

Logic Design

Unit - 1

The basic gates - review of basic logic gates, positive & negative logic, introduction to HDL.

Combinational logic circuits - SOPs method, TT to K-map, pairs, quads & octets, K-map simplification, don't care condⁿs, POS methods, POS simplification by Quine-McCluskey method, Hazards & Hazard covers, HDL implementation models.

Introduction: Digital electronic circuits operate with voltages of 2-level levels namely logic low & logic high. The range of voltages corresponding to logic low is represented with '0'. Similarly, that with logic is high is '1'.

Logic gate: the basic digital electronic circuit that has one or more inputs & single output. Hence the logic gates are the building blocks of any digital sys.

Classification of logic gates: 3 categories
- basic, universal & special gates.

Basic gates: AND, OR & NOT gates.

AND Universal gates: NAND, NOR

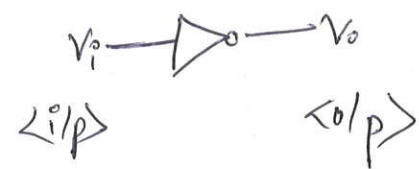
Special gates: EX-OR, EX-NOR.

HDL: Hardware Description Language, is used to describe the structure & behaviour of electronic circuits. It is

The basic gates: - NOT, OR, AND

Three logic circuits, the OR, the AND & the inverter (NOT) gates can be used to produce any digital sys.

The Inverter (NOT gate):
Its 7404-TTL



V_i	V_o
L	H
H	L

V_i	V_o
0	1
1	0

TTL - transistor - transistor

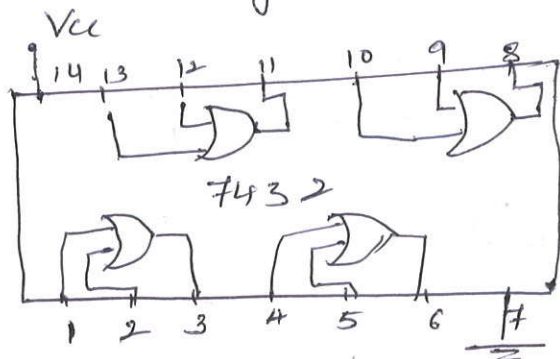
logic families
Transistor - Semiconductor device used to amplify
(1) Switch electronic signal & electric power

LED - Light Emitting Diode - to indicate a digital signal.

The OR gate: Fig shows the pin-diagram

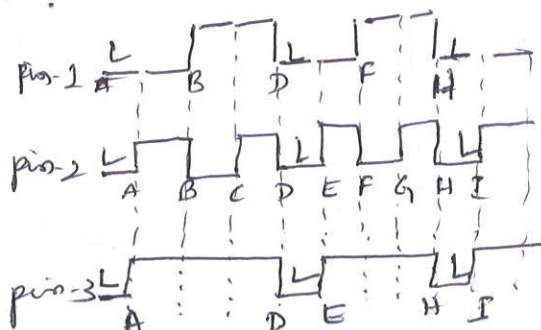
of a 7432, a TTL quad 2-i/p OR gate.

After connecting a supply voltage of +5V to pin-14 & a ground pin to pin-7, you can connect one more OR gates to other TTL device.



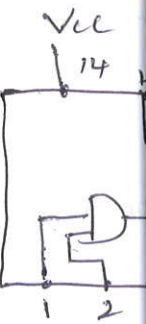
(a) pinout diagram

output (pin-3) is low only when both the inputs are low.



The A

a TTL voltage connect



(a) pin -

Inverter

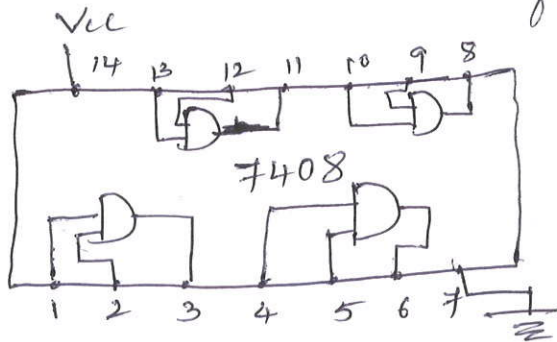
Boolean

* N

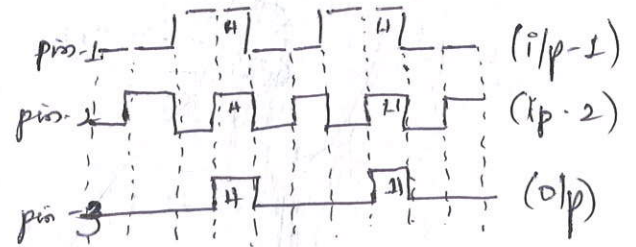
* OR

* AND

The AND-gate: Fig. shows a pin-out diagram of 7408, a TTL quad-2 i/p AND gate. After connecting a supply voltage of +5V to pin-14 and ground to pin-7, you can connect one or more of AND gates to other TTL device.



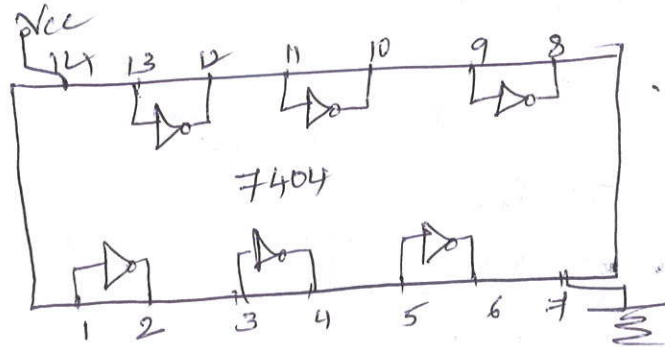
(a) pin-out diagram



O/p is high when both the i/ps are high.

(b) Timing diagram

Inverter:



6 NOT gates.
i.e., 6 inverters

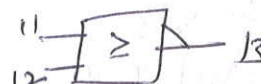
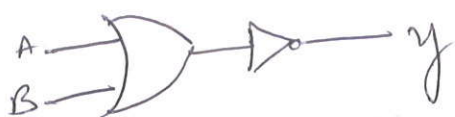
Boolean Algebra:

* NOT operation: $y = \bar{A}$ (y equal NOT A)
 $y = 0 = 1$
 $y = 1 = 0$

* OR operation: $y = A + B$
 $y = 1 + 0 = 1$ (where $A=1$ & $B=0$)
 $y = 0 + 1 = 1$ (where $A=0$ & $B=1$)

* AND operation: $y = A \cdot B$
 $y = 1 \cdot 0 = 0$
 $y = 0 \cdot 1 = 0$

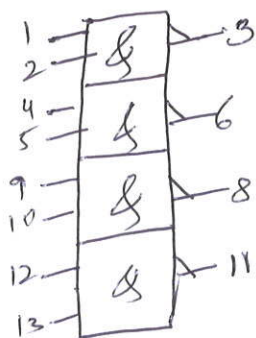
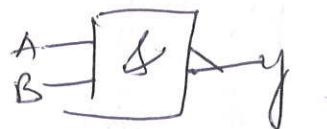
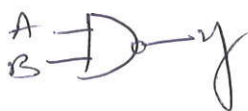
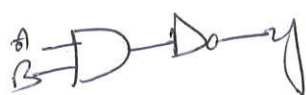
NOR gate Symbol : 7402



Equation

$$y = \overline{A + B} \quad (y \text{ equals NOT } A \text{ OR } B)$$

NAND gate Symbol :



7 - GND

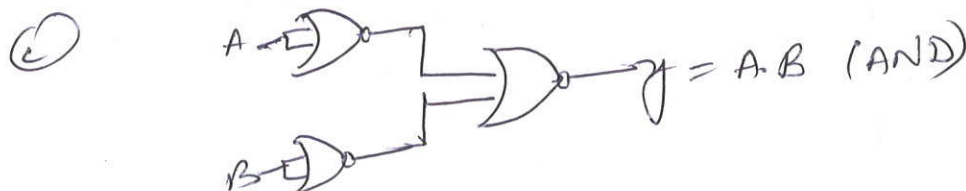
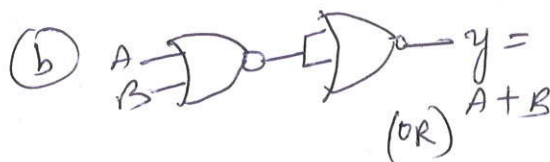
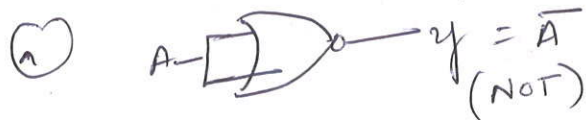
14 - VCC

$$y = \overline{A \cdot B} \quad (y \text{ equals NOT } A \text{ AND } B)$$

Universal logic gates - NOR, NAND :

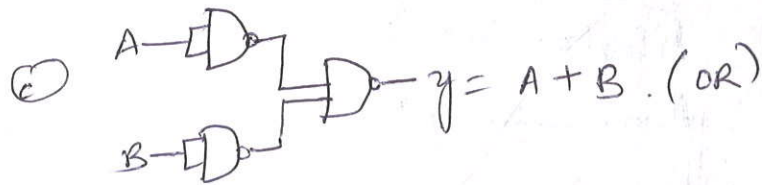
* Universality of NOR gate

↳ All other logic gates can be obtained from NOR gates



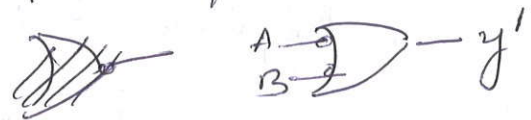
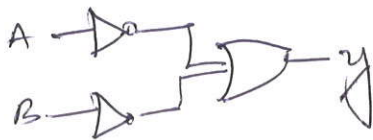
* Universality of NAND gate

↳ All other logic gates can be obtained from NAND gates.



