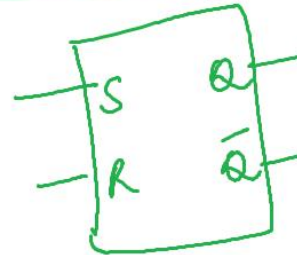
i) The SET & the RESET latch (SR Latch):

$\rightarrow$ 2 cross coupled NOR gates

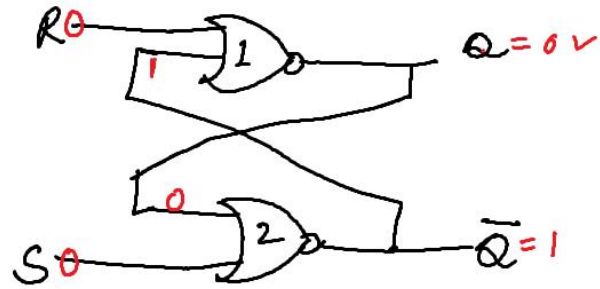
$\rightarrow$ Two i/p's (i) S-set
 (ii) R-reset

$\rightarrow$ two o/p's: (i) $\bar{Q}=0$ $\left\{ \begin{array}{l} \text{Normal for } \bar{Q}=1 \\ Q=0 \end{array} \right.$
 (ii) $Q=1$ $\left\{ \begin{array}{l} \bar{Q}=0 \\ Q=1 \end{array} \right.$

Block diagram:



SR Latch:



2i/μ:

S	R	Q?
0	0	No change
0	1	Reset
1	0	Set
1	1	?

TT of NOR

S	R	Q	Q-bar
0	0	0	1
0	1	0	0
1	0	1	0
1	1	0	0

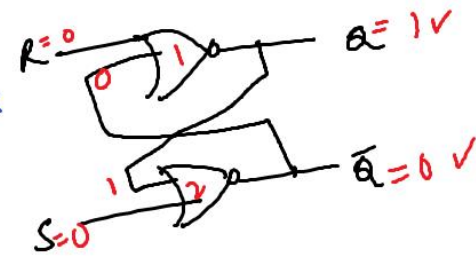
When 1 of the i/p is 1 $\Rightarrow$ op = 0.

Case 1: $S=0$ and $R=0$. (Initially assume $Q=0$)

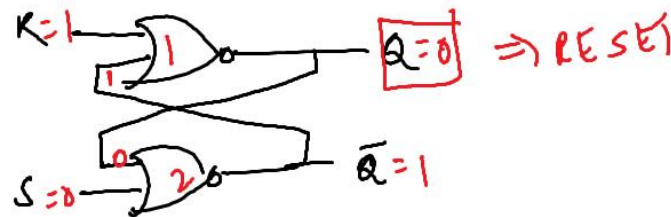
$\bar{Q}=1$ when $Q=0 \rightarrow$ stable $\Rightarrow$ No change

$S=0$ and $R=0$ (Initially assume $Q=1$)

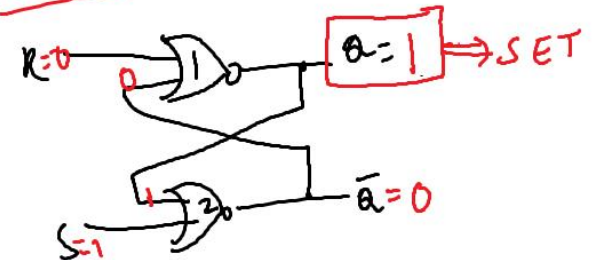
$\bar{Q}=0$ when $Q=1 \rightarrow$ stable $\Rightarrow$ No change



Case 2: $S=0$ and $R=1$



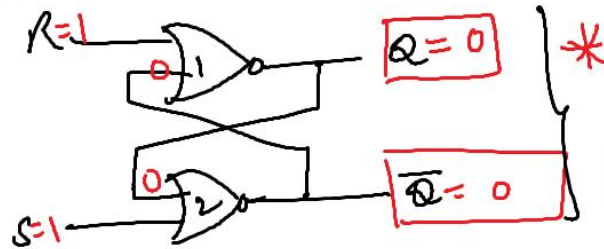
Case 3: $S=1$ and $R=0$



SR contd...

Case (iv) : $S=1$ and $R=1$

NOT
 $0 \ 0 \rightarrow 1$
 $1 \ 0 \rightarrow 0$
 $0 \ 1 \rightarrow 0$
 $1 \ 1 \rightarrow 0$



* Unable to determine $\Rightarrow$ unpredictable behavior.

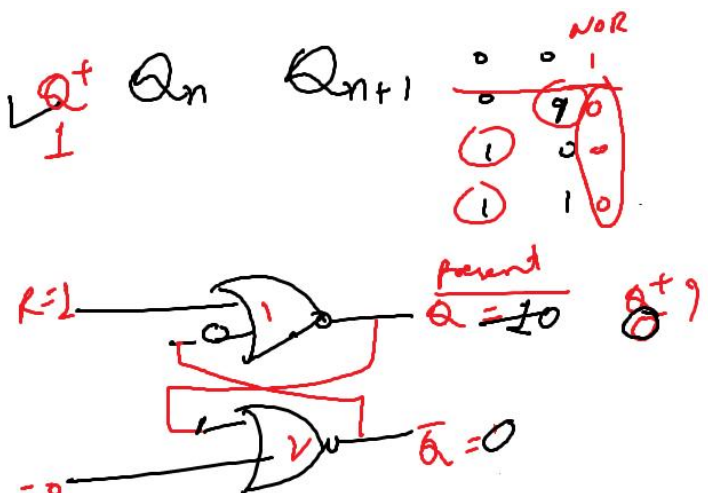
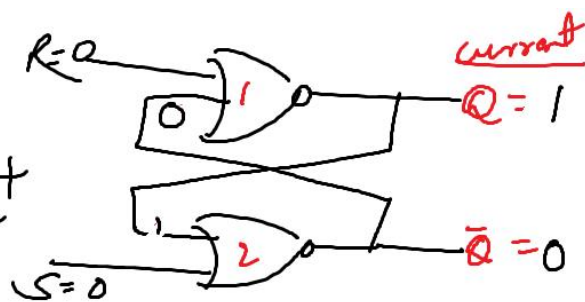
S	R	Q
0	0	No change
0	1	Reset
1	0	Set

1 1 * $\rightarrow$ Unpredictable
 forbidden
 if condⁿ

$Q=0 \Rightarrow \bar{Q}=1$
 $Q=1 \Rightarrow \bar{Q}=0$ } Q & $\bar{Q}$ are complementary.

In this case $S=1$ & $R=1$, Q & $\bar{Q}$ are NOT complementary (Metastable state).

S	R	Q	o/p
0	0	0	present state = Q
0	1	0	
1	0	1	next state = Q ⁺
1	1	*	forbidden state



$Q=0, \bar{Q}=1 \Rightarrow$ stable state

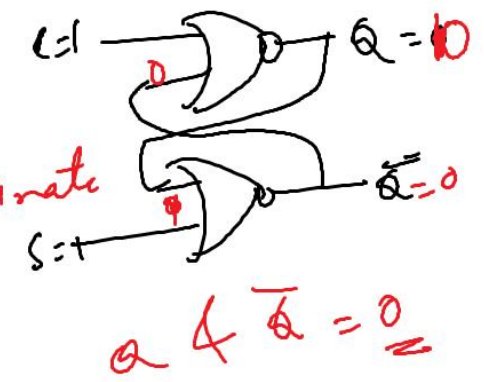
$Q=1, \bar{Q}=0 \Rightarrow$ stable state

S = set
R = reset

S	R	present Q	next state Q ⁺
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1

S	R	Q	Q ⁺
1	1	0	0
1	1	1	0

Indeterminate
forbidden state



S	R	o/p Q
0	0	Q
0	1	0
1	0	1
1	1	*

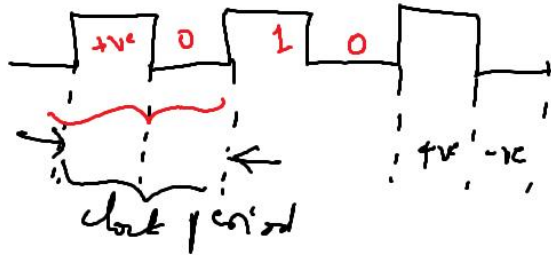
JK flip flop:

S	R	current state Q	Q^+ → Next state
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

{ Q }
 { Reset }
 { Set }
 { Invalid }
 forbidden

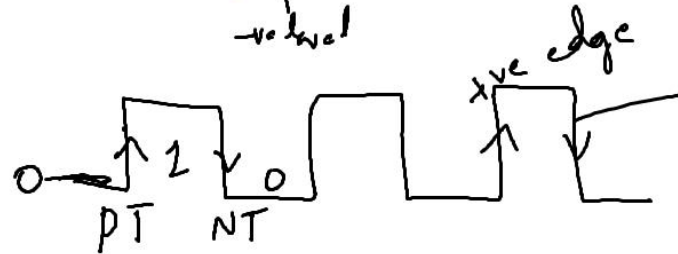
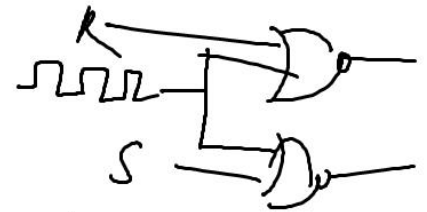
Invalid } Disadvantage of SR flip flop.

Clock Signal : is a particular type of signal that oscillate b/w a HIGH & LOW state & it is utilized to co-ordinate the actions of the circuit (ckt). It is produced by the clock-generator.



Synchronous per in clock

Digital signal - Square wave
Analog " - Sine wave

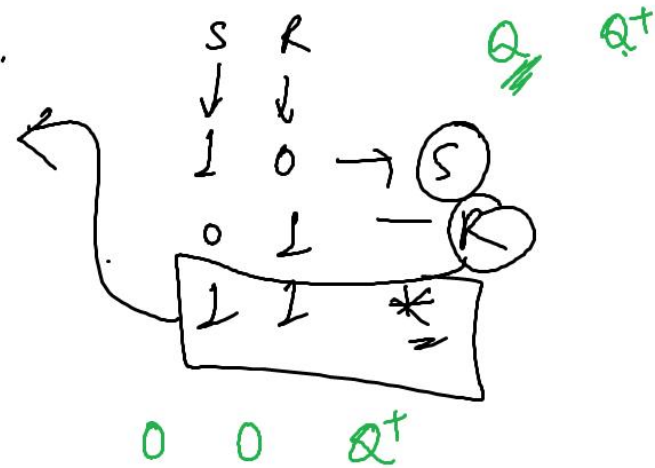
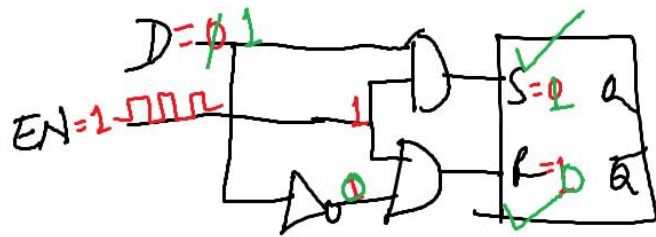


-ve edge

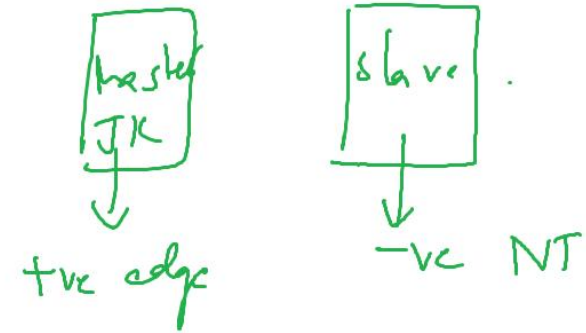
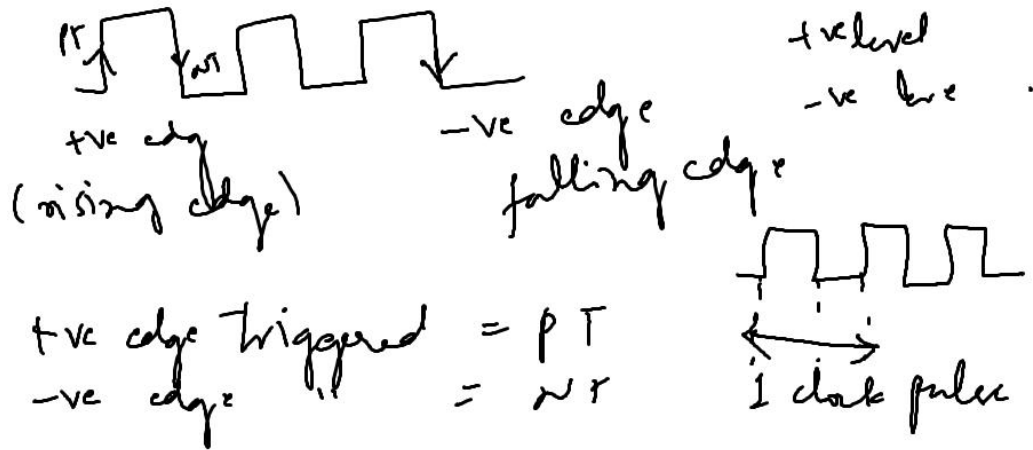
S	R	EN	Q
0	0	1	0
0	1	1	1
1	0	1	0
1	1	1	1

clocked SR FF

clocked D Flip flop:
JK flip flop
Data flip-flop : D



EN	D	Q ⁺
0	X	Last state(Q)
1	0	0 ✓
1	1	1 ✓



JK flip flop: Overcome the shortcoming of SR forbidden state.

Edge-triggered flip flop: t+ve edge trigger JK ff

: PT JK ff

Various representⁿ of F.F:

- ① Characteristic eqⁿ
- ② FSM = state transⁿ diagram
- ③ Excitation table

Characteristic eqⁿ: (J.K)
 3 vari^s J, K, Q $\rightarrow Q^+$

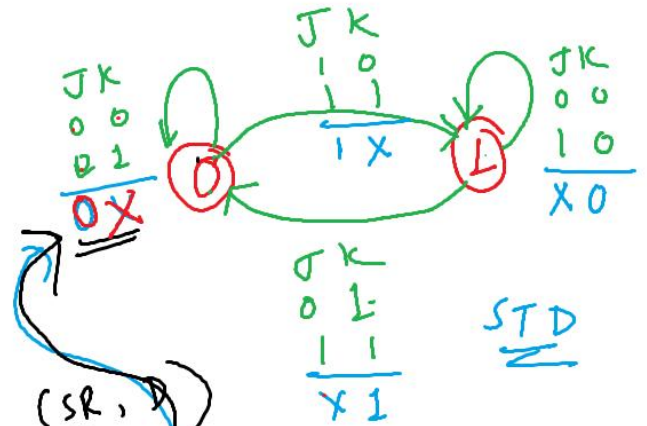
J \ K	00	01	11	10
0	0	0	1	1
1	1	0	0	1

$$Q^+ = \bar{Q}J + Q\bar{K}$$

$$\therefore \boxed{Q^+ = J\bar{Q} + \bar{K}Q}$$

J	K	Q^+	Q^+
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

next o/p SR: $S + \bar{R}Q$
 $D: D$



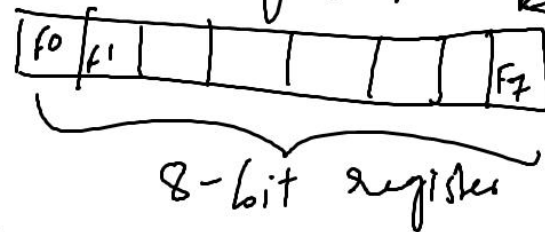
② State Transⁿ diagram

③ Excitation table:

Q	Q^+	J	K	S	R	D
0	0	0	X	0	X	0
0	1	1	X	1	0	1
1	0	X	1	0	1	0
1	1	X	0	X	0	1

$\boxed{0} \rightarrow 1\text{-bit of information}$

Register: Collection of flip-flops



8-bit register

= 1 char A

$A = A \ll 2$ twice
left

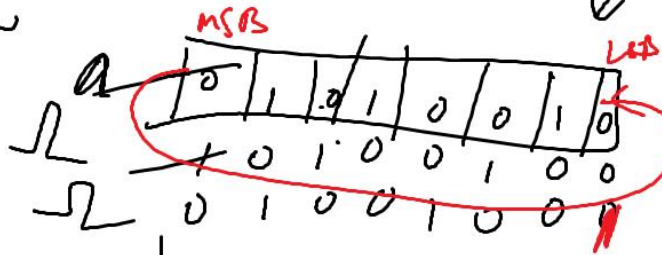
4 types of SR

S I S D $\Rightarrow$ serial-in $\Rightarrow$ serial-out
S I P D $\Rightarrow$
P I S D $\Rightarrow$
P I P D $\Rightarrow$ parallel out