NOR gate:

* Bubbled OR-gate: fig. shows inverter on the i/p lines of an AND gate.



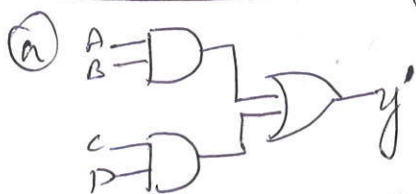
* De Morgan's second theorem.



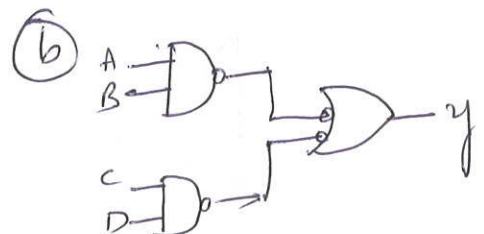
$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

The complement of a product is equal to the sum of the complements.

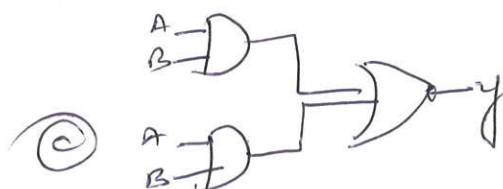
AND-OR-NOT gate:



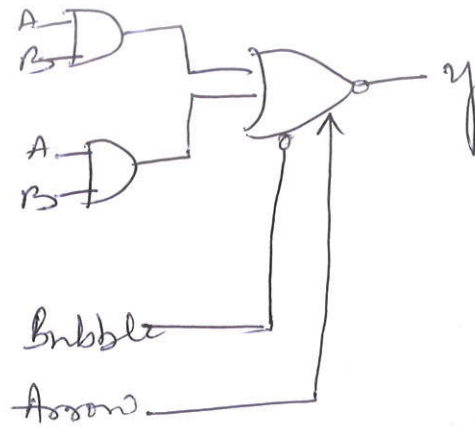
AND-OR circuit



NAND-NAND circuit



* Expandable AND-OR-NOT gate:



The widest AND-OR-NOT gate available in 7400 series is 4-wide. The 2 additional i/p's, labeled bubble & arrow allow it for 6 @ 8-wide circuit.

Positive & Negative logic

Use of binary 0 for low-voltage & a binary 1 for high voltage is called positive logic.

Use of binary 1 for low & binary 0 for high voltage is called negative logic.

Don't care conditions in logic design

- In some digital systems, certain i/p condⁿ never occur during normal operⁿ; therefore, the corresponding output never appears. Since the o/p never appears, it is indicated by X in the TT. The X is called a don't-care condition. The designer can assign either 0 (or) 1 to the o/p, whichever is optional, & produces a simpler logic circuit. Thus don't care condⁿs provide flexibility.

eg: Odd-parity^{bit} generator for (Binary Coded Decimal) BCD (0-9).

w	x	y	z	parity bit
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1 X
1	0	1	1	0
1	1	0	0	1 X
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0 X

$$p(w, x, y, z) = \sum m(0, 3, 5, 6, 9) + dc(10, 11, 12, 13, 14, 15)$$

The

p/w

p/w, x,

Realizat

XOR

The normal odd bit parity generator is,

$$P(w, x, y, z) = \sum m(0, 3, 5, 6, 9, 10, 12, 15)$$

$$= \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}x\bar{y}z$$

$$= \bar{w}\bar{x}(\bar{y}\bar{z} + y\bar{z}) + \bar{w}x(\bar{y}\bar{z} + y\bar{z}) + \bar{w}x(\bar{y}z + y\bar{z}) + \bar{w}x(\bar{y}z + yz)$$

$$= \bar{w}\bar{x}(\overline{y \oplus z}) + \bar{w}x(y \oplus z) + \bar{w}x(\overline{y \oplus z}) + \bar{w}x(y \oplus z)$$

$$= (\overline{y \oplus z})(\bar{w}\bar{x} + \bar{w}x) + y \oplus z(\bar{w}x + \bar{w}x)$$

$$= (\overline{y \oplus z})(\overline{w \oplus x}) + y \oplus z(w \oplus x)$$

$$\Rightarrow (\overline{w \oplus x})(\overline{y \oplus z}) + (\overline{w \oplus x})(y \oplus z) \quad \begin{array}{l} \nearrow \bar{A}\bar{B} + AB \\ \downarrow A \oplus B \end{array}$$

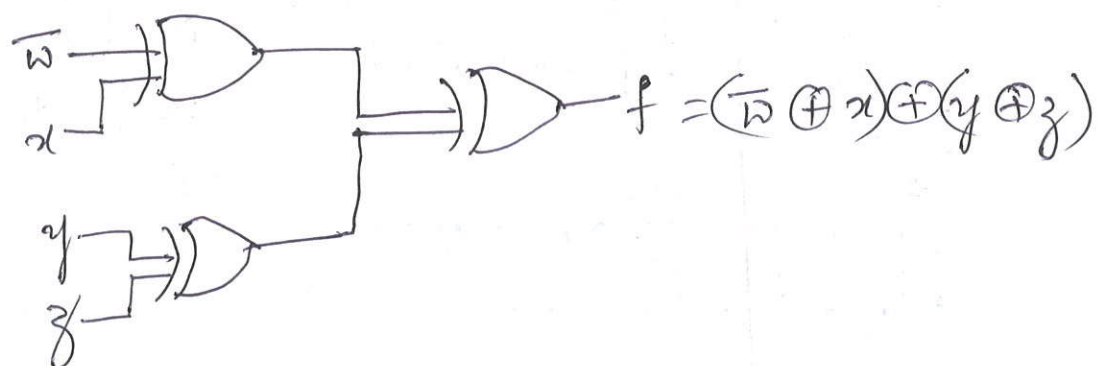
$$= \overline{(\overline{w \oplus x}) \oplus (y \oplus z)}$$

$$= \overline{w \oplus x} \oplus (y \oplus z)$$

$$\therefore \overline{A \oplus B} = A \oplus B$$

$$P(w, x, y, z) = (\overline{w \oplus x}) \oplus (y \oplus z)$$

Realization of odd-bit parity generator using only XOR gates



Using K-map for single-1:

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	X	X	X	X
10	0	1	X	X

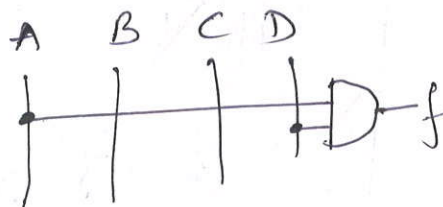
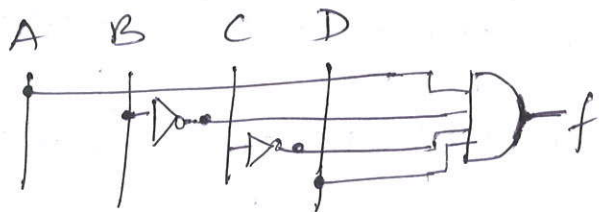
Without don't care cond,

$$f = A \bar{B} \bar{C} D \rightarrow \langle i \rangle$$

Now with don't care cond,

$$f = AD \rightarrow \langle ii \rangle$$

Realizⁿ of $\langle i \rangle$ & $\langle ii \rangle$



Product-of-Sum method: POS

- Given a TT, you identify the fⁿdamental sum needed for a logic-design.
- Then by ANDing these sums, you get the POS equatⁿ corresponding to the TT.
- With SOP, the fⁿdamental sum produces an o/p '0' for the corresponding i/p condⁿ.

A	B	C	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

$$\rightarrow A + B + C$$

$$\rightarrow A + \bar{B} + \bar{C}$$

$$\rightarrow \bar{A} + \bar{B} + C$$

Locate each o/p '0' in the TT & write down its fundamental sum.

Conver

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POS

→ Conver
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Conversion b/w SOP and POS:

- We have seen that SOP representⁿ is obtained by considering "ones" in the TT while POS comes considering "zeros".
- In SOP, each 1 @ output gives one AND term which is finally ORed.
- In POS, each zero gives one OR term which is finally ANDed.
- Thus SOP and POS occupy complementary locatⁿs in a TT and one representⁿ can be obtained from the other by
 - ↳ identifying complementary bcⁿs
 - ↳ changing min-term to max-term (00) reverse & finally
 - ↳ changing summation by product (or) reverse

POS Simplificatⁿ

- Convert the TT into k-map. After grouping the 1s, write the SOP eqnⁿ & draw the NAND-NAND circuit. This is the SOP for 'y'.
- Complement the k-map. Group the 1s, write the SOP eqnⁿ, & draw the NAND-NAND circuit for $\bar{y}$. This is the complementary NAND-NAND circuit.
- Convert the complementary NAND-NAND circuit to a dual NOR-NOR circuit by changing all NAND gates to NOR gates & complementing all signals. What remains is the POS solⁿ for 'y'.
- Compare the NAND-NAND ckt with the NOR-NOR ckt (step-3). One can use whichever ckt you prefer.

Simplification by Quine-McCluskey Method

Algorithm:

- 1) List all minterms in the binary form.
- 2) Arrange the minterms according to number of 1's.
- 3) Compare each binary m with every term in the adjacent next higher category and if they differ only by one position, put a check mark & copy the term in the next column with '-' in the position that they differed.
- 4) Apply the same process described in step-3 for the resultant column & continue these cycles until a single pass through cycle yields no further eliminatⁿ of literals.
- 5) List all the prime-implicants.
- 6) Select the minimum no of prime implicants which must cover all the minterms.

Eg: 1) Simplify the fol Boolean f's by using Quine McCluskey method. $f(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$

Solⁿ ~~Start~~

Step-1: List all minterms in binary form as column (a)

Step 2: Arrange ~~all~~ minterms according to no of 1's, as in col-(b).

minterms	Bin-represent ⁿ	minterms	binary represent ⁿ
m_0	0 0 0 0	m_0	0 0 0 0 ✓
m_2	0 0 1 0	m_2	0 0 1 0 ✓
m_3	0 0 1 1	m_8	1 0 0 0 ✓
m_6	0 1 1 0	m_3	0 0 1 1 ✓
m_7	0 1 1 1	m_6	0 1 1 0 ✓
m_8	1 0 0 0	m_{10}	1 0 1 0 ✓
m_{10}	1 0 1 0	m_{12}	1 1 0 0 ✓
m_{12}	1 1 0 0	m_7	0 1 1 1 ✓
m_{13}	1 1 0 1	m_{13}	1 1 0 1 ✓

col-(a)

col-(b)

Step-3: Compare each binary no with every term in the adjacent next higher category and if they differ only by one position, put a check mark and copy the term in the next column, with '-' in the position that they differed.

Step 4: Apply the same process as in Step-3 for the resultant column & continue these cycles until no classⁿ of literal occurs.

min-terms	Bin-rep ⁿ
0, 2	0 0 - 0 ✓
0, 8	- 0 0 0 ✓
2, 3	0 0 1 - ✓
2, 6	0 - 1 0 ✓
2, 10	- 0 1 0 ✓
8, 10	1 0 - 0 ✓
8, 12	1 - 0 0 ✓
3, 7	0 - 1 1 ✓
6, 7	0 1 1 - ✓
12, 13	1 1 0 -

col-(C)

min-terms	Bin-represent ⁿ
0, 2, 8, 10	- 0 - 0
2, 3, 6, 7	0 - 1 -

col-(A)

Step-5: List the prime implicants

Prime implicants	Bin-represent ⁿ
$A\bar{C}\bar{D}$ 8, 12	1 - 0 0
$AB\bar{C}$ 12, 13	1 1 0 -
$\bar{B}\bar{D}$ 0, 2, 8, 10	- 0 - 0
$\bar{A}C$ 2, 3, 6, 7	0 - 1 -

Step-6: Select the min-no of ~~prime-terms~~ prime-implicants which must cover all the min-terms.

Prime Implicants	m ₀ (col-1)	m ₂ (col-2)	m ₃ (col-3)	m ₆ (col-4)	m ₇ (col-5)	m ₈ (col-6)	m ₁₀ (col-7)	m ₁₂ (col-8)	m ₁₃ (col-9)
$A\bar{C}\bar{D}$ (8, 12)									
$AB\bar{C}$ (12, 13) ✓								⊙	⊙
$\bar{B}\bar{D}$ (0, 2, 8, 10) ✓	⊙	⊙				⊙	⊙		
$\bar{A}C$ (2, 3, 6, 7) ✓		⊙	⊙	⊙	⊙				

Search for single dot columns & select the prime implicants corresponding to that dot by putting the check mark in front of it. Search for multi-dot columns one-by-one. If the corresponding min-terms is already included in the final expressⁿ, ignore the min-terms & go to next-dot-colⁿ.

$$F(A, B, C, D) = AB\bar{C} + \bar{B}\bar{D} + \bar{A}C$$

on the
differs
copy
the position

resultant
literal

counts

counts
0
-
0
-

which

m_{13}
$(\omega - q)$
$\odot$

is corresponding
Search
information
turns ϵ

Enrico McCluskey using don't care terms

Unit - 2

Data - Processing Circuits

- Logic cts that process binary data

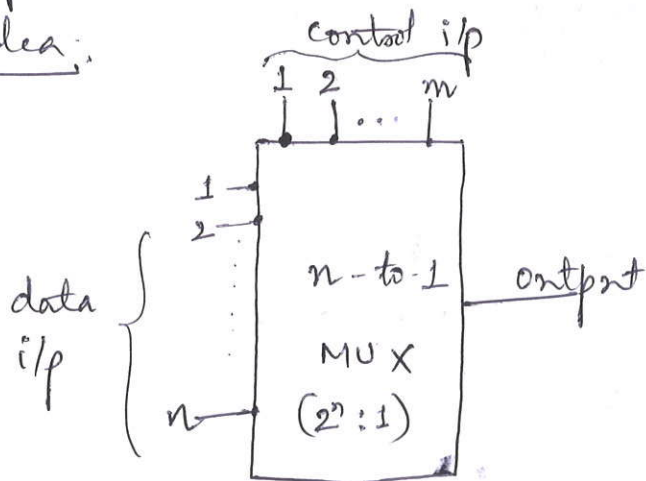
Eg: MUX, De-Mux, 1-of-16 decoder, BCD to decimal decoders, Seven segment decoders, encoders, EX-OR gates, parity checkers/generators, magnitude comparators, read-only memories, PLAs & PALs.

Multiplexers: - data selector

- means many into one.

- is a ckt with many i/p ~~but~~ only 1-o/p. By applying control signals, we can steer any i/p to the o/p. Thus it is also called a data selector & control i/p's are termed as select i/p's.

General idea:



The ckt has n -i/p signals
 m -control signal
& 1-o/p signal

Note: m -control signals can select at most 2^m i/p signals that $n \leq 2^m$.

i.e., $\boxed{2^m \text{ to } 1 \text{ MUX}}$

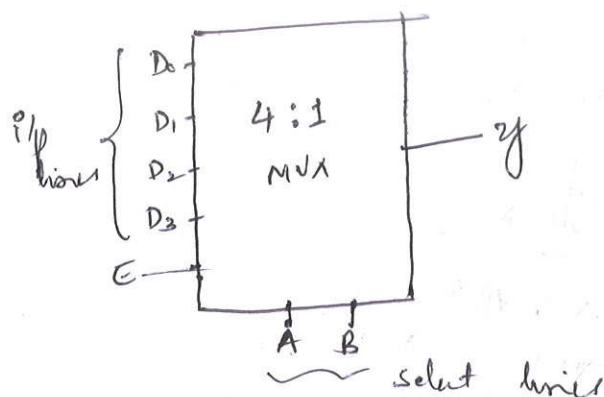
Ckt

Logic

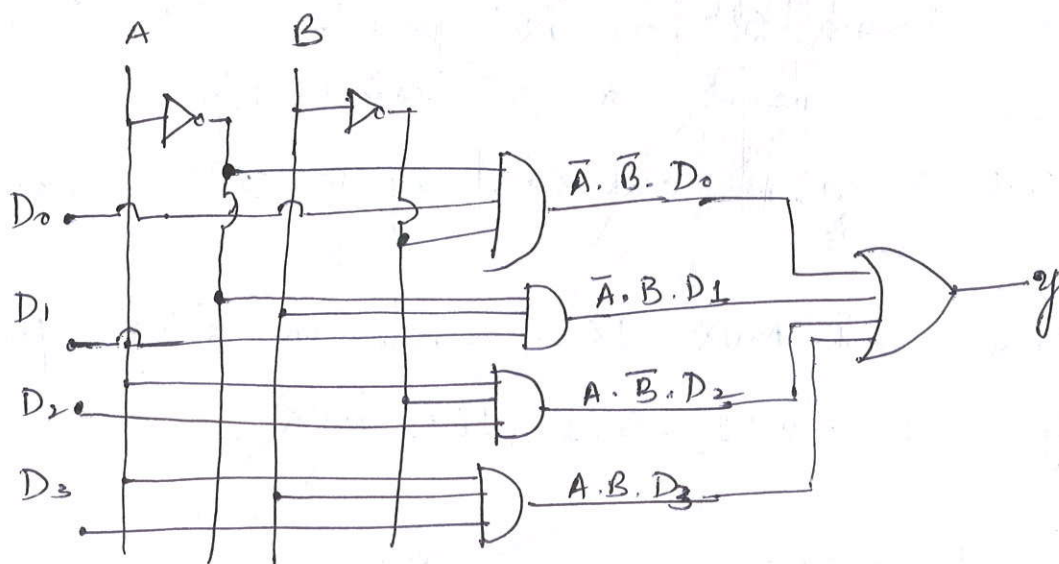
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Ckt diagram: 4-to-1 MUX

Fⁿ Table / Truth Table:



A	B	y
0	0	D ₀
0	1	D ₁
1	0	D ₂
1	1	D ₃



Logic eqnⁿ of the above ckt:

$$y = \bar{A}\bar{B}.D_0 + \bar{A}.B.D_1 + A.\bar{B}.D_2 + A.B.D_3$$

If $A=0$ and $B=0$

$$= \bar{0}.\bar{0}.D_0 + \bar{0}.0.D_1 + 0.\bar{0}.D_2 + 0.0.D_3$$

$$= 1.1.D_0 + 1.0.D_1 + 0.1.D_2 + 0.0.D_3$$

$$= 1.D_0 + 0 + 0 + 0$$

$$\boxed{y = D_0}$$

In other words, for $AB=00$, the first AND gate to which D_0 is connected remains active & equal to D_0 & all other AND gates are in-active with o/p held @ logic 0. Thus, mux o/p 'y' is same as D_0 .