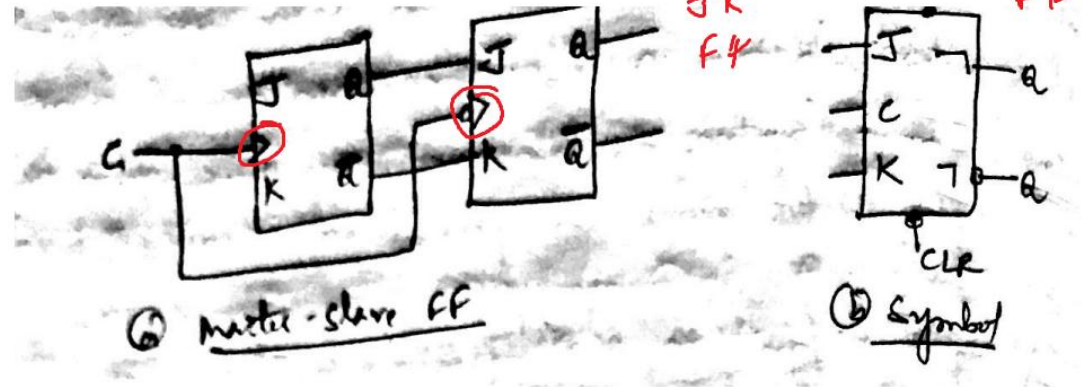
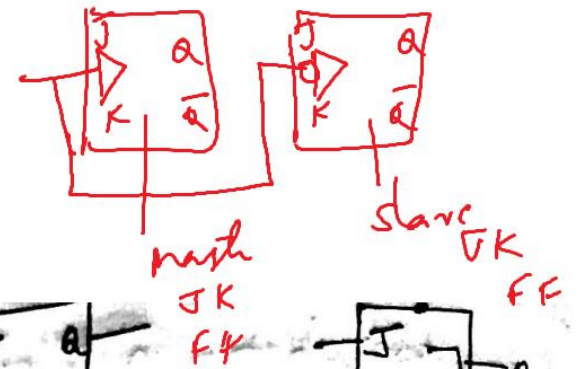
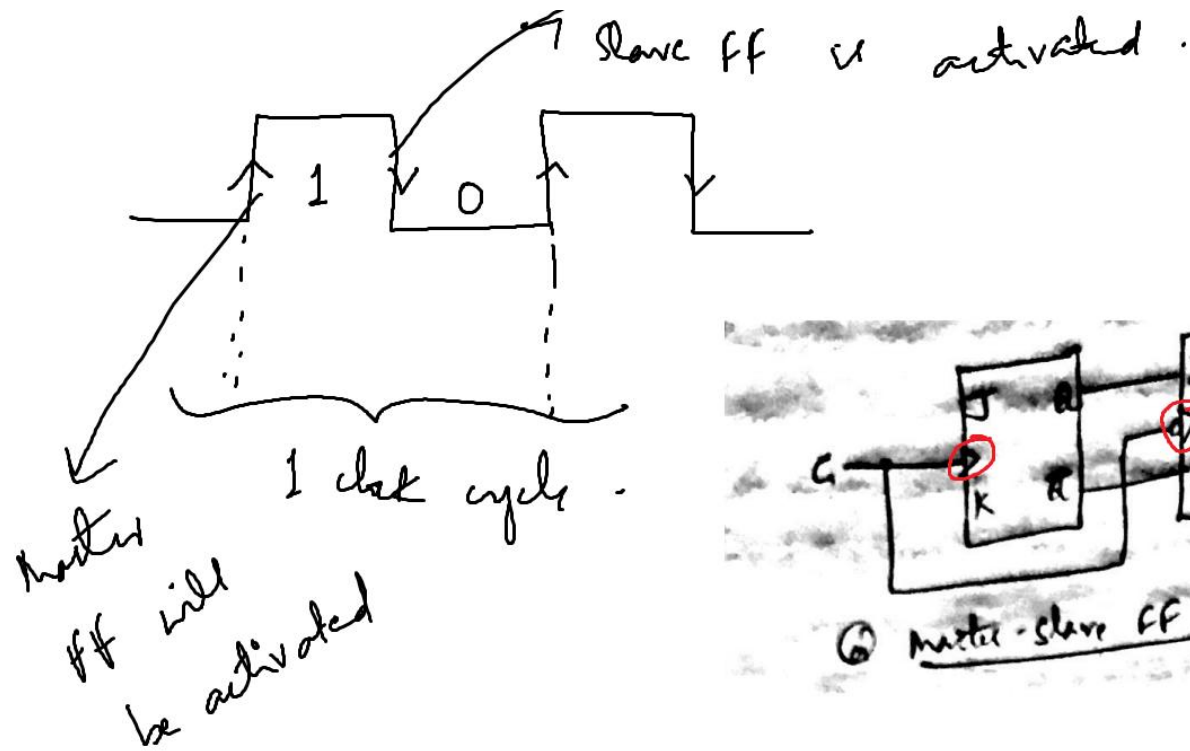
52
0101 0010

AL

Arithmetic (Co) logic

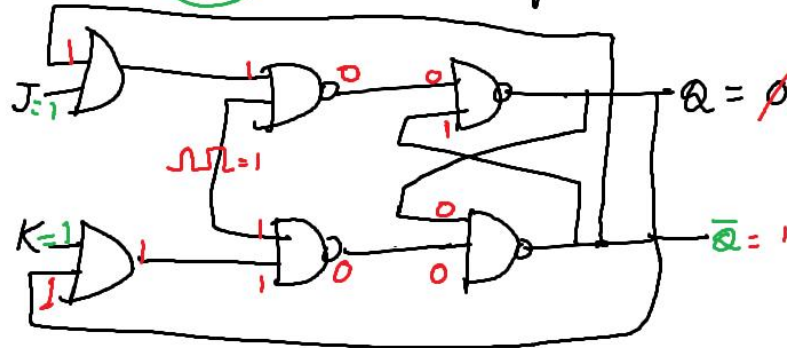


1st shift
2nd shift



JK flip flop: $\overline{S}\overline{R}$ $\rightarrow$ using NAND

SR D D

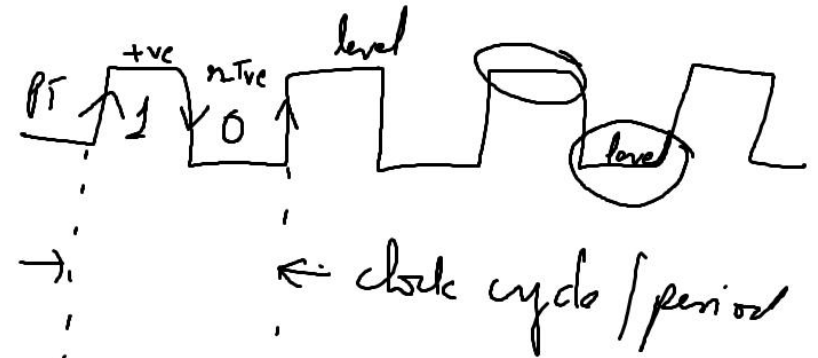
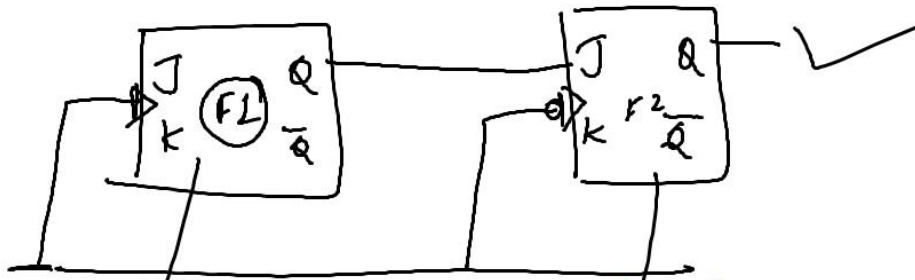


$Q = 01 = \text{toggle}$ ✓

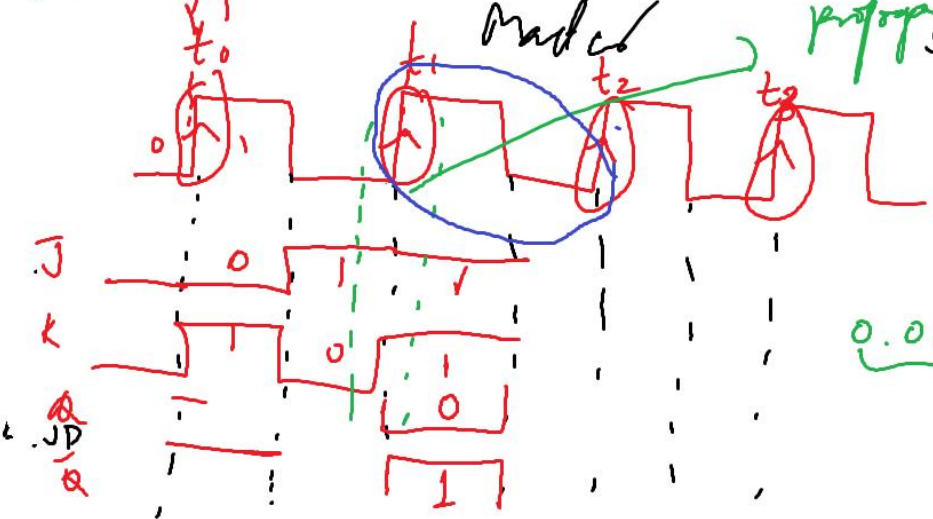
$J=1, K=1, Q=0$ what is Q^+ = ? ①

J	K	Q	Q ⁺	
0	0	0	0	② (last state)
0	0	1	1	
0	1	0	0	③ - reset
0	1	1	0	
1	0	0	1	④ set
1	0	1	1	
1	1	0	1	toggle - ⑤
1	1	1	0	

J K Master-Slave :

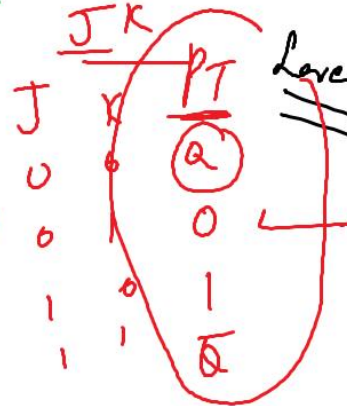


+ve edge to

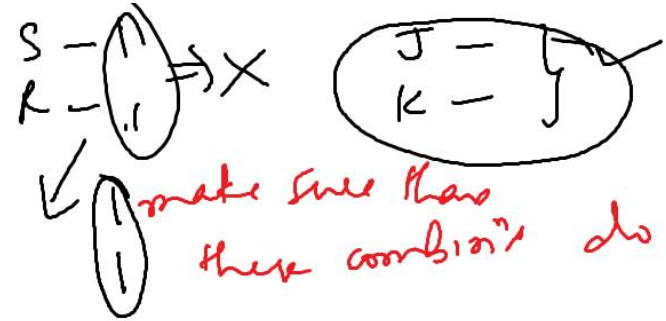
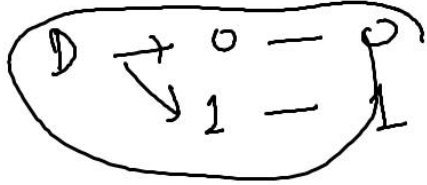


propagation delay.

0.000001



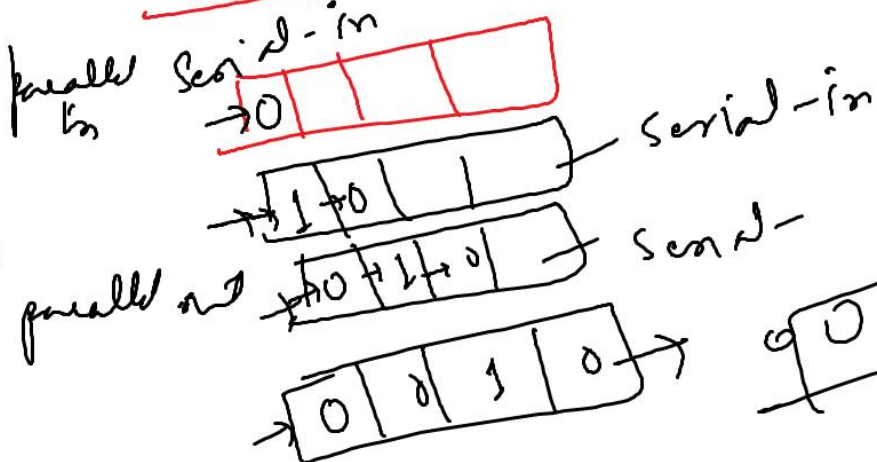
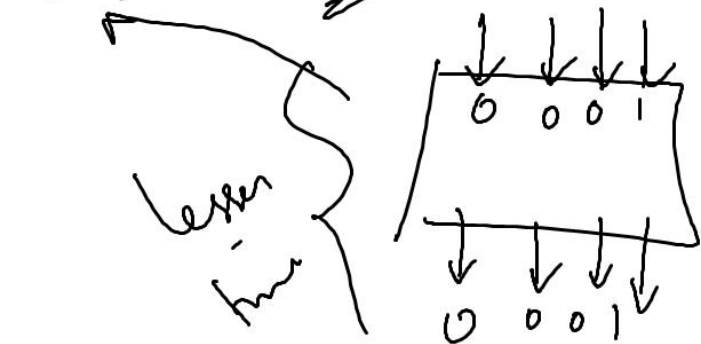
Edge triggered
Level triggered



make sure that these combinations do NOT exist

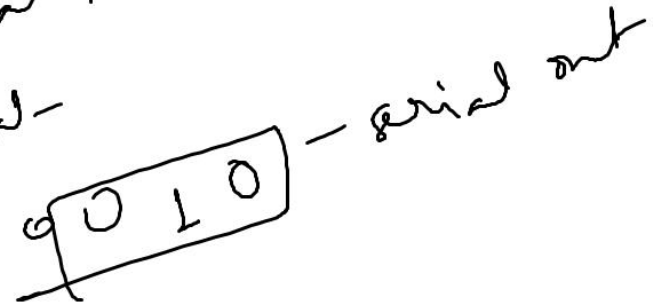
F1 | f2 | f3 | f4

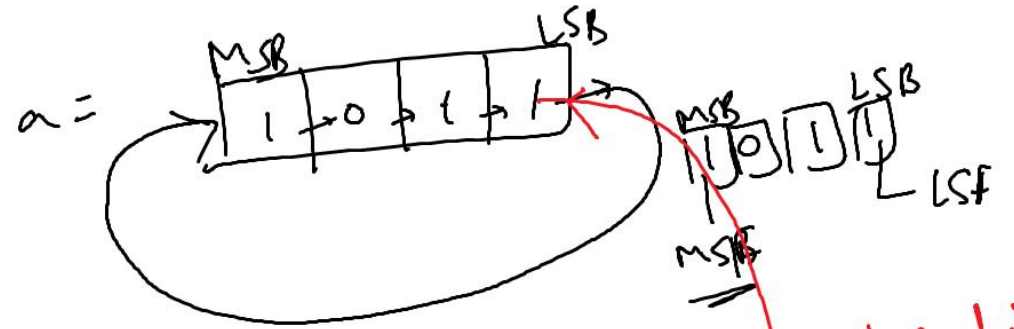
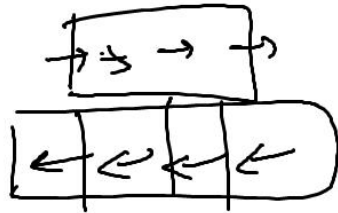
Not is more NIBBLE



Serial-in

Serial-





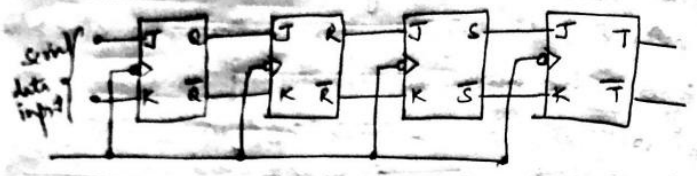
Ring @
 ↓
Ring counter

$a = a771$

$a771 \ 1 \ 1 \ 0 \ 1 \leftarrow 1^{\text{st}}$
 $a771 \ 1 \ 1 \ 1 \ 0$
 $3^{\text{rd}} \ 0 \ 1 \ 1 \ 1$
 $4^{\text{th}} \ 1 \ 0 \ 1 \ 1$
 $5^{\text{th}} \ 1 \ 0 \ 1 \ 1$

original i/p

Serial shift reg:
 The FFs used to construct registers are usually edge triggered JK, SR (or) D types.