∴ If $B=0$, then $y=0$ and ∴ $D_0=1$, $y=1$.

1111, for $AB=01$, 2nd AND gate will be active & all other AND gates remain inactive. Thus o/p $y=D1$. Following same procedure we can complete the TT.

A low -
if equal

On the
& forces
the value

Strobe/ Enable	
L	
L	
L	
:	
:	
:	
L	
H	

Bubbles

If we want 5:1 MUX, then how many select lines are reqd.?

- There is no 5th combination possible with 2 select lines & hence we need a 3rd select i/p.
- With 3_{select} i/p, we can select up-to $2^3 = 8$ data i/p's.
- Commercial MUX ICs come in integer power of 2, eg: 2:1, 4:1, 8:1, 16:1 MUX.

16-to-1 MUX:

The i/p bits are labelled D_0 to D_{15} . Only one of these is transmitted to o/p, which one depends on the value of control i/p ABCD.

Eg: When $ABCD = 0000$, the upper AND gate is enabled while all other AND gates are disabled. Therefore, data bit D_0 is transmitted to the o/p giving $y = D_0$. Similarly $y = D_{15}$, when $ABCD = 1111$.

The 74150: is a 16-to-1 TTL MUX.

Pins - 1 to 18 and 26-23 - are data lines

Pins - 11, 13, 14, 15 are control lines - ABCD.

Pins - 10 is the o/p.

active
is o/p
complete

A low-strobe enables the multiplexer ($\overline{E}$), so that o/p y equals the complement of the i/p-data bit:
i.e., $y = \overline{D_n}$ where n = decimal equivalent of ABCD.

select lines

On the other hand, a high strobe ~~also~~ disables the MUX & forces the o/p into the high state. With a high strobe, the value of ABCD does NOT matter.

select lines

a i/p.

ver of 2,

one of
depends

gate

is all

omitted

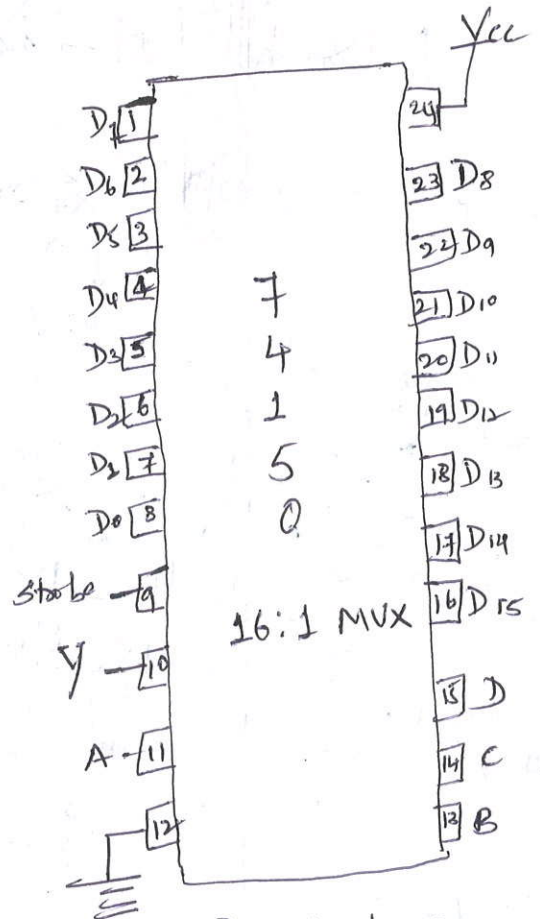
D₁₅,

lines

D.

Strobe/ Enable	A	B	C	D	Y
L	L	L	L	L	$\overline{D_0}$
L	L	L	L	H	$\overline{D_1}$
L	L	L	H	L	$\overline{D_2}$
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
L	H	H	H	H	$\overline{D_{15}}$
H	X	X	X	X	H

74150 Truth Table



Pinout diagram of 74150

Bubbles on Signal line:

$y \Rightarrow$ is the o/p that is the complement of the selected data-bit.

$y \Rightarrow$ bubble on i/p pin, means the signal is active low.

Universal logic ckt :

* MUX is sometimes called as universal logic ckt because 2ⁿ-to-1 MUX can be used as design soln for any n-vari TT.

* Ex: TT can be realized using 8-to-1 MUX.
 $f(A, B, C, D)$

Let's consider A, B & C vari $\rightarrow$ to be fed as Select line.
 fourth vari D $\rightarrow$ has to be present as a data line.

Ex: ① $f(w, x, y, z) = \sum m(0, 1, 5, 6, 7, 9, 12, 15)$
 $\downarrow$
 write this in the form of TT
 (or) K-map.

		yz			
wx	00	01	11	10	
	00	1	1	0	I_0
	01	0	1	1	I_1
	11	1	0	0	I_2
	10	0	1	0	I_3

Submaps :

$w=0$
 $x=0$
 $y=0$

0	1
1	1

I_0

$w=0$
 $x=0$
 $y=1$

1	0
0	0

I_1

$w=0$
 $x=1$
 $y=0$

0	1
0	1

I_2

$w=0$
 $x=1$
 $y=1$

1	0
1	1

I_3

$w=1$
 $x=0$
 $y=0$

0	1
0	1

I_4

$w=1$
 $x=0$
 $y=1$

1	0
0	0

I_5

$w=1$
 $x=1$
 $y=0$

0	1
1	0

I_6

$w=1$
 $x=1$
 $y=1$

1	0
1	0

I_7

$I_0 =$
 $I_1 =$
 $I_2 =$

② Realization

00
 01
 11
 10

I_0 sub

I_1 sub

I_2 sub

give ckt
sols for

$$I_0 = \bar{y} + z = 0 + 1 = 1$$

$$I_1 = 0$$

$$I_2 = z$$

$$I_3 = z + \bar{y} = 1$$

$$I_4 = z$$

$$I_5 = 0$$

$$I_6 = \bar{y}$$

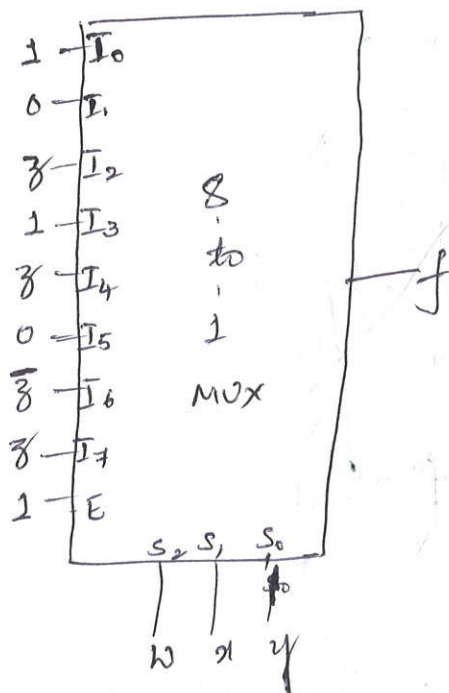
$$I_7 = z$$

$$E = 1$$

Select line
data
line.

15)

TT



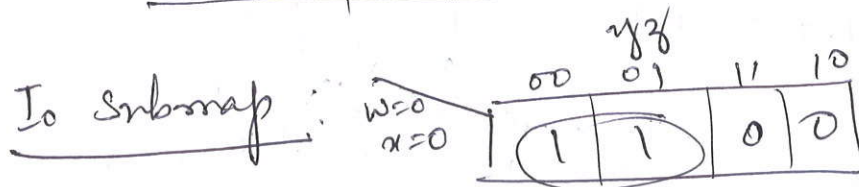
(2)

Realization of 4-var boolean f using 4 to 1 mux

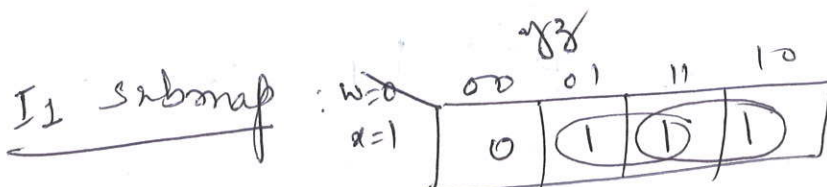
$$f(w, x, y, z) = \sum m(0, 1, 5, 6, 7, 9, 13, 14)$$

	00	01	11	10	
00	1	1	0	0	I_0
01	0	1	1	1	I_1
11	0	1	0	1	I_3
10	0	1	0	0	I_2

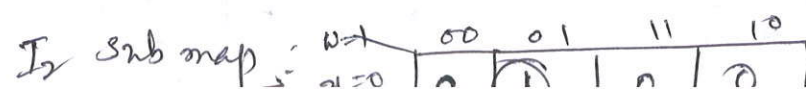
wx — select lines
yz — data lines



$$I_0 = \bar{y}$$



$$I_1 = z + y$$



$$I_2 = \bar{y}z$$

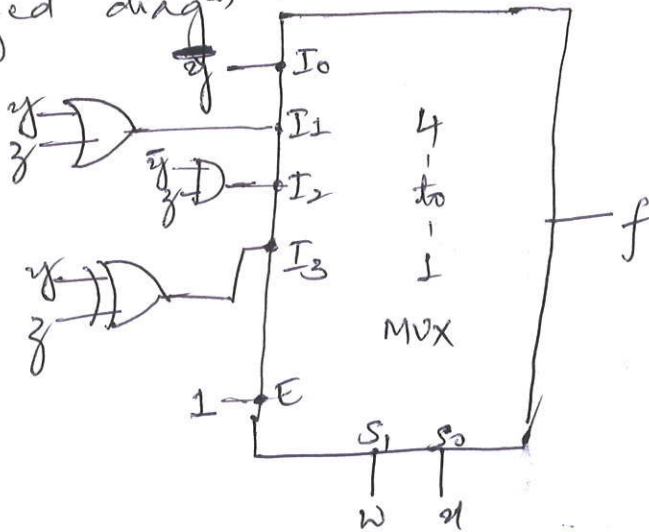
I_3 submap: $w=1$ $x=1$

	00	01	11	10
y	0	1	0	1

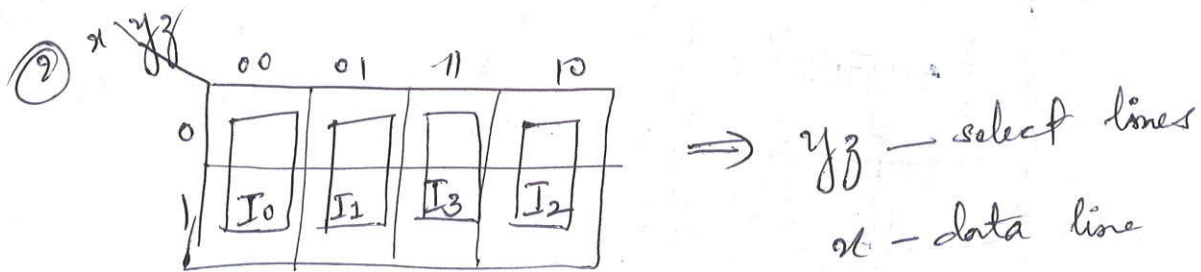
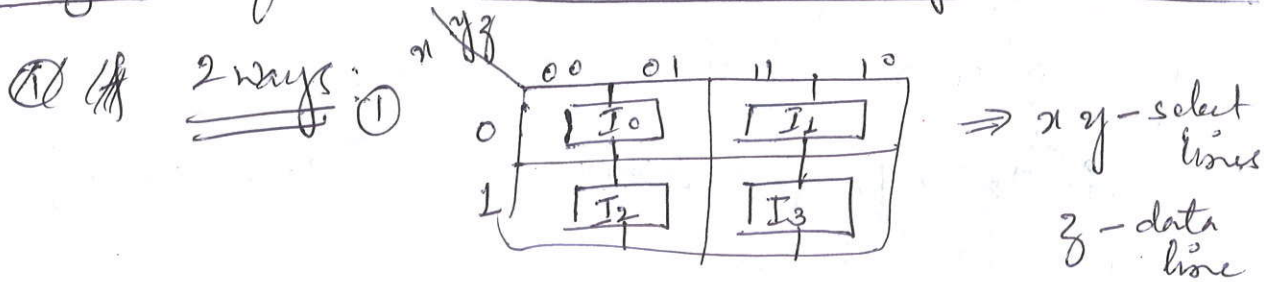
$$= \bar{y}z + y\bar{z}$$

$$I_3 = y \oplus z$$

Realized diagram



Realization of 3 vari boolean f^s using 4-to-1 MUX



Eg: First method:

$$f(x, y, z) = \sum m(0, 2, 3, 5)$$

	00	01	11	10
x	0	1	1	0
y	0	1	0	1

I_0 - map: $x=0$ $y=0$

	0	1
z	1	0

$$= \bar{z}$$

I_1 - map: $x=0$ $y=1$

	0	1
z	1	1

$$= 1$$

I_2 - map: $x=1$ $y=0$

	0	1
z	0	1

$$= z$$

I_3 - map: $x=1$ $y=1$

	0	1
z	0	0

$$= 0$$

Second

Nibble

$\rightarrow$ of 4-b
nibbles

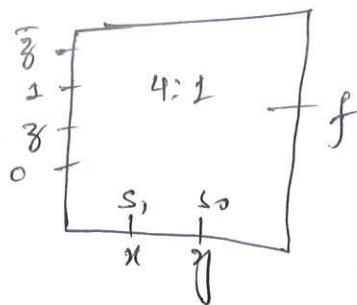
$\rightarrow$ Input
 B_3, B_2
det

$\rightarrow$ When

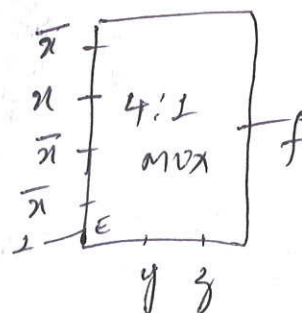
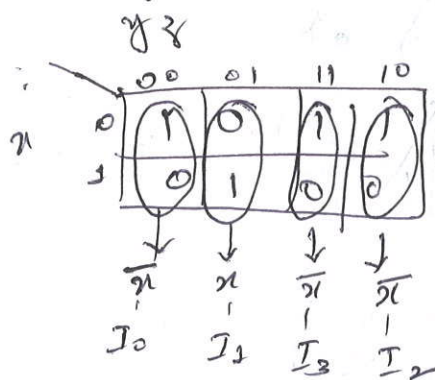
$\rightarrow$ when
active

$\rightarrow$ i.e.,
to the

$$I_0 = \bar{S} \quad I_1 = 1 \quad I_2 = S \quad I_3 = 0$$



Second method



Nibble MUX

- 1 MUX

- select lines

- data line

- 4 lines

line

→ In computing (or) digital, nibble means combination of 4-bit. Eg. 0010 (or) 0001 etc/.

→ Sometimes, we want to select one of 2 i/p nibbles. So use a nibble-MUX.

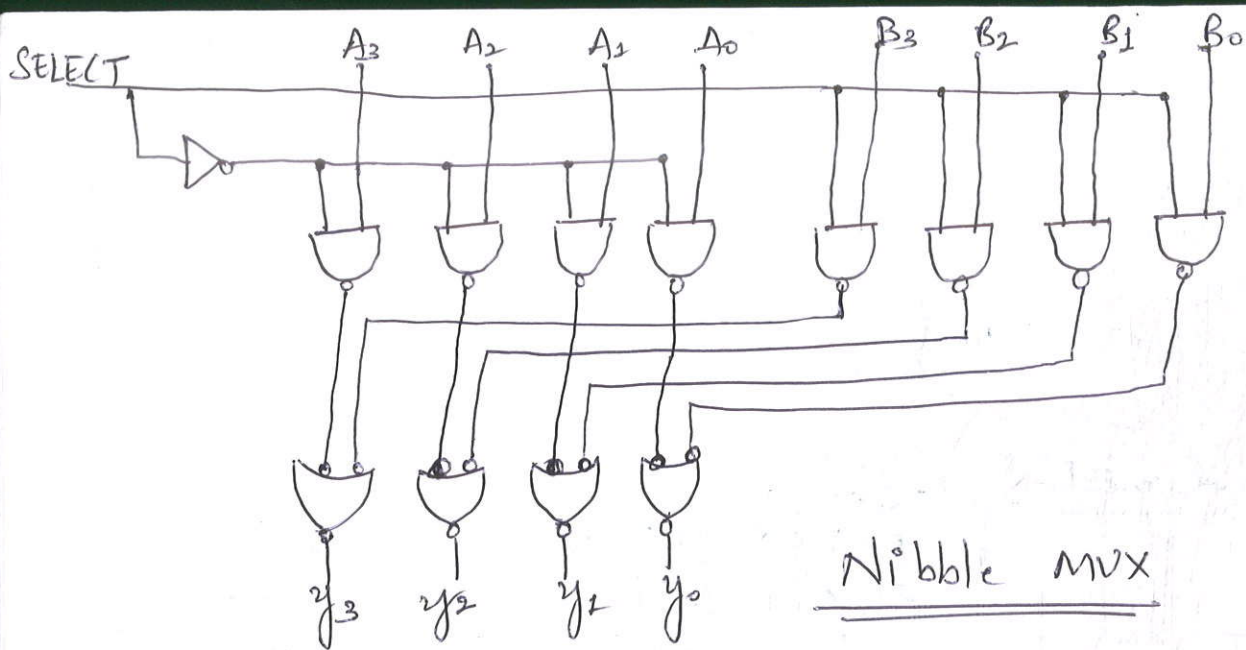
→ Input on left is A_3, A_2, A_1, A_0 & on right is B_3, B_2, B_1, B_0 . The control signal labeled SELECT det which i/p-nibble is transmitted to the o/p.

→ When $SELECT = 0$, 4-NAND gates ^{on LEFT} are activated.
i.e., $y_3 \cdot y_2 \cdot y_1 \cdot y_0 = A_3 \cdot A_2 \cdot A_1 \cdot A_0$

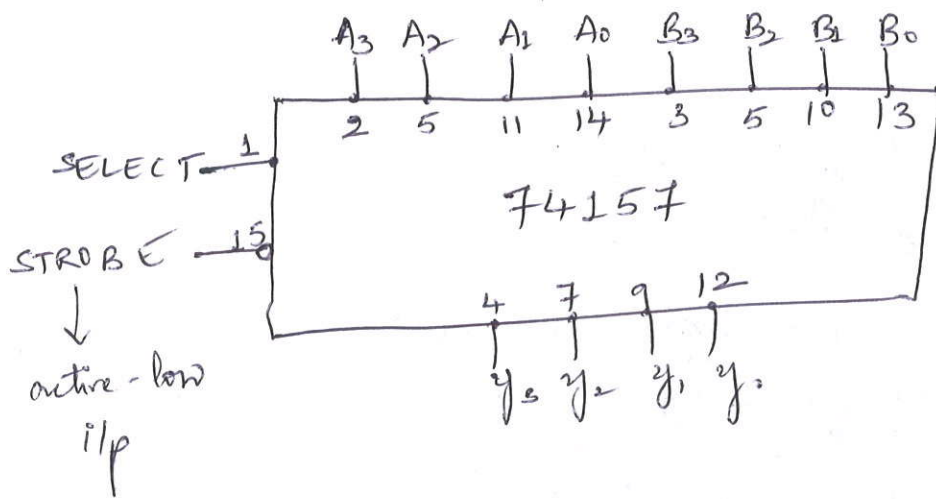
→ When $SELECT = 1$, 4-NAND gates on RIGHT are activated.

$$i.e., y_3 \cdot y_2 \cdot y_1 \cdot y_0 = B_3 \cdot B_2 \cdot B_1 \cdot B_0$$

→ i.e., when $SELECT = 0$, left nibble is steered to the o/p, else right nibble is steered when it is high.