OR
 use D-FF (but not mandatory)

Shift left mode: fig shown below is serial-in serial-out shift-left reg. (SISO-SL)

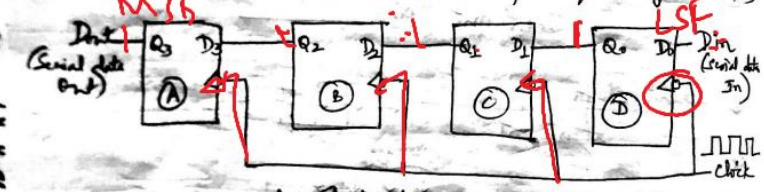


fig: Shift-left reg. D flip flop

Serial-In $\Rightarrow$ Serial-Out
 4-FF $\Rightarrow$ 4-bit shift reg

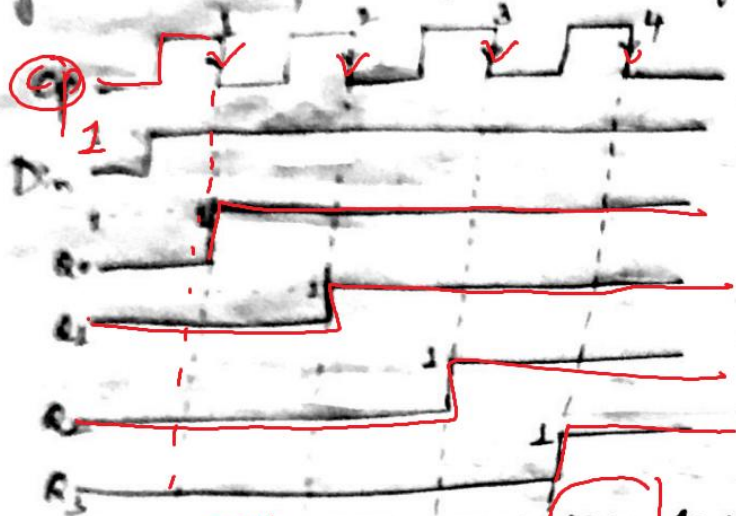


Din $\Rightarrow$ Data i/p

	D_4	D_3	D_2	D_1	D_0
Din	Q_3	Q_2	Q_1	Q_0	
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
	0	0	0	0	
	0	0	0	1	1
	0	0	1	1	1
	0	1	1	1	

CP $\downarrow$
 $\downarrow$
 $\downarrow$
 $\downarrow$

Fig shows waveforms for shift left reg.



Waveforms - timing diagram

Givone

0000, 0001, 0011, 0111, 1111 (data in reg) 4-bit reg

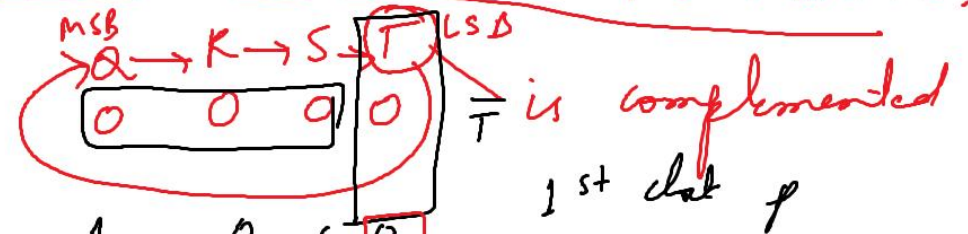
fig waveforms for shift left reg.

clock	Flip-Flops
Q R S T	
0	0 0 0 0
1	1 0 0 0
2	1 1 0 0
3	1 1 1 0
4	1 1 1 1
5	0 1 1 1
6	0 0 1 1
7	0 0 0 1
8	0 0 0 0
9	1 0 0 0

fig (b) state Table of Johnson counter

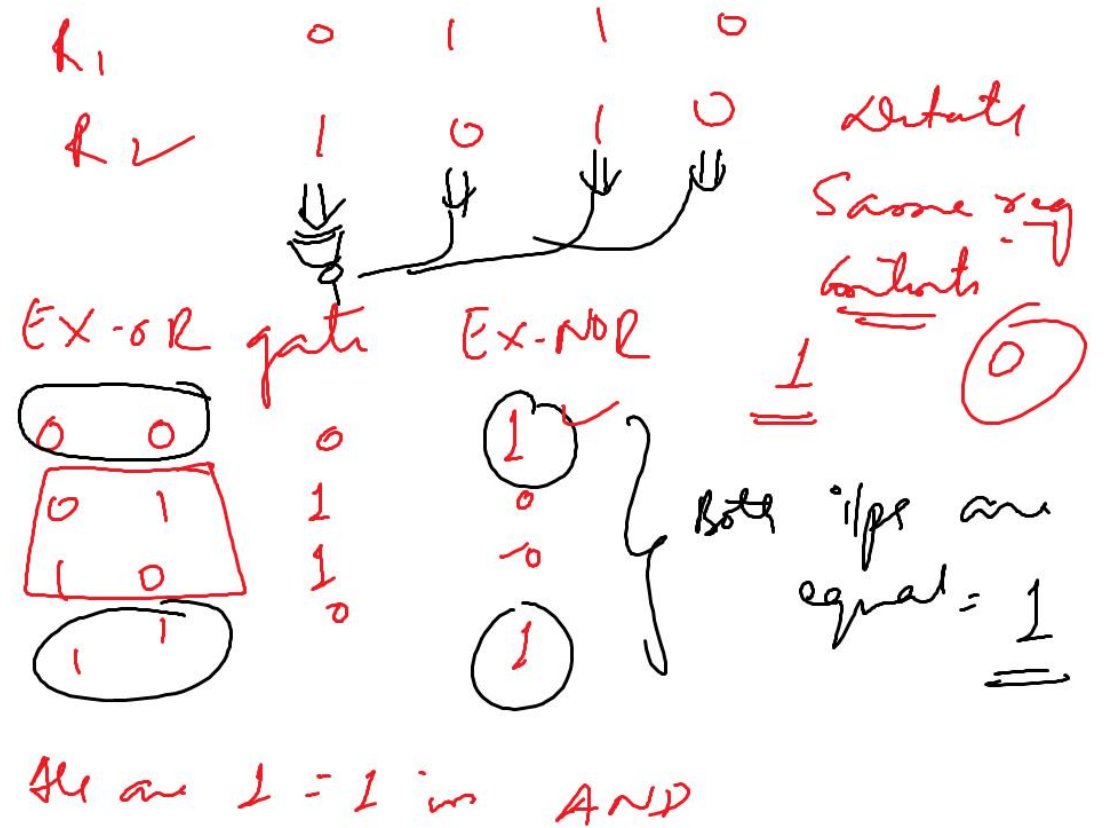
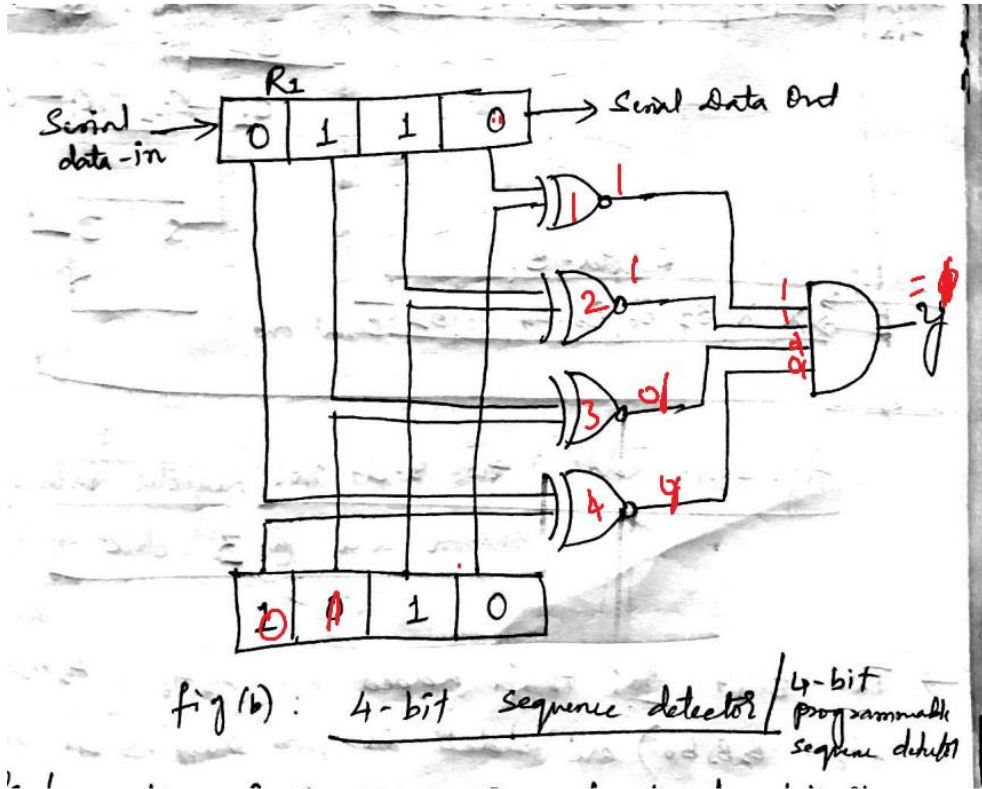
4-flops $\Rightarrow$ 8 clock cycles.
 N-flops $\Rightarrow$ 2N clock pulses.

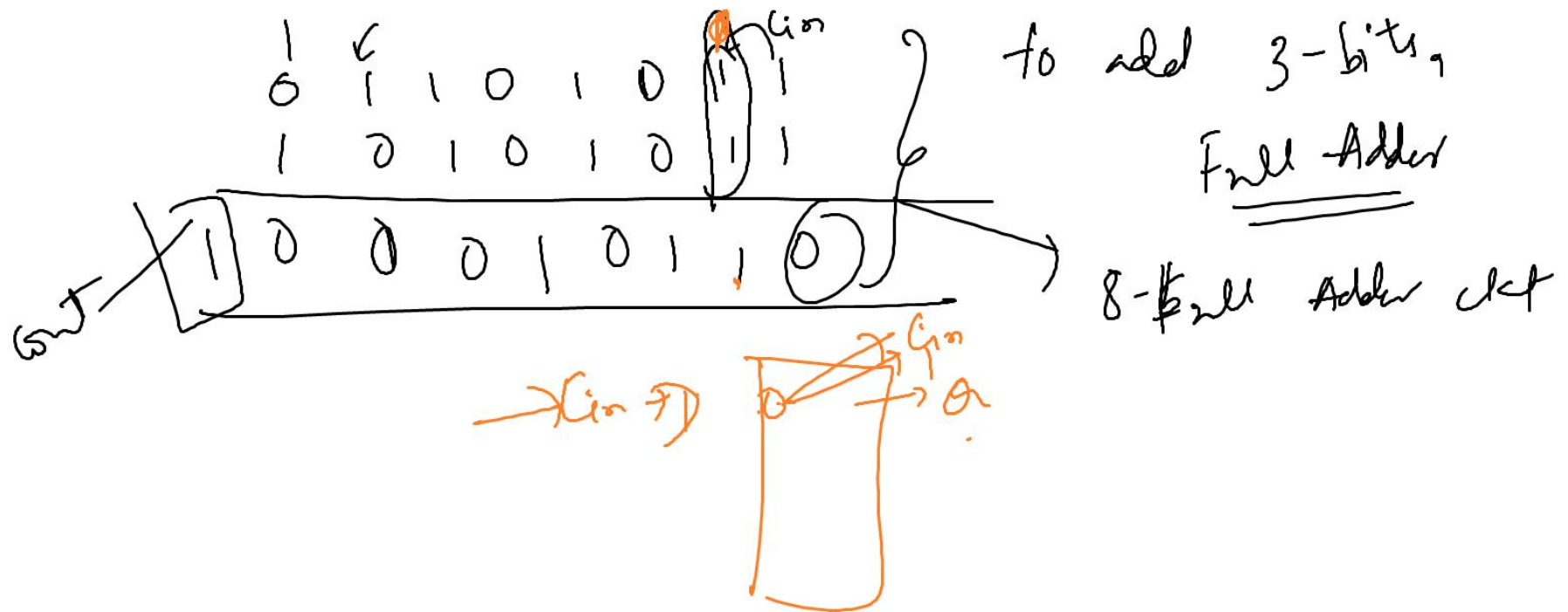
Switched tail / twisted tail (Johnson counter).



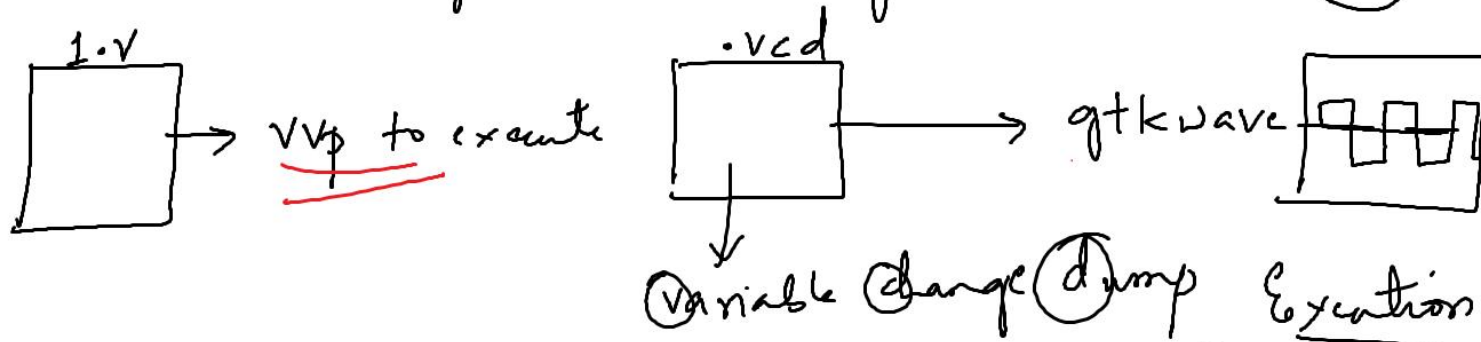
1	0	0	0
1	1	0	0
1	1	1	0
1	1	1	1

Q 1 1 1 1 } 4th clock cycle
 Q R S T
 A





icarus : iverilog $\Rightarrow$ icarus verilog
- verilog simulation & synthesis



1.v $\Rightarrow$ cd (change directory)
 $\Rightarrow$ cd `iverilog`
 $\Rightarrow$ `c:\iverilog\`: cd `bin`
 $\Rightarrow$ `c:\iverilog\bin`:

Execution of iverilog :

- 1) `iverilog -o some-file 1.v`
- 2) `Vvp some-file`
 `test.vcd` $\rightarrow$ in your source
 pgm only.
- 3) `gtkwave test.vcd`

Writing verilog programs: AND, OR, NOT ops

```
module gates (input a, b, output [2:0] y);  
    assign y[2] = a & b; // AND multiline /*.....*/  
    assign y[1] = a | b; // OR comments  
    assign y[0] = ~a; // NOT  
endmodule
```

```
module gates_tb;  
    wires [2:0] y;  
    reg a, b;  
    gates aon(a, b, y);  
    initial  
    begin  
        $dumpfile ("gate_test.vcd");  
    end  
endmodule
```

```
class student  
{  
    public: int a=3;  
           float b=4.0;  
};  
test bench  
main() -  
module instantiation  
{  
    student s1; // instn of class  
    s1.a;  
    s1.b;  
    cont  
}
```

$a[2] \rightarrow a[0], a[1], a[2]$
 $y[2:0] \rightarrow y[2], y[1], y[0]$

```
$ dump vars (0, gates - tb);
```

```
a = 1'b0;
```

```
b = 1'b0;
```

```
#50; // time delay - 50 units of time
```

```
a = 1'b0;
```

```
b = 1'b1;
```

```
#50;
```

```
a = 1'b1;
```

```
b = 1'b0;
```

```
#50;
```

```
a = 1'b1;
```

```
b = 1'b1;
```

```
#50;
```

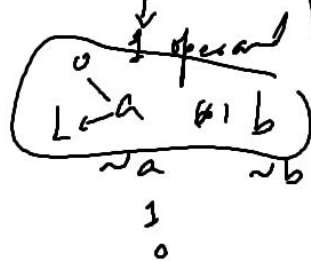
```
end  
endmodule
```

a	b	a AND b	out
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Syntax:

<size> <radix> <value>

NOT:



```
module gates (
endmodule
```

```
module u1vc (
endmodule
```

```
module ff (
endmodule
```



size: 1-bit

radix: base: binary: b

value: 0 1

```
$ dump vars (<levels>, <module>);
```

0 TB ~~main~~ ()

1, TB ()

2, main ()

main ()

```
{
  state s1;

```

```
s1.add();
```

```
s1.sub();
```

```
s1.mul();
}
```


Tc/C++ → IDE

C:\Users\bruce> ~~cd~~ ^{change} ^{directory} _{parent}

C:\Users> cd ..