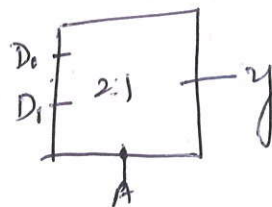
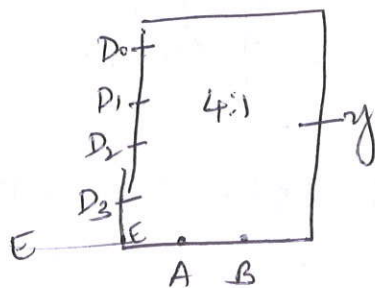
Pin-diagram: 74157



Worked Examples:

1) Show how 4:1 MUX can be obtained using only 2:1 MUX

Soln:



Logic eqⁿ for 2:1 MUX : $y = \bar{A} \cdot D_0 + A \cdot D_1 \rightarrow (1)$

Logic eqⁿ for 4:1 MUX : $y = \bar{A} \cdot \bar{B} \cdot D_0 + \bar{A} \cdot B \cdot D_1 + A \cdot \bar{B} \cdot D_2 + A \cdot B \cdot D_3$

The
Compa
b
bracketed
these
i/p's
mux.

(2) De

Soln:

$\Rightarrow$ 4 so
cho

$\Rightarrow$ 2-to
of the
MUX
what
5th

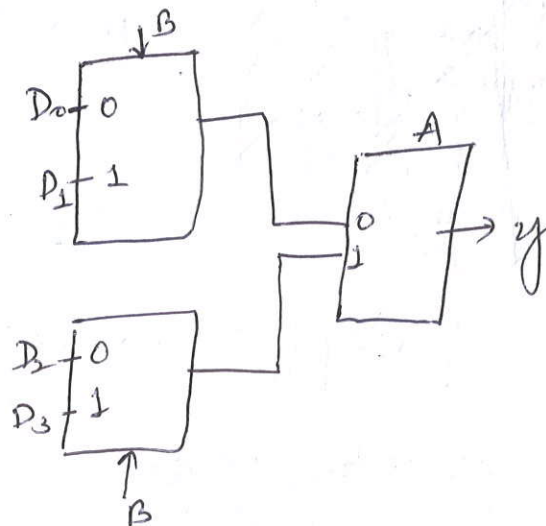


The logic eqⁿ of 4:1 MUX can be re-written as

$$y = \bar{A}(\bar{B}D_0 + BD_1) + A(\bar{B}D_2 + BD_3) \rightarrow (2)$$

Compare eqⁿ (1) & (2),

We need ~~2~~ two 2:1 MUX to realize the two bracketed terms where B serves as select i/p. The o/p of these 2 MUXs can be sent to a 3rd MUX as data i/p's where A serves as select i/p & we get 4:1 MUX.

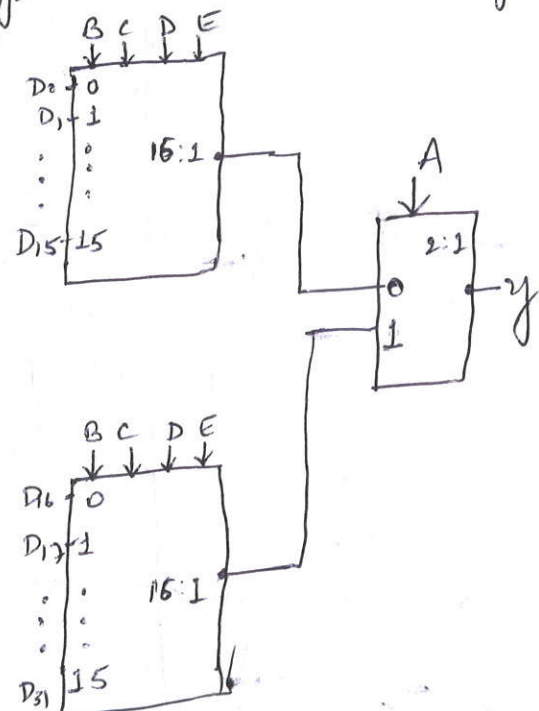


(2) Design a 32-to-1 MUX using two 16:1 MUX & one 2-to-1 MUX.

Solⁿ: A 32-to-1 MUX requires $\log_2 32 = 5$ select lines say, ABCDE.

⇒ 4 select lines BCDE choose 16-to-1 MUX o/p's.

⇒ 2-to-1 MUX chooses one of the o/p of 2 16-to-1 MUX depending on what appears in the 5th select line - A



using

→ (1)

B · D₁ +

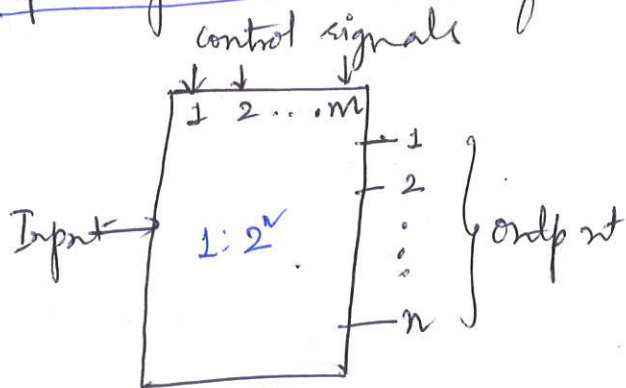
B · D₃

Demultiplexers :

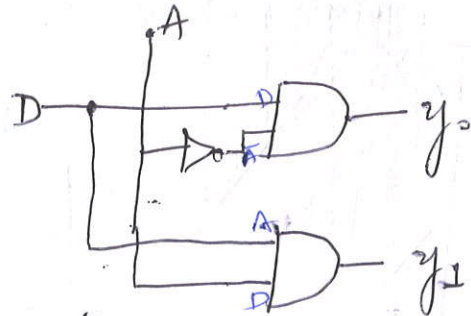
→ Demultiplex means one into many.

→ A demultiplexer is a logic circuit with one i/p & many o/p's.

→ By applying control signals, we can steer the i/p signal to one of the o/p lines.



(a) Block diagⁿ



(b) Logic ckt
 $1:2 \Rightarrow \bar{A}D + AD$
 1-to-2 mux

For ~~availability~~ available 2^m

1-to-16 MUX: 74154 is a 1-to-16 DeMUX

- The ckt has 1 i/p signal
 m -control @ select signals
 n -o/p signals,
 where $n \leq 2^m$

→ The i/p bit is labeled - D
 This data bit (D) is transmitted to the data bit of the o/p lines. This o/p depends on the value of ABCD, the control i/p.

→ When ABCD = 0000, The upper AND gate is enabled while all other AND gates are disabled.
 $\therefore$ D is transmitted to y_0 o/p, giving $y_0 = D$.

→ If
 if
 i.e.
 All

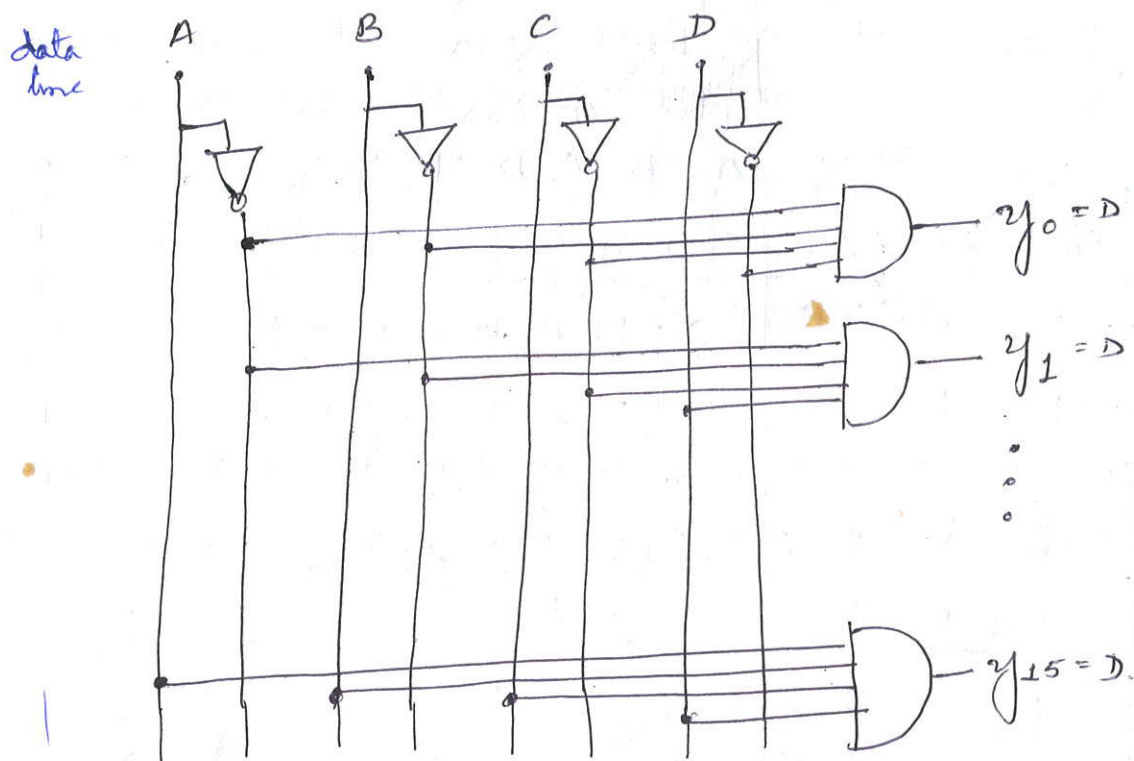
→ If the
 all go
 & no

List of

→ If D is low, y_0 is low
 if D is high, y_0 is high.

i.e., the value of y_0 depends on the value of D.
 All other o/p are in the low-state.

→ If the control - nibble is changed to ABCD = 1111,
 all gates are disabled except bottom AND gate,
 & resulting y_{15} o/p, i.e., $y_{15} = D$.



1 to 16 decoder

List of commercially available DeMUX ICs :

IC NO	DEMUX Type	Decoder Type
74154	1-to-16	4-to-16
74138	1-to-8	3-to-8
74155	1-to-4	2-to-4

The 74154 : is a 1-to-16 deMux.

Pin 18 - i/p data D. - active low ✓

Pins 20 to 23 - control bits ABCD

Pins 1 to 11 and 13 to 17 - O/p lines.

Pin 19 - STROBE - active-low i/p. ✓

Pin 24 & Pin 12 - Vcc & ground resp.

Truth Table of 1-to-16 DeMux : When the strobe is low, the control-i/p det. which o/p lines are low, similarly with high state.

Strobe	Data	A	B	C	D	y ₀	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇	y ₈	y ₉	y ₁₀	y ₁₁	y ₁₂	y ₁₃	y ₁₄	y ₁₅
→ L	H	X	X	X	X	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
→ H	L	X	X	X	X	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
→ H	H	X	X	X	X	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H
L/D	L/D	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	1		0														
0	0	0	0	1	0			0													
0	0	0	0	1	1				0												
0	0	0	1	0	0					0											
0	0	0	1	0	1						0										
0	0	0	1	1	0							0									
0	0	0	1	1	1								0								
0	0	1	0	0	0									0							
0	0	1	0	0	1										0						
0	0	1	0	1	0											0					
0	0	1	0	1	1												0				
0	0	1	1	0	0													0			
0	0	1	1	0	1														0		
0	0	1	1	1	0															0	
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Pin / b

* Data

Strobe

Worked

b e

Soln

For

if A

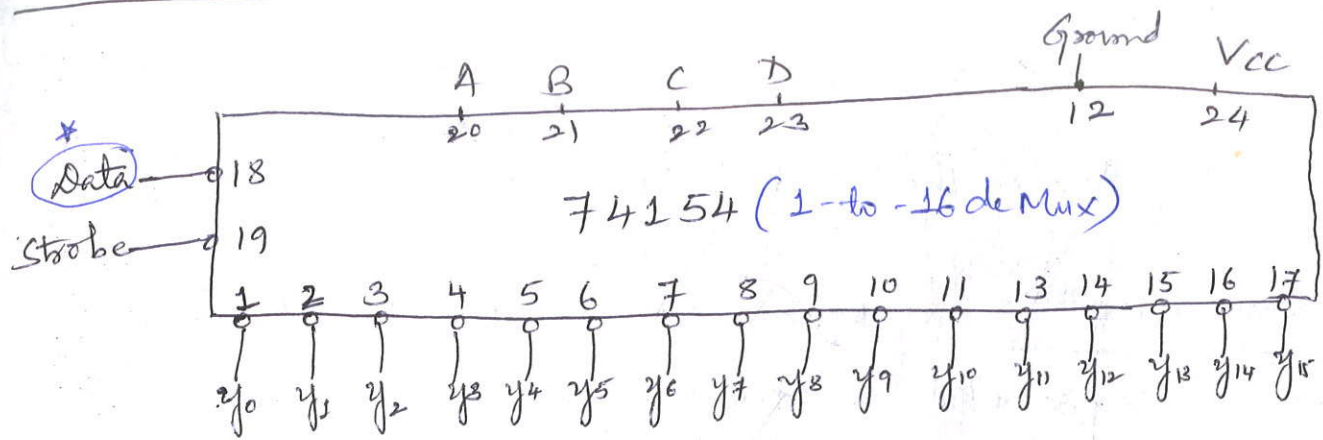
if A=

1 to 32

from

1 to 16

Pin / logic diagram of 74154 - deMux



Worked Example: Show how two 1-to-16 MUX can be connected to get 1-to-32 deMUX.

Solⁿ: 1-to-32 deMUX has 5 select @ vars A B C D E.

Four of them B C D E - fed to two 1-to-16 deMUX.

Fifth A - used to select one of these two deMUX through strobe-i/p.

If $A = 0$, the top 74154 is chosen.

& B C D E directs data to one of the 15 o/p's of the IC.

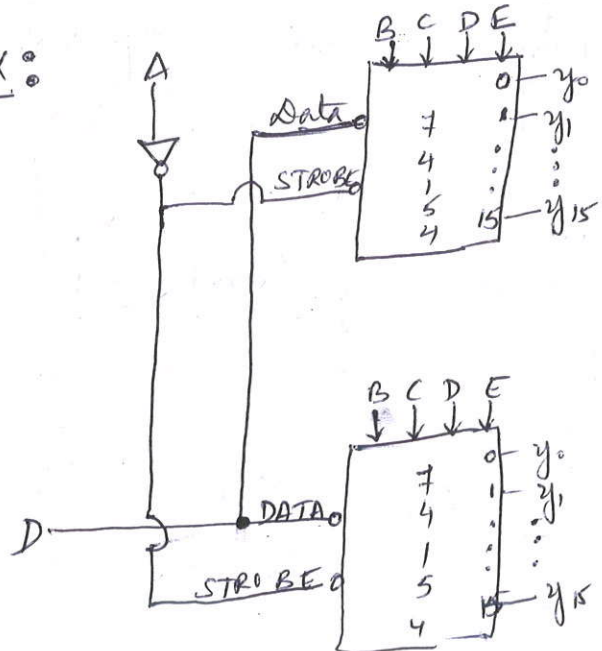
If $A = 1$, the top 74154 is chosen,

& depending on the value of B C D E, the data is directed to one of the 15 o/p's of the IC.

1 to 32 MUX:

from

1 to 16 MUX



1 of 16 decoder : 74154 - with NO data i/p

* A decoder is similar to demux, with one exceptⁿ there is NO data input.

* Inputs are the control bits - ABCD

Logic Diagram : is same as demux except D-line.

* The logic ckt is called 1 of 16 decoder because only 1 of the 16 o/p lines is high.

* Binary to Decimal decoder : ✓

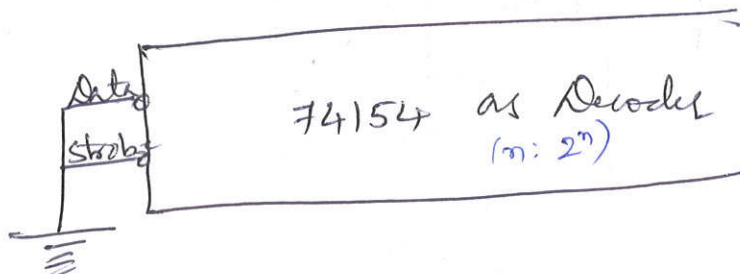
ABCD - possibilities from 0000 to 1111.

The subscript of the high o/p always equal to the decimal equivalent of ABCD. For this reason, the ckt is sometimes called a binary-to-decimal decoder.

4-line to 16-line decoder : ✓ Since it has 4-i/p lines & 16-o/p lines.

Decoder - demultiplexer : 74154 is called a decoder-demux, because it can be used either.

To use 74154 as decoder, just ground the i/p DATA & STROBE. Then, the selected o/p line is in low-state.



Using 74154 - Demux as Decoder

Worked

① Read
ex mult

f1

f2

f

Soln :

BCD

- The

no.