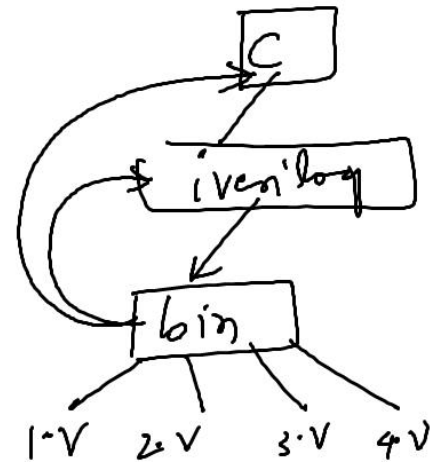
C:\> cd iverilog

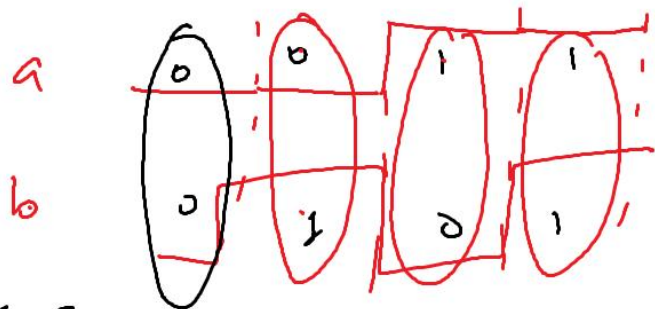
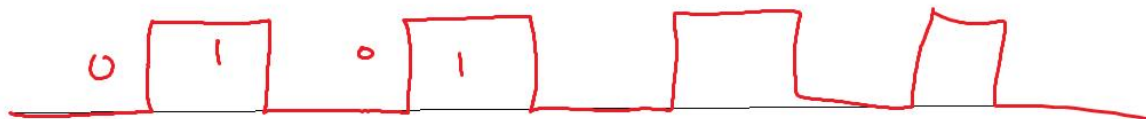
C:\iverilog> cd bin

C:\iverilog\bin>

C:\Tc\bin\I.c

C:\iverilog\bin





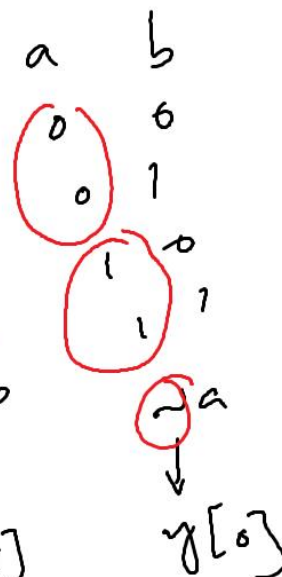
$y[2:0]: y[2] y[1] y[0]$

$= 001$

$= 010$

$\neq 011$

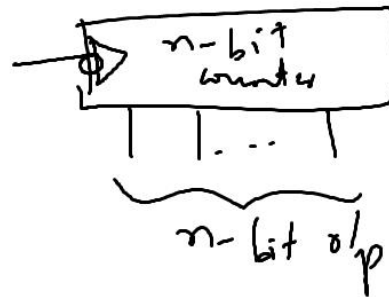
$\neq 110$



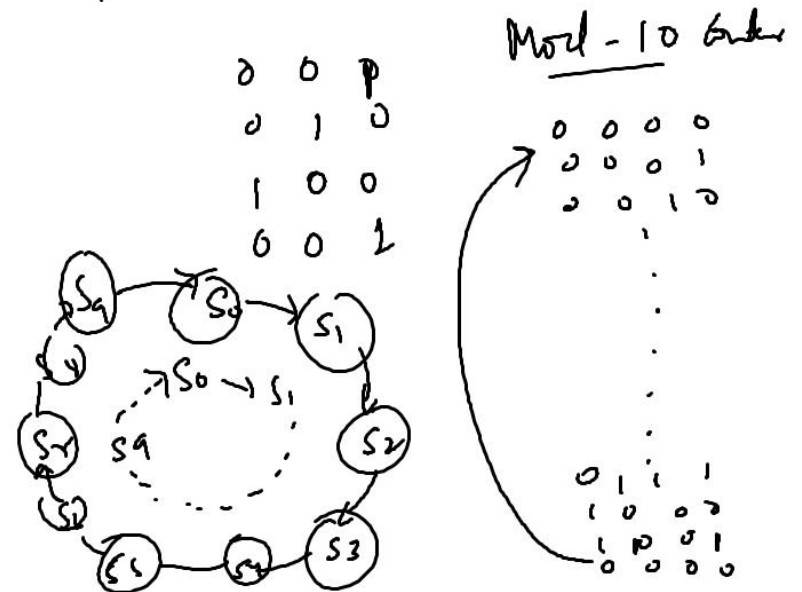
Counters : FF $\rightarrow$ registers $\rightarrow$ Counter.
 is also a register capable of counting the no. of clock pulses arriving @ its input.
 group of FF
 storing & shifting the data



(a) $\uparrow$ - n-bit counter



(b) $\uparrow$ - n-bit counter



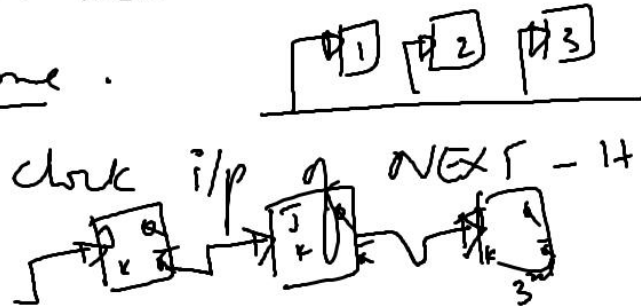
The n -bit binary counter has n -flip flops and has 2^n distinct states of o/p
 Eg. 2-bit binary counter has 2-FFs and 2^2 -distinct o/p state
 - 4 o/p states

111th 3-bit binary counter has 3-FFs - 2^3 -o/p states
 00, 01, 10, 11
 0 1 2 3
 8-o/p states
 000, 001, ... 111
 0 ... $2^n - 1$
 (n=3)

2 types i) Synchronous Counter
 ii) Asynchronous "

Synch: When a counter is clocked such that each FF in the counter is triggered @ the SAME-time.

Asynch: Each FF is connected to the clock i/p of NEXT-HIGHER-ORDER-FF



Differences/Comparison

Asynchronous counter

1) O/p of First FF drives the clock for the next FF

2) All the FFs are NOT clocked simultaneously

3) Speed: Slow (propagation - time - delay)

4) Logic ckt is very simple even for more no. of states

5) Cost is LESS

Synchronous counter

1) No connection b/w the o/p of 1st FF & clock i/p of the next FF

2) All the FFs are clocked simultaneously

3) Fast

4) Design involves Complex logic ckt as no. of states increases.

5) Cost is MORE

Modulus of Counter: Total no. of counts (or) states a counter can indicate is called a Modulus.

Eg: Modulus of a 4-stage counter would be $(2^4) = 16$

Eg: Mod-6 Counter $\Rightarrow$

0	0	0	✓	0
0	0	1	✓	⋮
0	1	0	✓	⋮
0	1	1	✓	⋮
1	0	0	✓	⋮
1	0	1	✓	↙

$$\begin{array}{r} 2 \overline{) 6} \\ \underline{2 \quad 3} \quad -8 \\ 1 \quad -1 \end{array}$$

$$\begin{array}{c} \downarrow \\ 00006 \\ \vdots \\ 111112 \end{array}$$

Mod-4 counter: $0 \rightarrow 3 \Rightarrow$

0	0
0	1
1	0
1	1

Asynchronous Counters: (Binary Ripple Counter)

SISO

$V_{CC}=1$

CLK

(Transⁿ - 0 to 1)
PT

$\uparrow$ K $\oplus$ $\bar{Q}$ - prev

If the Transⁿ is from 1-0 $\Rightarrow$ the NEXT
0-1 $\Rightarrow$ NEXT FF will TRIGGER FF will NOT

be triggered
NEXT FF will NOT trigger.

Down Counter:

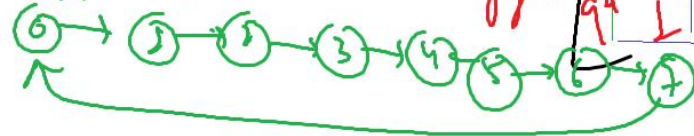
Up Counter

Up Counter: PT - $\bar{Q}_A$
NT - Q_A

Down Counter: PT - Q_A
NT - $\bar{Q}_A$

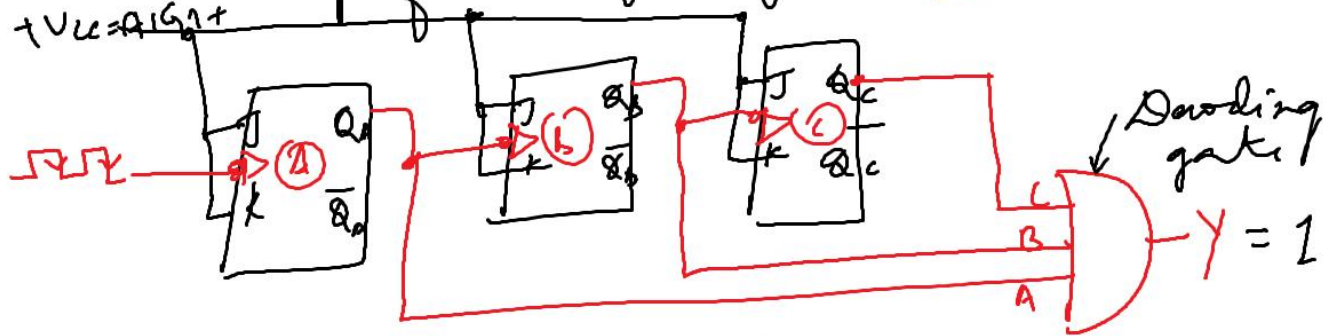
Up Counter

CLK	$\bar{Q}_C$	$\bar{Q}_B$	$\bar{Q}_A$	Q_C	Q_B	Q_A
Initially	1	1	1	0	0	0
1 st CLK	1	1	0	0	0	1
2 nd	1	0	1	0	1	0
3 rd	1	0	0	0	1	1
4 th	0	1	1	1	0	0
5 th	0	1	0	1	0	1
6 th	0	0	1	1	1	0
7 th	0	0	0	1	1	1
8 th	1	1	1	0	0	0
9 th	1	1	0	0	0	1



Decoding gates : indicate whether the counter has reached a particular state.

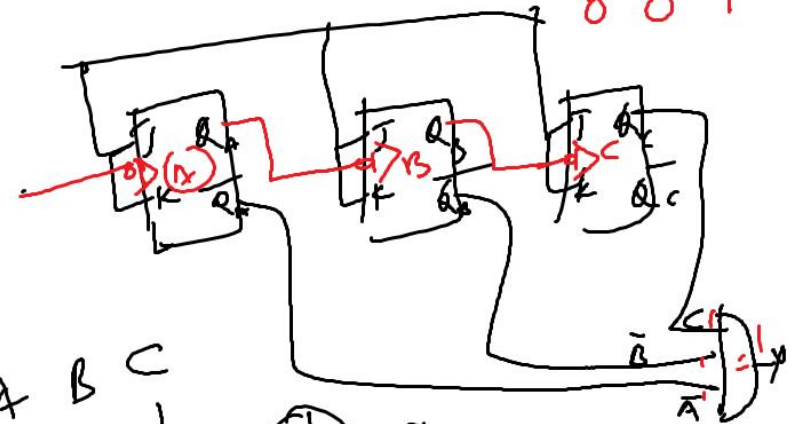
→ O/p of the counter are connected to the AND-gate as i/p's and the o/p of AND-gate goes HIGH for a particular state.



$C = 1$, $B = 1$ and $A = 1$

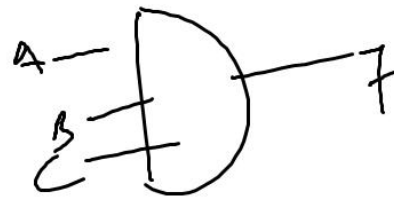
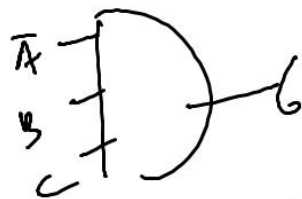
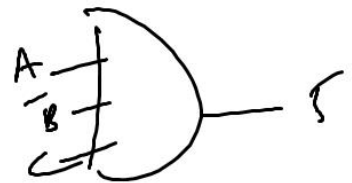
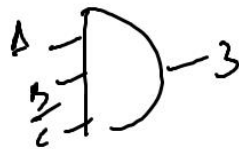
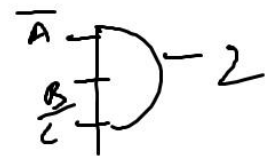
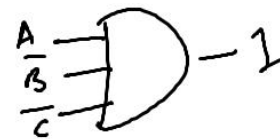
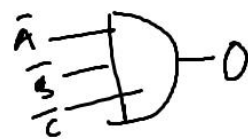
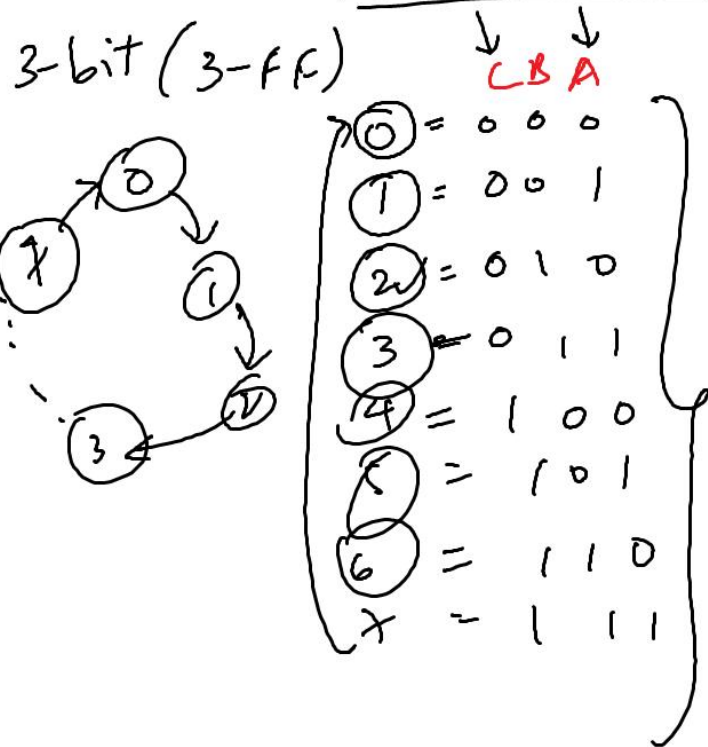
(a) Decoding gate to indicate the state $7_{(10)} = 111_{(2)}$

A B C
1 1 1



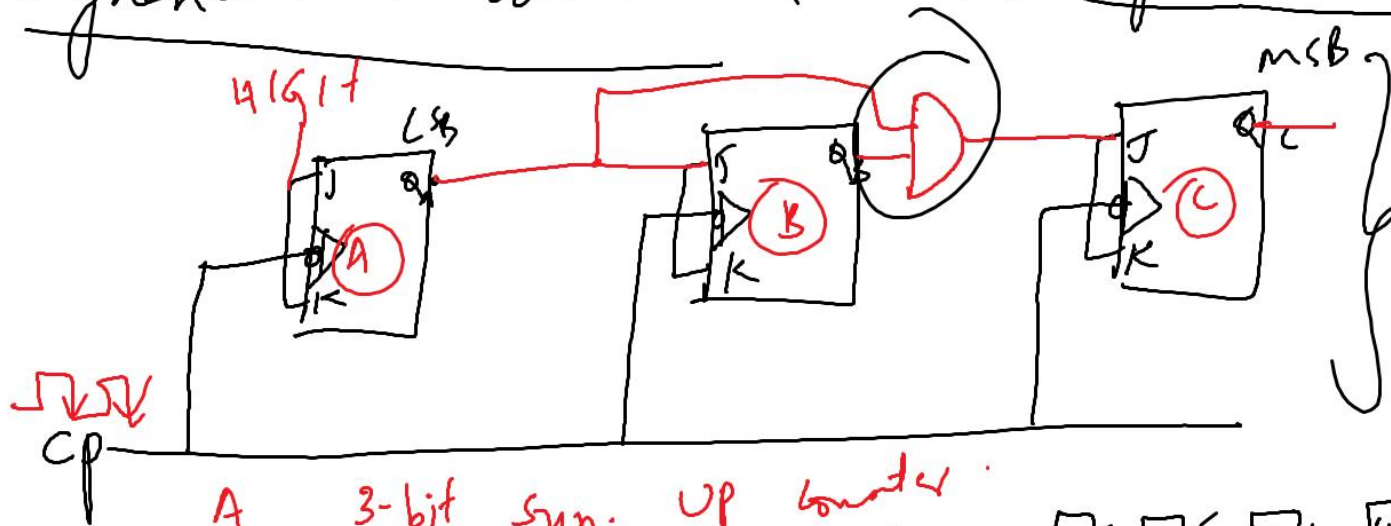
(b) Decoding gate to indicate state $4_{(10)} = 100_{(2)}$

|||ly, we can connect corresponding o/p to decoding gate i/p to indicate the desired state

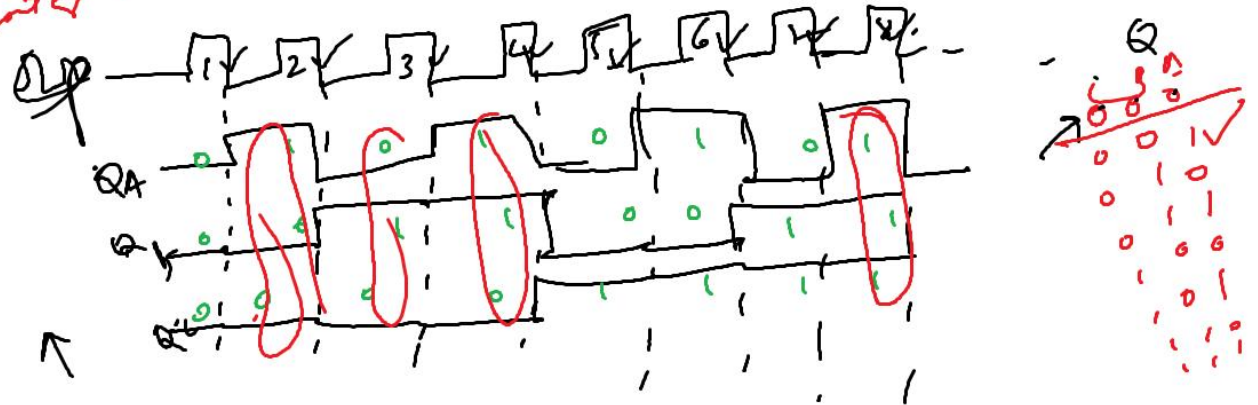
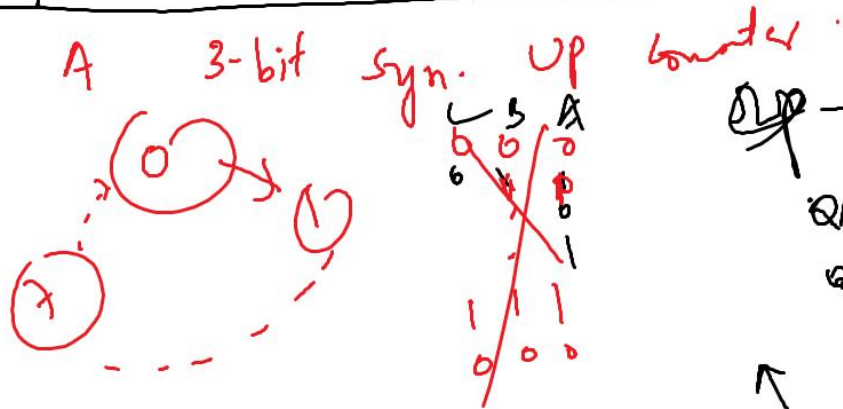


Decoding gate : I - can design Synchronous counter

Synchronous Counter : (3-bit) Synchronous Binary Up counter :



Working of this, we should know the Design of a synchronous counter.



Design of Synchronous Counters:

1) Determine the no of FFs reqd.

If $n \rightarrow$ no. of FF, then 3-FFs - 3 bits
 $N \rightarrow$ no. of states (2^3 - states)
 i.e., $2^n \geq N$ in the counter

2) Choose the type of the FF to be used

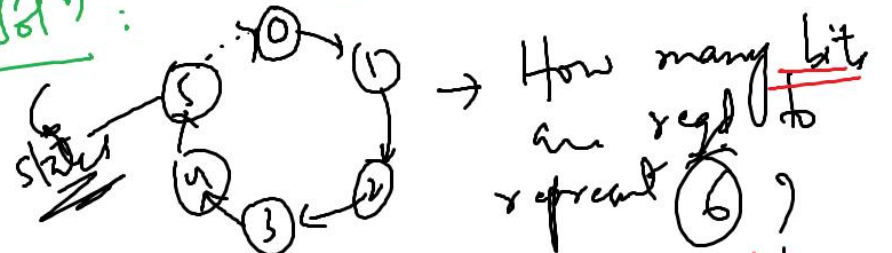
3) Using the excitation table for selected FF, determine the excitation table for the counter.

4) Use K-map (or) any other simplification method to derive the FF fns.

5) Draw the logic diagram of Sych. Counter.

Ex: Design a synchronous
Mod-6 Counter using
clocked JK ff.

Soln:



$$2^n \geq 6 \text{ - no. of states.}$$

$$8 \geq 6 \therefore \boxed{n=3}$$

$$2^3 \geq 6$$

$$\begin{array}{r} 2 \overline{) 6} \\ \underline{4} \\ 2 \end{array}$$

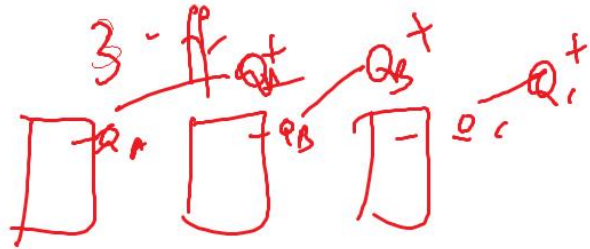
$110 = 3 \text{ bits}$

② JK - FF is chosen

③ Excitation table of JK-ff, del ex'itn table of Counter (Trans Table)

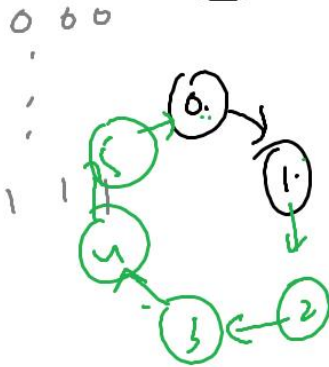
Q	Q^+	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

state transition table
diag JK



Present state			Next state			Flip-flop - ilps					
Q_A	Q_B	Q_C	Q_A^+	Q_B^+	Q_C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	1	0	1	X	0	0	X	1	X
1	0	1	0	0	0	X	1	0	X	X	1
1	1	0	X	X	X	X	X	X	X	X	X
1	1	1	X	X	X	X	X	X	X	X	X

④ Use k-map as simplification method.



J_A

Q_B	Q_C	00	01	11	10
Q_A	0	0	0	1	0
	1	X	X	X	X

$\therefore J_A = Q_B \cdot Q_C$

K_A

Q_B	Q_C	00	01	11	10
Q_A	0	X	X	X	X
	1	0	1	X	X

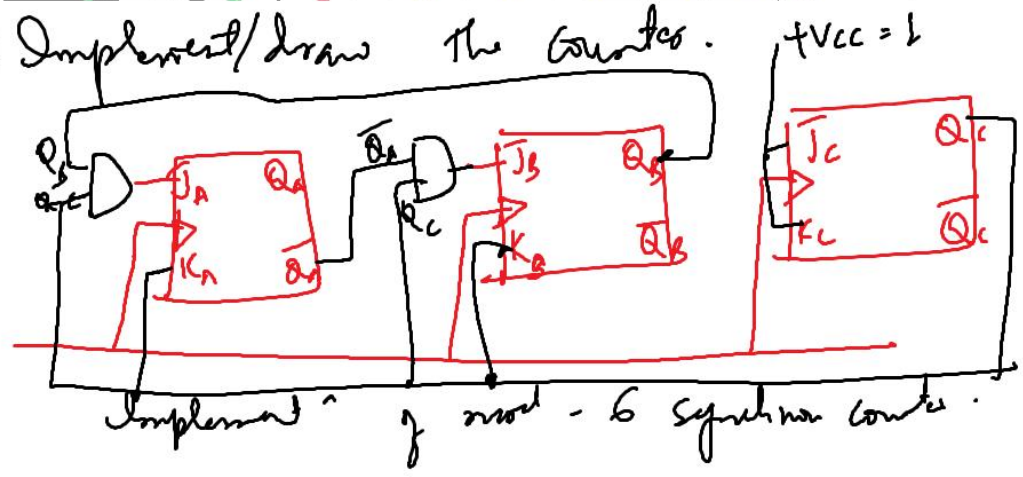
$\therefore K_A = Q_C$

Pcount	Next state			Flip-flop - J/Ks								
	Q_A	Q_B	Q_C	Q_A^+	Q_B^+	Q_C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	0	0	1	0	X	0	X	1	X
1	0	0	1	0	1	0	0	X	1	X	X	1
2	0	1	0	0	1	1	0	X	X	0	1	X
3	0	1	1	1	0	0	1	X	X	1	X	1
4	1	0	0	1	0	1	X	0	0	X	1	X
5	1	0	1	0	0	0	X	1	X	X	X	1
6	1	1	0	X	X	X	X	X	X	X	X	X
7	1	1	1	X	X	X	X	X	X	X	X	X

$J_A = Q_B Q_C$
 $K_A = Q_C$
 $J_B = Q_A$
 $J_C = Q_A Q_B Q_C$
 $K_C = Q_A Q_B$

$J_B = \overline{Q_A} \cdot Q_C$ (Deadly gate)
 $K_B = Q_C$
 $K_C = 1$

5) Implement/draw the Counter.



$\therefore J_C = 1$
 To avoid crossing of lines - represent the counter by signal named Q_A, Q_B, Q_C by logic diagram.

Design of a Synchronous Mod-6 counter using clocked $\overline{T}$ flip flop

1) finding the no of ffs $\Rightarrow$ mod-6

$$\underline{3\text{-bit}} = \underline{3\text{-ff}}$$

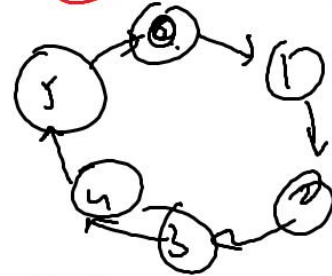
$$\therefore n = \underline{3}$$

$$N = 6$$

$$2^n \geq N$$

$$2^3 \geq 6$$

$$8 \geq 6$$



$$\begin{array}{r} 2 \overline{) 6} \\ \underline{3} \\ 1 \end{array}$$

$$6_{(10)} = \overline{1} \overline{1} 0_{(2)}$$

2) Draw the Excitable table of a $\overline{T}$ -ff

Q	Q ⁺	$\overline{T}$
0	0	0
0	1	1
1	0	1
1	1	0

state trans
diag
 $\overline{T}$ -ff

Q	Q ⁺	D
0	0	0
0	1	1
1	0	0
1	1	1

D-ff
Excitⁿ
ff

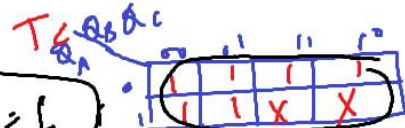
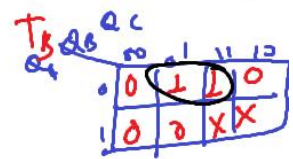
3) Draw the Excitable Table for Counter

(T-ff) Transition table of mod-6 counter

Present state	Next state	$\overline{T}_A$	$\overline{T}_B$	$\overline{T}_C$
Q _A Q _B Q _C	Q _A ⁺ Q _B ⁺ Q _C ⁺			
0 0 0	0 1 1	0	0	1
0 0 1	0 1 0	0	1	1
0 1 0	1 1 1	0	0	1
0 1 1	1 0 1	1	1	1
1 0 0	1 0 1	0	0	1
1 0 1	1 0 0	0	1	1
1 1 0	X X X	X	X	X
1 1 1	X X X	X	X	X

4) K-map: ff fns.

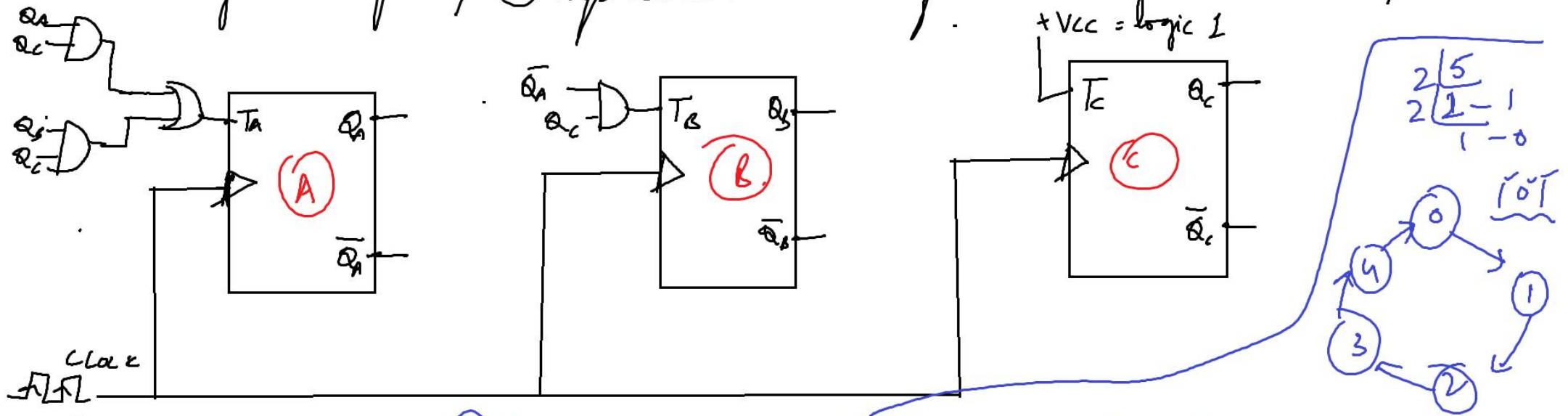
$$T_A = 1 \quad T_B = 1 \quad T_C = 1$$



$$T_A = Q_A \cdot Q_C + Q_B \cdot Q_C \quad T_B = \overline{Q}_A \cdot Q_C \quad T_C = 1$$

$$T_A = Q_A \cdot Q_C + \bar{Q}_B \cdot Q_C \quad T_B = \bar{Q}_A \cdot Q_C \quad T_C = 1$$

→ Draw a logic diagram / Implementation of mod-6 counter using T-f/f



Assignment: Design a mod-5 counter using D-f/f

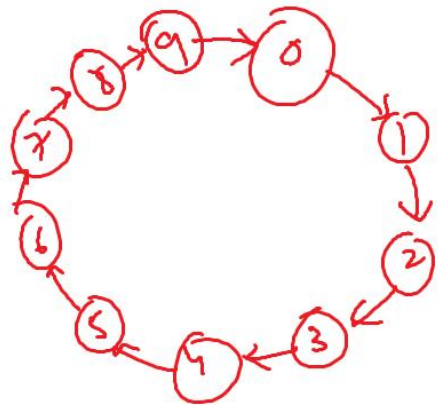
3-f/f $\Rightarrow 2^3 = 8$ bits 5-states $\Rightarrow 0, 1, 2, 3, 4, 0, 1, \dots$

② Design a synchronous Decade-counter using D-ff

$$\begin{array}{r} 2 \overline{) 10} \\ \underline{4} \\ 2 \overline{) 5} \\ \underline{2} \\ 1 \end{array}$$

10 $\Rightarrow$ mod-10 (0 $\rightarrow$ 9)

$\Rightarrow 10_{(10)} = \underline{1010}_{(2)} = 4\text{-bits} = \underline{4\text{-ff}} \Rightarrow 2^4 = \underline{16} \text{ combin?}$



D-ff Excitⁿ table



JK $\rightarrow$ toggled-olp
T-ff

D-ff

SR

③ Design a Synchron mod-4 counter using SR ff

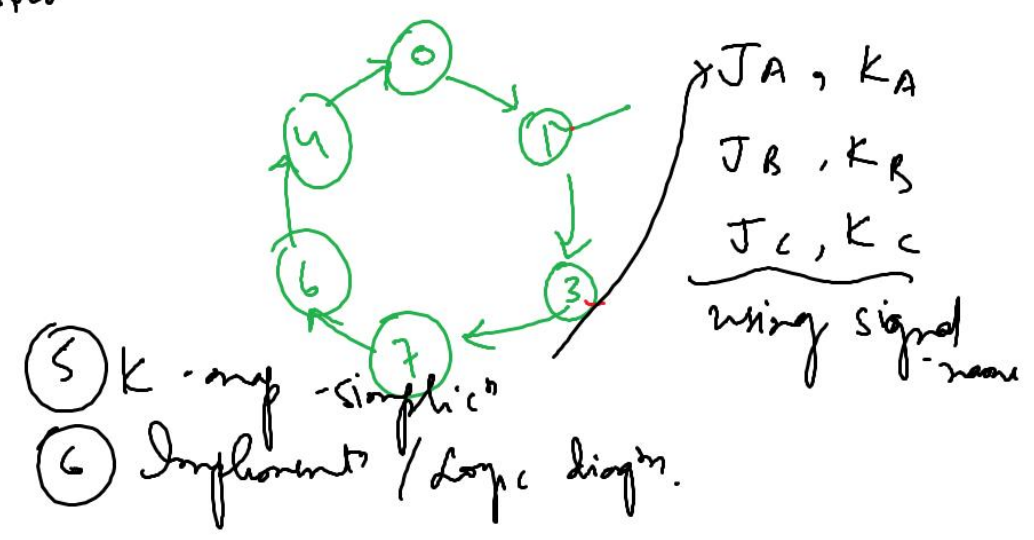
Design a Sync-counter with the sequence 0, 1, 3, 7, 6, 4, 0.

- Soln
- 1) Determining the no. of FF. = 3 ffs are reqd
 - 2) FF to be used: JK
 - 3) Dd Excn table of JK
 - 4) Dd. Excited table / Transition table of given counter

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$$2 \overline{) \begin{matrix} 7 \\ 3-1 \\ 1-1 \end{matrix}}$$

Present state			Next state			JK - flip-flop i/p's					
Q _A	Q _B	Q _C	Q _A ⁺	Q _B ⁺	Q _C ⁺	J _A	K _A	J _B	K _B	J _C	K _C
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	1	0	X	1	X	X	0
0	1	0	X	X	X	X	X	X	X	X	X
0	1	1	1	1	1	1	X	X	0	X	0
1	0	0	0	0	0	X	1	0	X	0	X
1	0	1	X	X	X	X	X	X	X	X	X
1	1	0	1	0	0	X	0	X	1	0	X
1	1	1	1	1	0	X	0	X	0	X	1



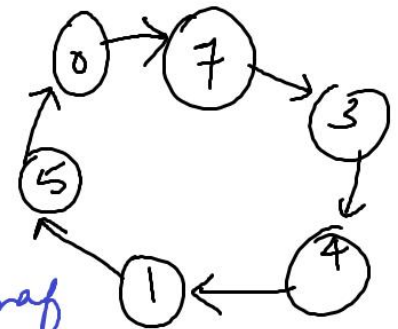
- 5) K-map - simplify
- 6) Implement / Logic diagram.

Design a Synch counter using JK to count the fol sequence.

7, 3, 4, 1, 5, 0, 7...

Soln: ① 3-ffs ② JK ③ JK Excitⁿ table
④ Excitable fol counted

	present state			next state			J-K-ff - i/p's					
	Q _a	Q _b	Q _c	Q _a ⁺	Q _b ⁺	Q _c ⁺	J _a	K _a	J _b	K _b	J _c	K _c
0	0	0	0	1	0	1						
1	0	0	1	X	X	X	X	X	X	X	X	X
2	0	1	0	1	0	0						
3	0	1	1	0	0	1						
4	1	0	0	0	0	0						
5	1	0	1	0	0	0						
6	1	1	0	X	X	X	X	X	X	X	X	X
7	1	1	1	0	1	1						

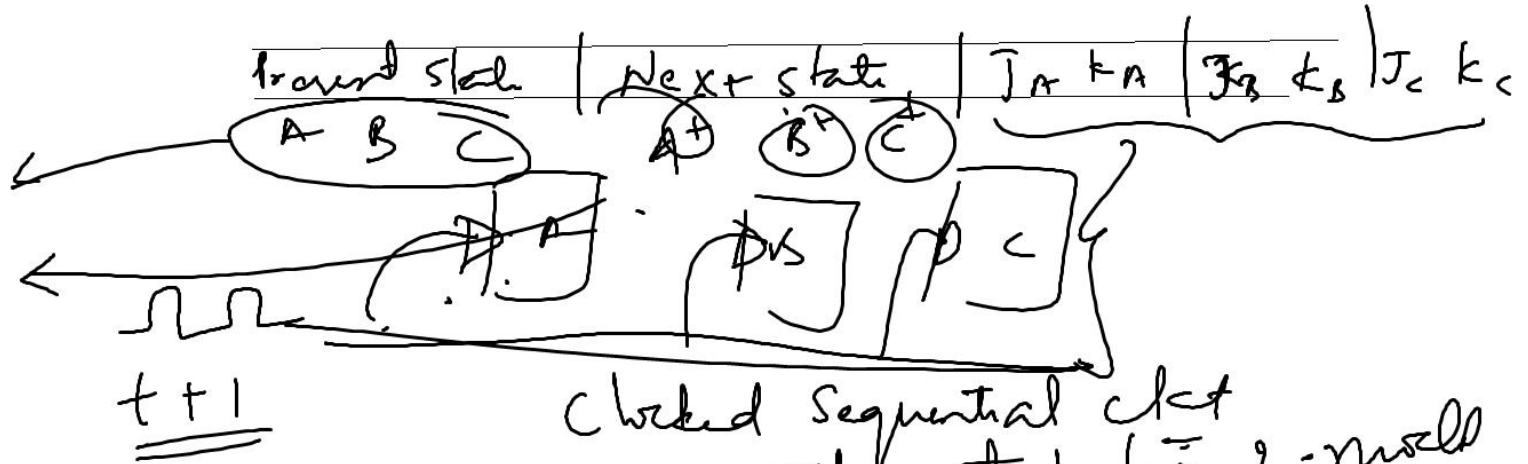


⑤ K-map
FF - i/p's for

⑥ Implementⁿ diagram
using signal-name.

Unit-5:

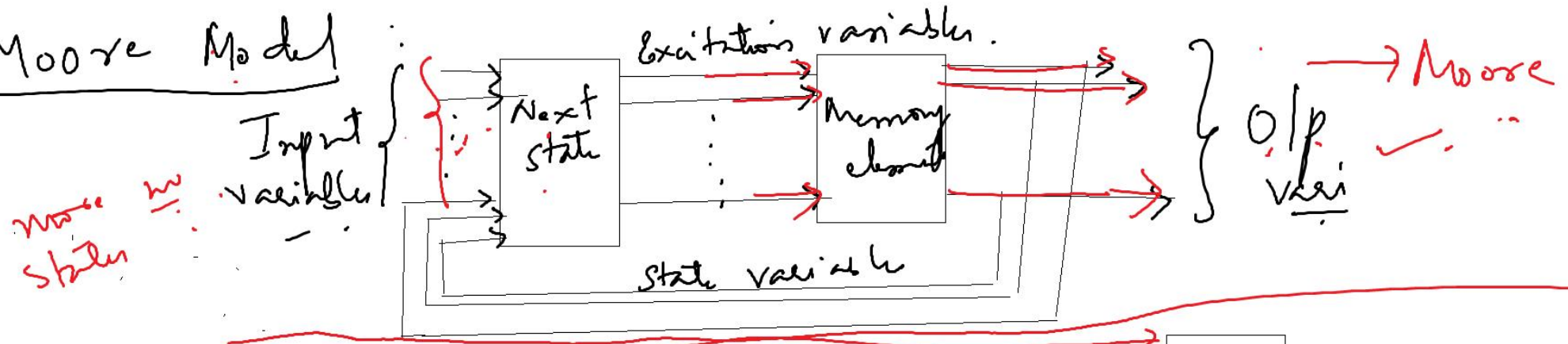
Present state :
Next state :



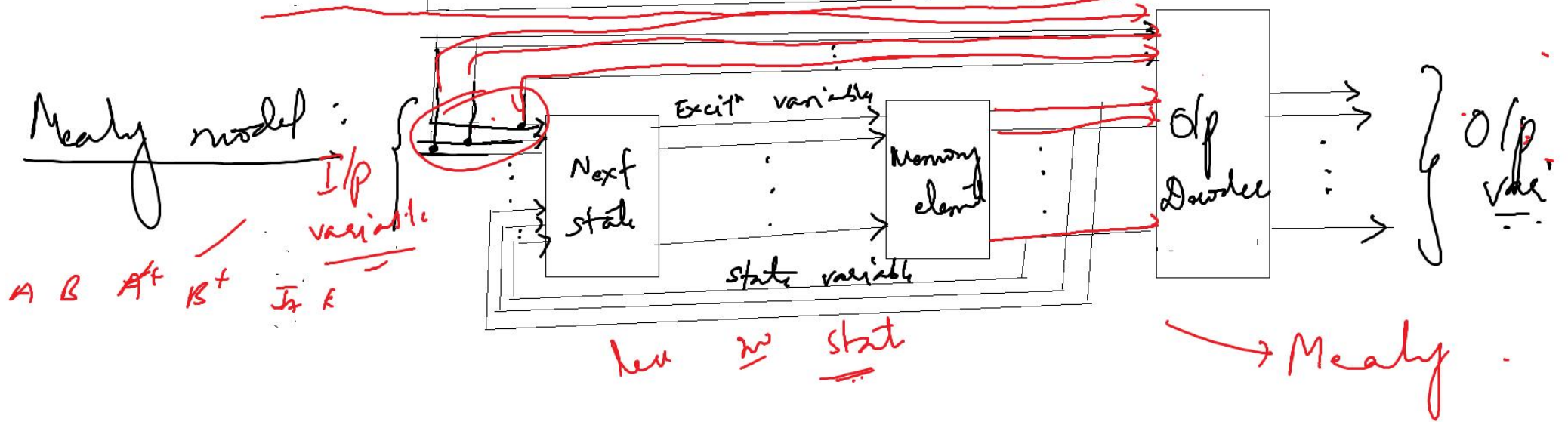
Clocked Sequential ckt
are represented by 2-model

1) Moore model } Represent Clocked / sync Seq ckt.
2) Mealy model }

Moore Model



Mealy model



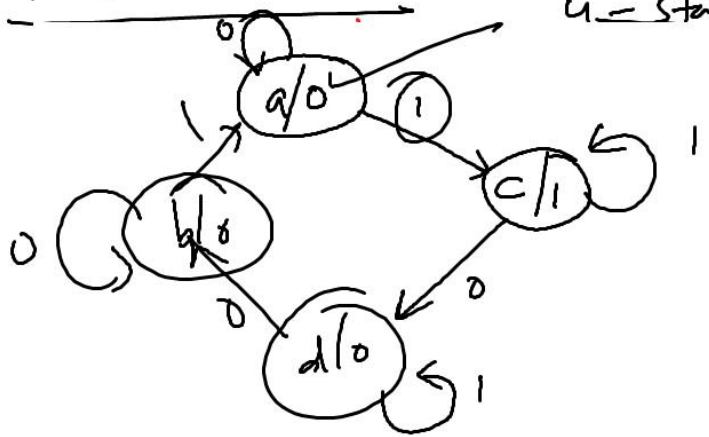
State M/c. I/p vari, O/p vari, state var, Exite var

State & state vari : $2^x = y$
 $x \rightarrow$ no. of FF.
 $y = \max$ no of possible stat

$2^2 = 4$ possible state.

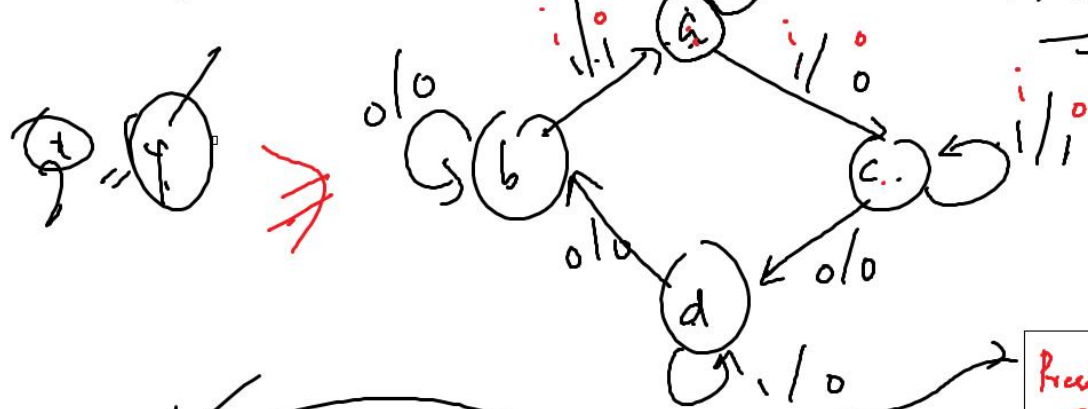
✓
State diag. in: Moore model 4-states

Node: state with op.



Each state: Node:
 a/ $\frac{I/S}{O/Y}$ $\xrightarrow{I/P}$ $\frac{O}{Y}$

Mealy model state diagram



Node : State
Arrow : i/o

Moore : s/y i/p.
Mealy : state i/o

$X \rightarrow i/p.$

State table

Transm table

Recent state A B	Next state X=0 X=1	Next state X=0 X=1	Output X=0 X=1
0 0	0 0	1 0	
0 1	0 1	0 0	
1 0	1 1	1 0	
1 1	0 1	1 1	

Recent state A B	Next state X=0 X=1	Next state X=0 X=1	Output X=0 X=1
0 0	a	c	0 0
0 1	b	a	0 1
1 0	c	c	0 1
1 1	d	d	0 0

Mealy model state diagram

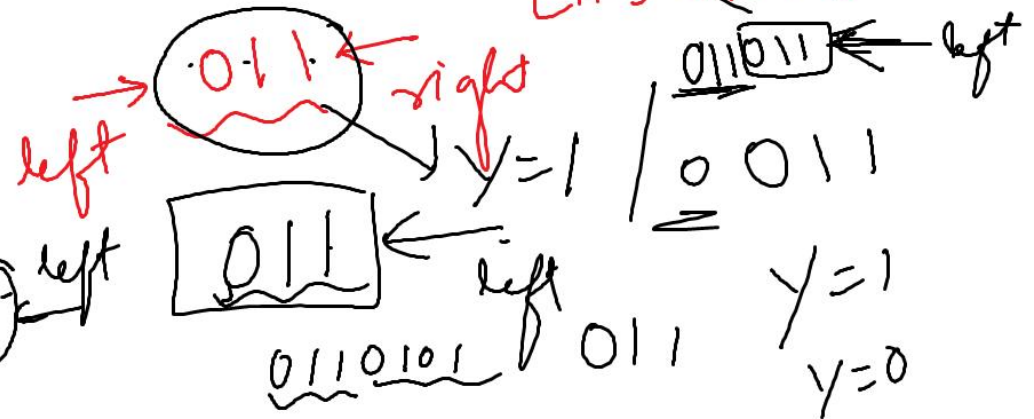
State Table

P.S	N.S $x=0$	N.S $x=1$	0/1 y
A B	AB	AB	
a	a	c	0 ✓
b	b	a	0 ✓
c	d	c	1 ✓
d	b	d	0 -

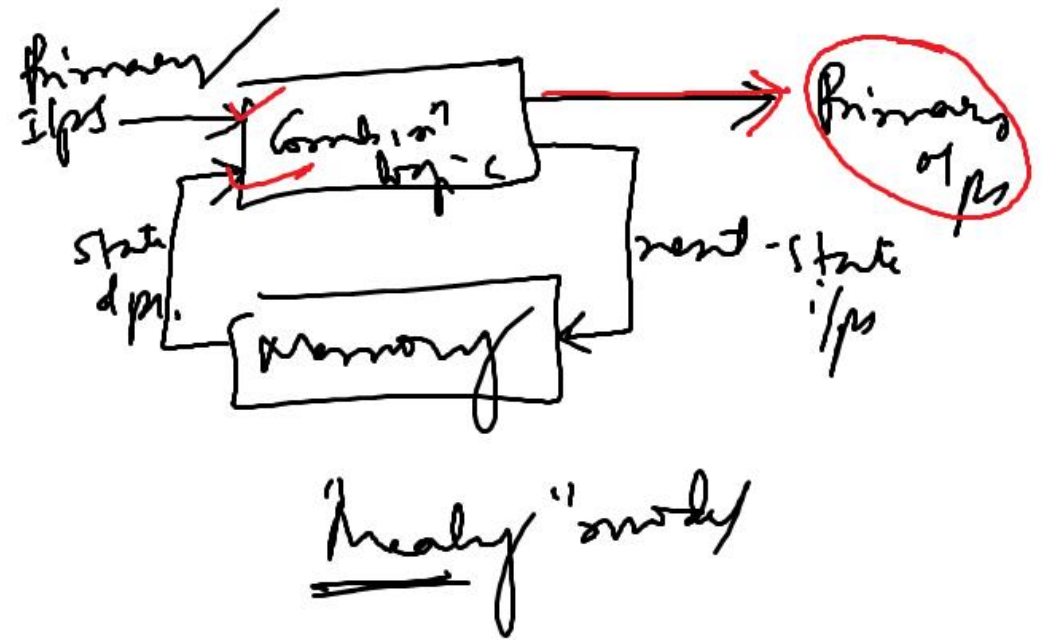
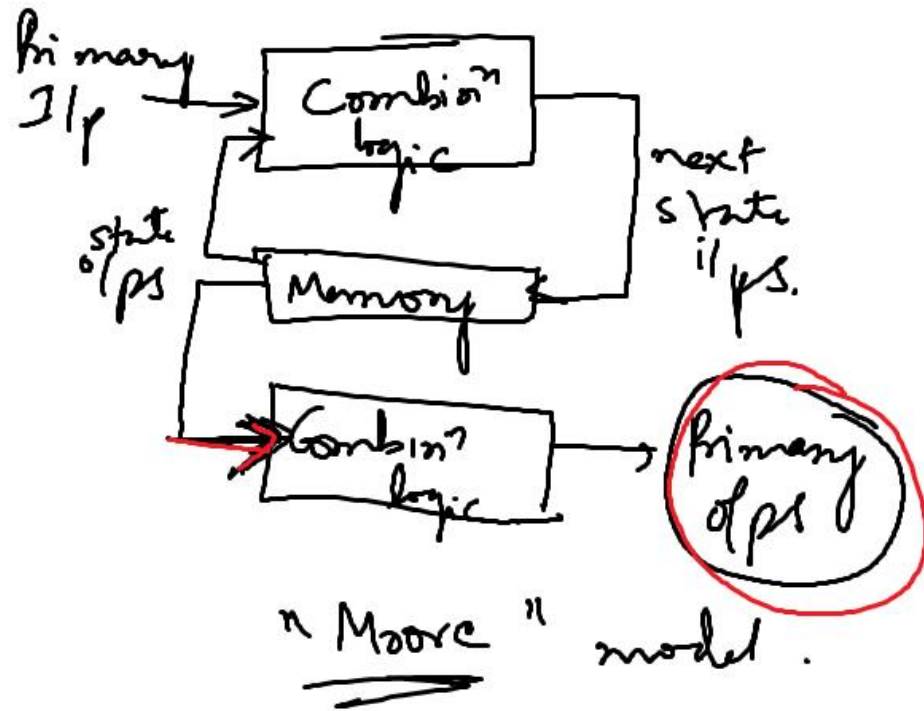


Sequence Detector : Moore / Mealy

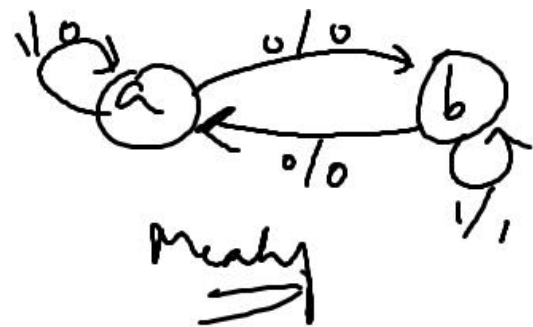
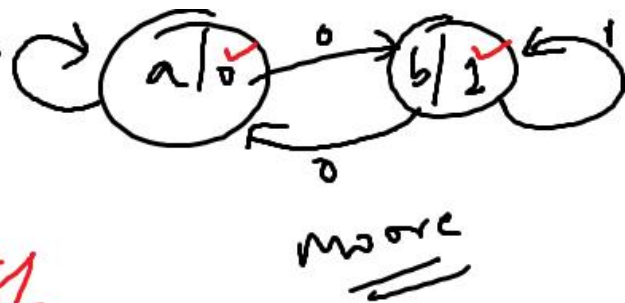
$\begin{array}{r} 111011 \\ \underline{y=0} \end{array}$ $\begin{array}{r} 011 \\ \underline{\hspace{1cm}} \\ 011 \end{array} \rightarrow y=1$ $\begin{array}{r} 011 \\ \text{---} \end{array} \rightarrow y=0$ $\begin{array}{r} 011 \\ \text{---} \end{array}$



Moore & Mealy



State transⁿ diagram :



Req^t : Sequence det^r atol

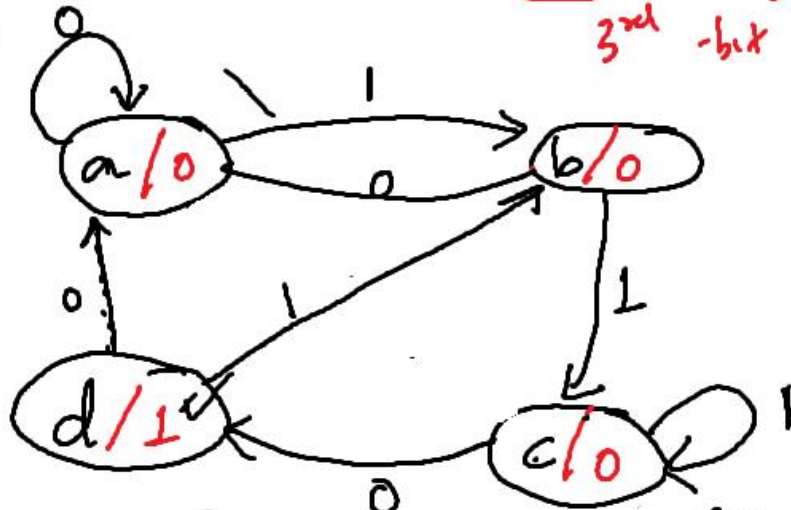
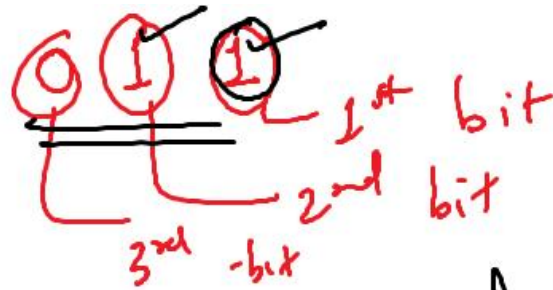
Binary Seqⁿ 011 ← left

$x = y$
— LHS RHS —

right $x = y$ ← left

Design a Seq. detector that receives a binary data stream @ its i/p x , & signals when a combination 011 arrives @ the i/p by making its o/p $y = \text{HIGH}$, which otherwise remains ($y = 0$) LOW. Consider the data is coming from LEFT.
i.e., 1st bit to be detected is 1, 2nd ... 1, 3rd ... 0 from i/p seq.

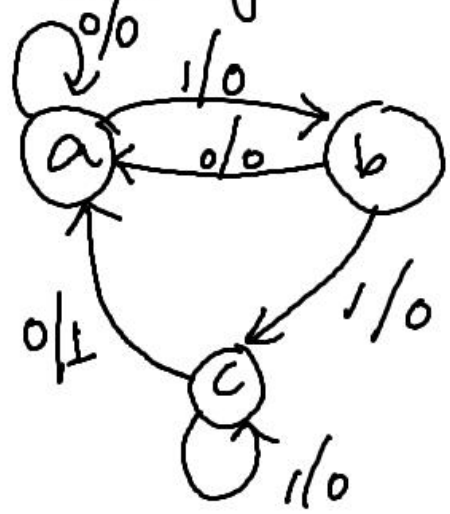
Moores model :



State transⁿ diagram of Seq. detector (011 ← left)

Let's see
Status

Mealy model



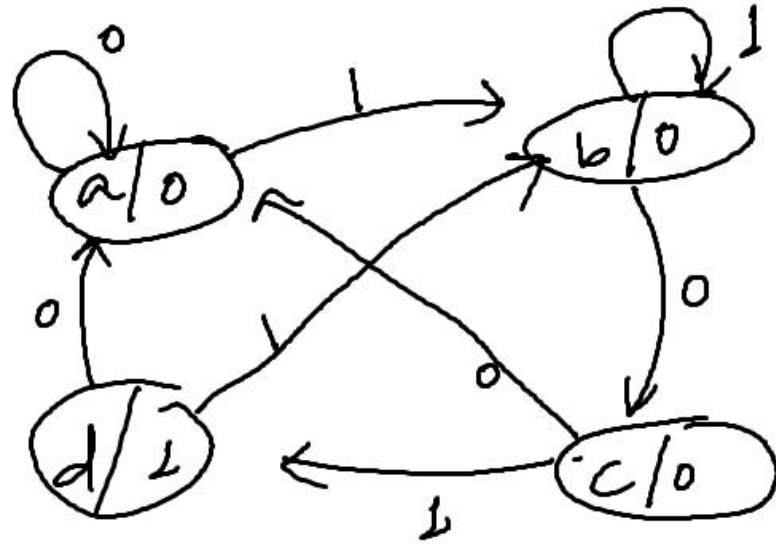
Mealy model

right $\begin{array}{c} 10 \\ \leftarrow \end{array} \boxed{} \leftarrow \text{left}$

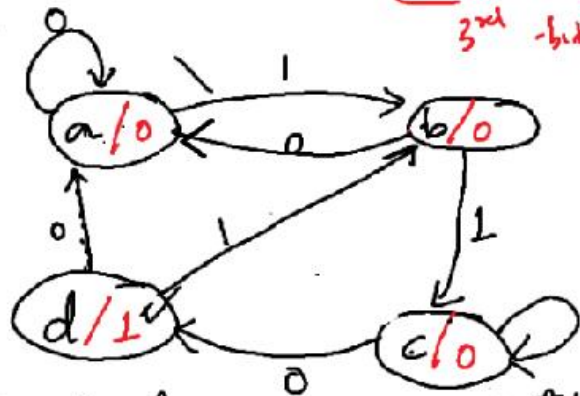
(a)

101011100

$\rightarrow$ 010 $\leftarrow$ left



State transition diagram:



trans: 0, 1, 0, 1, 0, 1, 0, 1

a - 0 0 ✓
b - 0 1 ✓
c - 1 0 ✓
d - 1 1 ✓

Present state		if	Next state		o/p
B_n	A_n	X	B_{n+1}	A_{n+1}	Y
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	0	1	1	0
0	0	1	0	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	1

State Synthesis
Moore
Extra table:

Flip Flops			
J_B	K_B	J_A	K_A
0	X	0	X
0	X	1	X
0	X	X	1
X	0	1	X
X	0	0	X
X	1	X	1
X	1	X	0

Q	Q^+	J	K
0	0	0	X
0	1	0	X
1	0	X	1
1	1	X	0

Present state		if	Next state		if	1-way lops			
B_n	A_n	X	B_{n+1}	A_{n+1}	Y	J_B	K_B	J_A	K_A
0	0	0	0	0	0	0	X	0	X
0	0	1	0	0	0	0	X	1	X
0	1	0	0	0	0	0	X	X	1
0	1	1	1	0	0	1	X	X	1
1	0	0	1	1	0	X	0	1	X
1	0	1	1	0	0	X	0	0	X
1	1	0	0	0	1	X	1	X	1
1	1	1	0	1	1	X	1	X	0

Moore
~~print~~
 X
 B_n
 A_n

B_n	A_n	0	1	1	0
0	0	0	0	X	X
0	1	0	1	X	X
1	0	1	1	X	X

$$J_B = X A_n$$

$$J_A = \bar{X} B_n + X \bar{B}_n \quad K_B = A_n$$

$$K_A = \bar{X} + \bar{B}_n$$

$$Y_n = A_n \cdot B_n$$

A_n	B_n	0	1
0	0	0	0
1	0	0	1

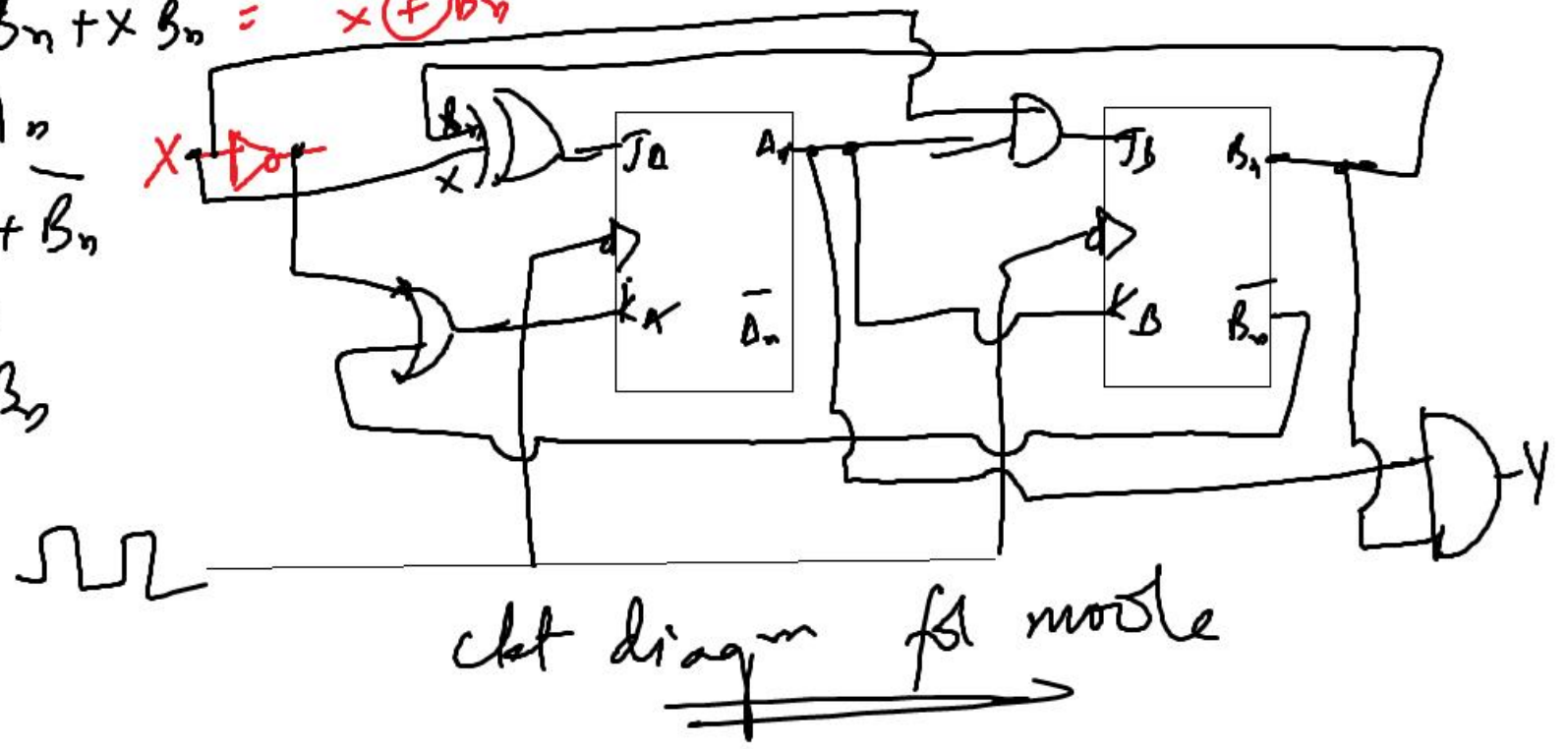
Moore: $J_A = \bar{X} B_n + X \bar{B}_n = X \oplus B_n$

$K_B = X A_n$

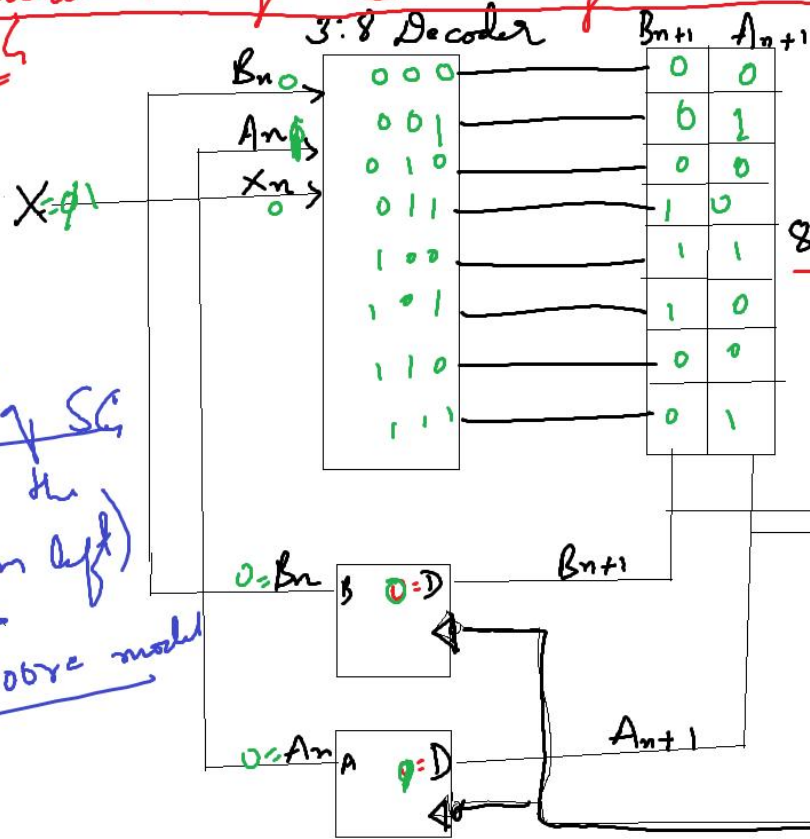
$K_A = \bar{X} + \bar{B}_n$

$K_B = A_n$

$Y = A_n \cdot B_n$



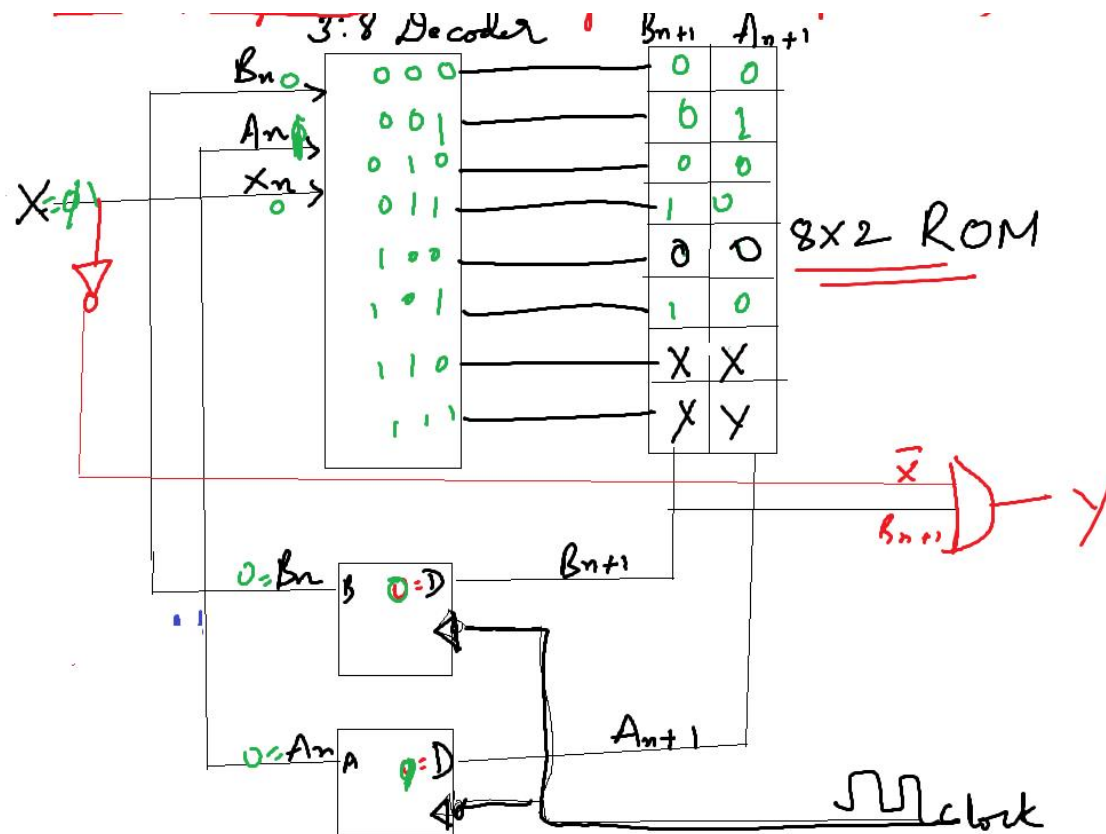
Implementation using Read Only Memory (ROM)



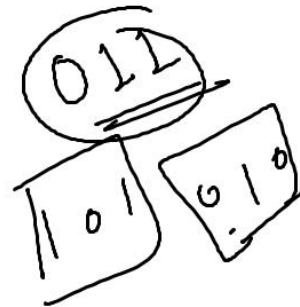
Initially B -ffs are cleared $\therefore B_n = 0 \& A_n = 0$
 $B_n = 0$
 $A_n = 0$
 $X = 0$ (or)

$BAX = 000$ is selected
 $BAX =$

Implementing SL that detects the seq 011 (from left) using Moore model



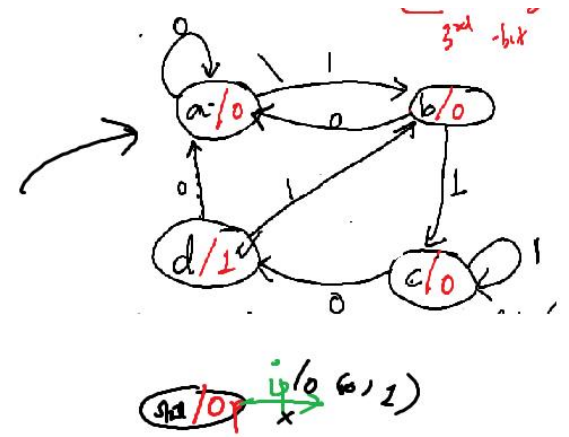
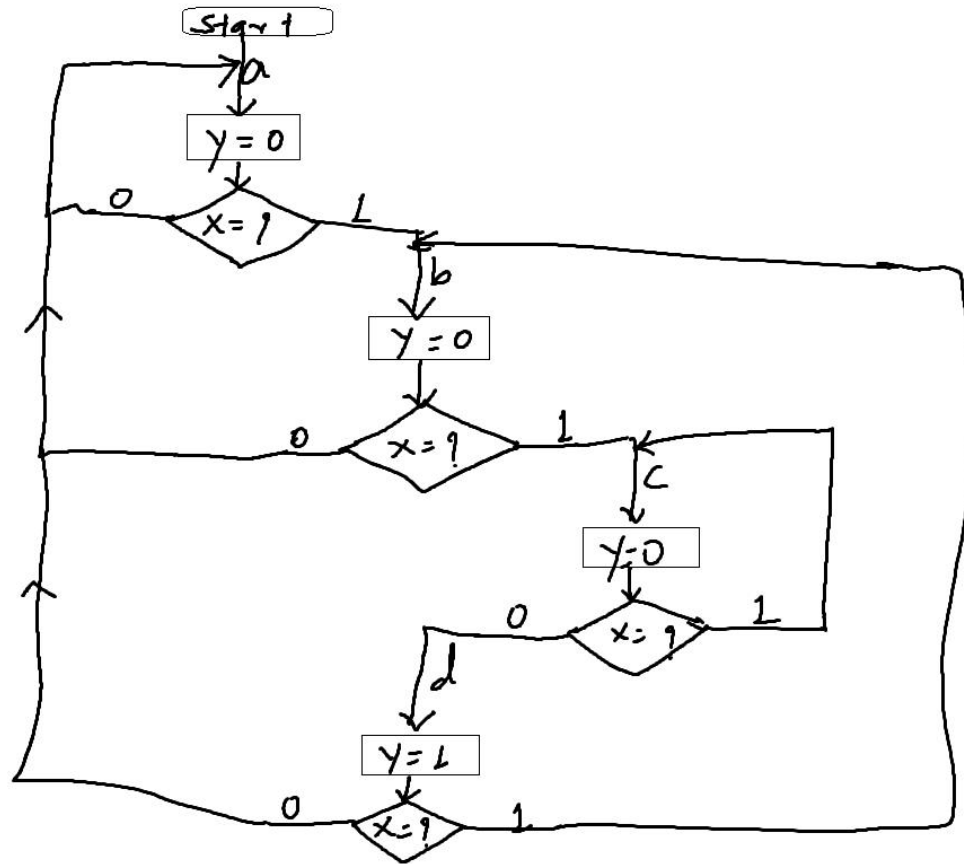
Mealy model: 3 - state
 pre iff + post skt
 6 - combin



$$Y = \bar{X} \cdot B_n$$

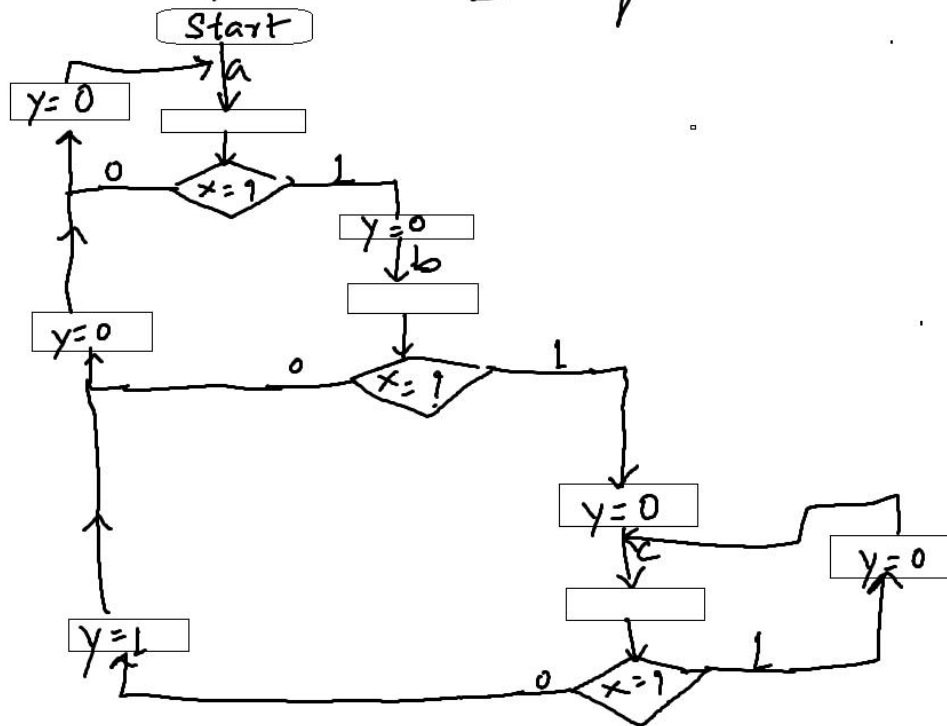
ROM - based implementation
 of sequence - detector (011 ← left)
 using Mealy - model

ASM: Algorithmic State Machine :

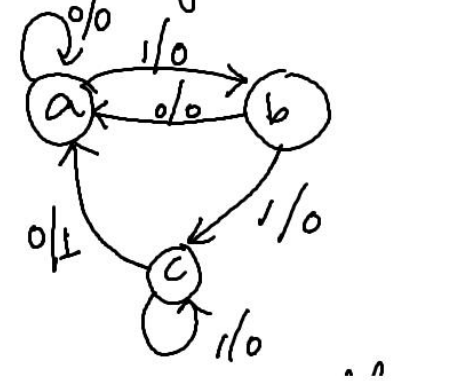


A SM chart of seq detailed
problem: Moore model

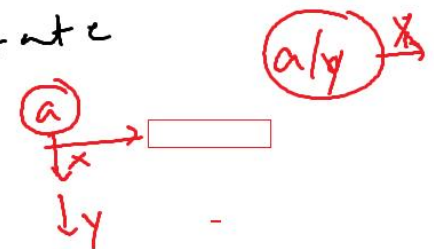
Asm chart of seq detector problem: Mealy model



Mealy model

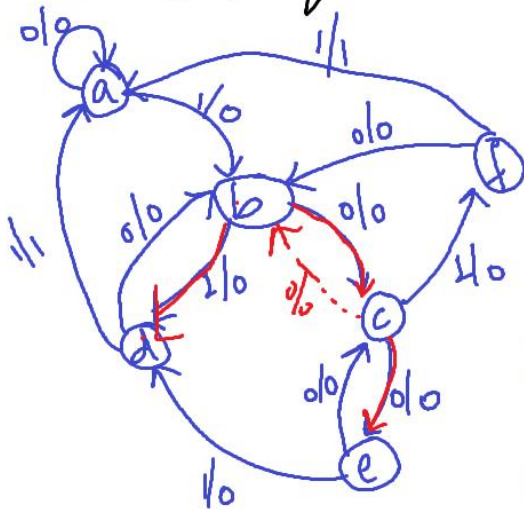


Stante



State Reduction Technique: Row-elimination method (Mealy model)

STD using Mealy:



S1:

Present State	Next State		Present O/p	
	x=0	x=1	x=0	x=1
a	a	b	0	0
<u>√b</u>	c✓	d✓	0	0
c	e	f	0	0
d	b	a	0	1
<u>√e</u>	c✓	d✓	a	0
f	b	a	0	1

remove row →

b = e, remove eth row

d = f, remove fth row

original table

next state
present o/p y
states are eq.

S4:

Present State	Next State		Present O/p	
	x=0	x=1	x=0	x=1
a	a	b	0	0
<u>√b</u>	c✓	d✓	0	0
<u>√c</u>	b	d	0	0
<u>√d</u>	b	a	0	1

b = c remove

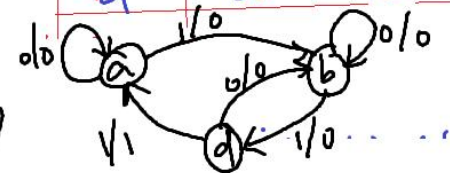
Present State	Next State		Present O/p	
	x=0	x=1	x=0	x=1
a	a	b	0	0
<u>√b</u>	b	d✓	0	0
<u>√d</u>	b	a	0	1

Final Simplified STD ⇒

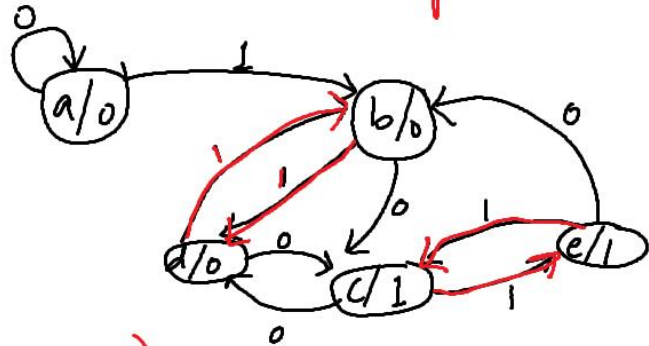
S2:

Present State	Next State		Present O/p	
	x=0	x=1	x=0	x=1
a	a	b	0	0
<u>√b</u>	c✓	d✓	0	0
c	e	f	0	0
<u>√d</u>	b✓	a✓	0	1

S3:



Reduce state-transⁿ diag^m (Moore model): row-elimⁿ meth.



a

Present state	Next state		Present o/p
	x=0	x=1	
a	a	b	0
b	c	d	0 ✓
c	d	e	1
d	c	b	0 ✓
e	b	c	1

original table

c

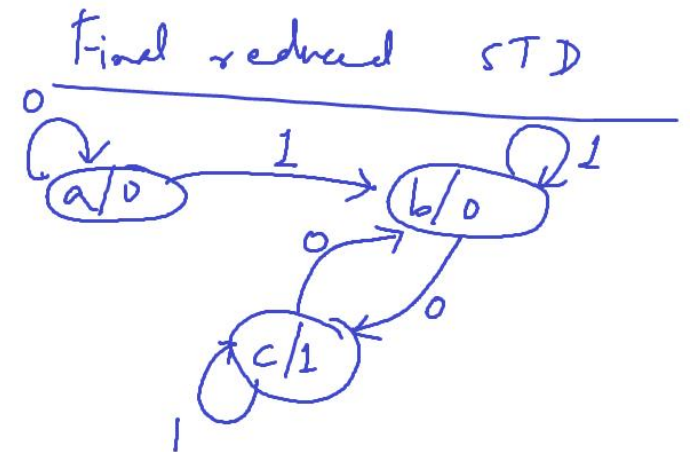
Present state	Next state		Present o/p
	x=0	x=1	
a	a	b	0
b	c	b	0 ✓
c	b	c	1 ✓

∴ b = d ∴ b on 1 ⇒ next state d } o/p = 0
d on 1 ⇒ ... b }

b

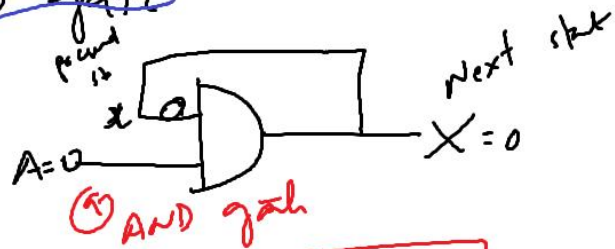
Present state	Next state		Present o/p
	x=0	x=1	
a	a	b	0
b	c	b	0 ✓
c	b	e	1 ✓
e	b	c	1 ✓

∴ c = e ,
∴ c on 1, ns = e } o/p = 1
e on 1, ns = c }
c = ~~e~~



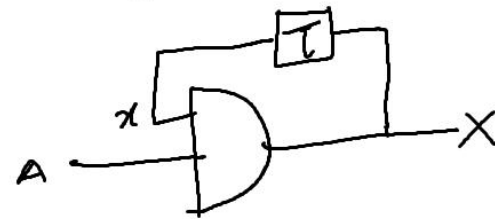
Asynchronous Seq ckt: Analysis of Asynch. Seq ckt

① AND gate



o/p $X=1$, when $A=1, x=1$
 $x=0$ when $A=0, x=0$
 $x=0$ when $A=0, x=1$
 $x=0$ when $A=0, x=0$

Unstable - stable state
 $0 \rightarrow 0$



$\rightleftarrows$ } deadlock

② AND gate with propagation delay.

x \ A	0		1	
	0	1	0	1
0	x	x		
1	x	x		

horizontaly vertically

x \ A	0		1	
	0	1	0	1
0	0	0		
1	0	1		

Truth table

$X = x \cdot A$

$$X = x \cdot A$$

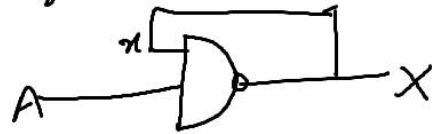
$X = x$, the i/p is stable
 $X \neq x$... unstable

$1 \rightarrow 1$

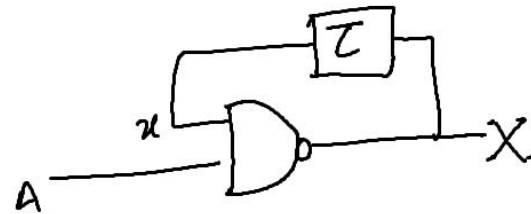
$0 \rightarrow 0$ unstable

stable - enable 0
 $1 \rightarrow 0$ $0 \rightarrow 1$

Analysis of NAND:



(a) NAND gate:



(b) NAND gate with propagation delay

NAND		
0	0	1
0	1	1
1	0	1
1	1	0

(c) Truth table

A	0	1
0	1	1
1	①	0

make transⁿ from 1) Unstable to stable
 $1 \rightarrow \textcircled{1}$ ② $0 \rightarrow \textcircled{0}$
 horizontally / vertically

2) Unstable: oscillations
 $(0-1) \text{ \& } (1-0)$