Eg:

Worked Example:

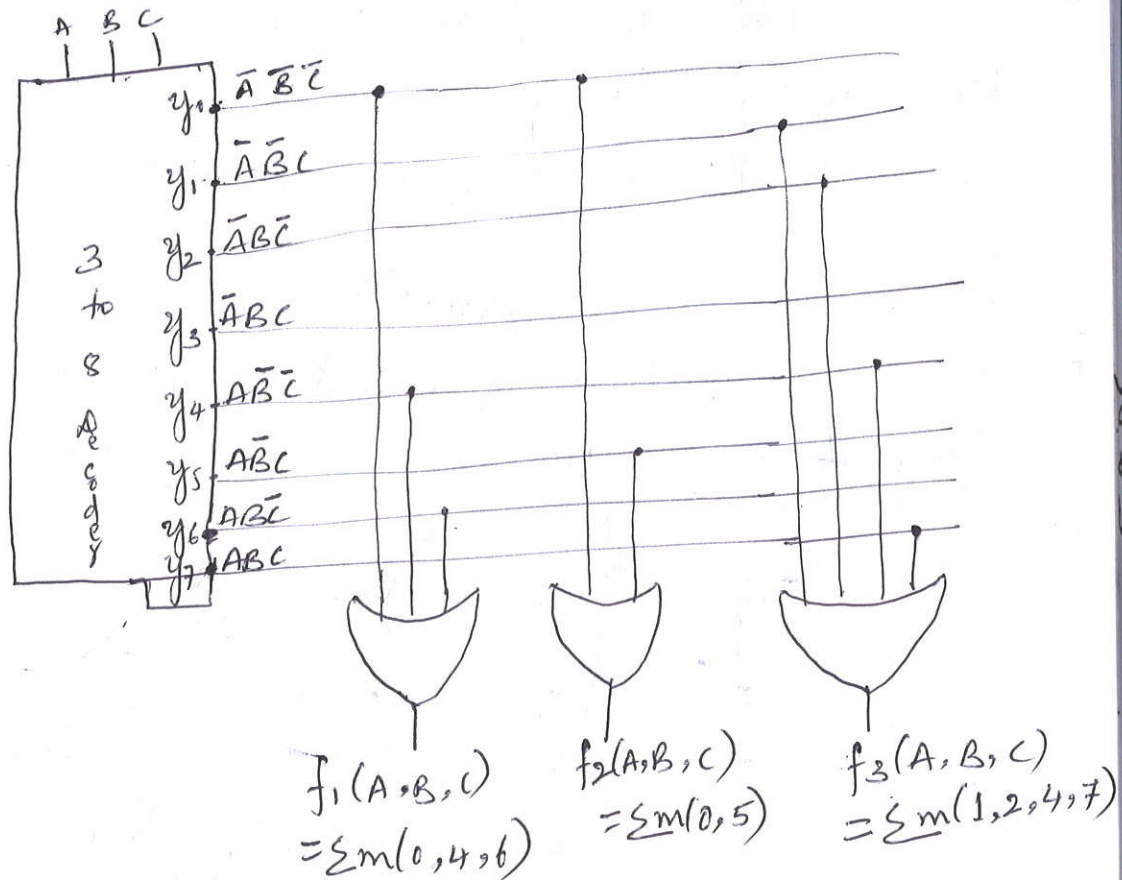
① Realize the foll Boolean fⁿ using 3-to-8 Decoder & multi-i/p OR gates.

$$f_1(A, B, C) = \sum m(0, 4, 6)$$

$$f_2(A, B, C) = \sum m(0, 5)$$

$$f_3(A, B, C) = \sum m(1, 2, 3, 7)$$

Solⁿ: Since @ the decoder o/p, we'll get all the min-terms, we use them as shown to get reqd boolean fⁿs.



BCD - to - Decimal Decoder:

BCD - binary coded decimal.

- The BCD code expresses each digit in a decimal no. by its nibble equivalent.

Eg, The decimal (429) is changed to BCD as

④ 0100 ② 0010 ⑨ 1001

∴ BCD code 0100 0010 1001 is equivalent to $429_{(10)}$.

The 74

⇒ Convert the decimal no 8963 to its BCD form.

8	9	6	3
↓	↓	↓	↓
1000	1001	0110	0011

⇒ Converting to decimal form ~~from~~ BCD.

0101	0111	1000
↓	↓	↓
5	7	8

⇒ BCD digits are from 0000 to 1001 (0 through 9). High decimal being coded is 9. All combinations above this (1010 to 1111) can NOT exist in BCD code/form.

Note:

7445

BCD-to decimal decoder: 1 of 10-decoder
 Only 1 of the 10 o/p lines is high.

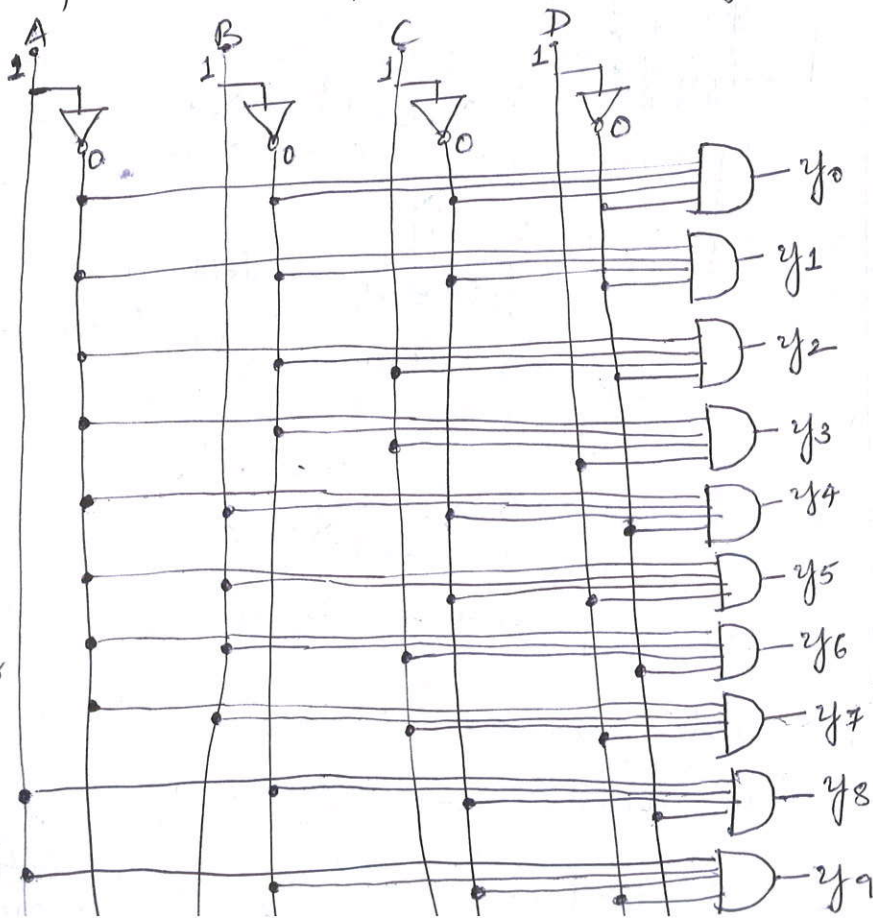
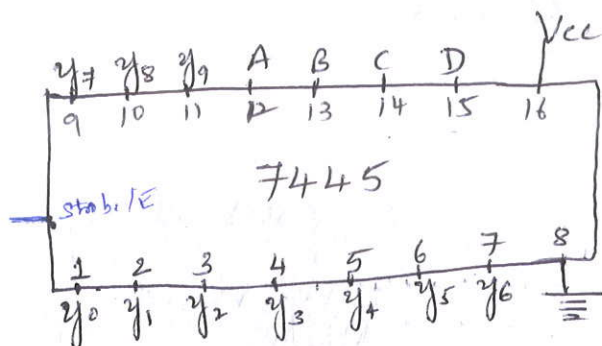


fig:
 (1) of (10) decoders

Decimal NO	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
These combinations forces o/p lines to HIGH state	



Note: An invalid BCD i/p (1000 to 1111) forces all o/p lines into the high state.

7445 Truth Table : BCD to decimal decoder

Decimal NO	Inputs				Outputs									
	A	B	C	D	y ₀	y ₁	y ₂	y ₃	y ₄	y ₅	y ₆	y ₇	y ₈	y ₉
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
1	0	0	0	1	1	0	1	1	1	1	1	1	1	1
2	0	0	1	0	1	1	0	1	1	1	1	1	1	1
3	0	0	1	1	1	1	1	0	1	1	1	1	1	1
4	0	1	0	0	1	1	1	1	0	1	1	1	1	1
5	0	1	0	1	1	1	1	1	1	0	1	1	1	1
6	0	1	1	0	1	1	1	1	1	1	0	1	1	1
7	0	1	1	1	1	1	1	1	1	1	1	0	1	1
8	1	0	0	0	1	1	1	1	1	1	1	1	0	1
9	1	0	0	1	1	1	1	1	1	1	1	1	1	0
	1	0	1	0	1	1	1	1	1	1	1	1	1	1
	1	0	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	0	0	1	1	1	1	1	1	1	1	1	1
	1	1	0	1	1	1	1	1	1	1	1	1	1	1
	1	1	1	0	1	1	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1	1	1	1	1	1	1

These combin's forces
o/p lines to H16# state

SEVEN - Segment Decoder :

The 7446 TTL - a 7-segment decoder driver

- In most practical appls, 7-segment displays are used to give visual indication of o/p states of digital ICs such as decade counters, latches etc.
- These o/p's are usually in 4-bit BCD form, & are thus NOT suitable for directly driving 7-segment displays. The special BCD-to-7-segment decoder/driver ICs are used to convert the BCD signal into form suitable for driving these displays

Digit	Segments Activated	Display
0	a, b, c, d, e, f	
1	b, c	
2	b, c, d, e, f	
3	b, c, d, e, f, g	
4	a, b, c, d, f, g	
5	a, b, c, d, e, f, g	
6	a, b, c, d, e, f, g	
7	a, b, c, d, e, f, g	
8	a, b, c, d, e, f, g	
9	a, b, c, d, e, f, g	

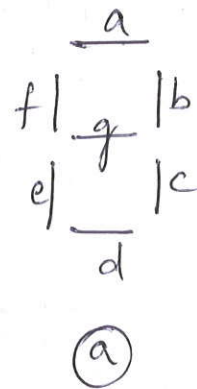


fig (a) & (b) Seven segment indicator

The

- Tr
- th
- Each
- pu



(a) 7446

→ Logic
7446
input
output
for
BCD
the
of 7446
LEDs
as a
appear

The 7446 & the 7448

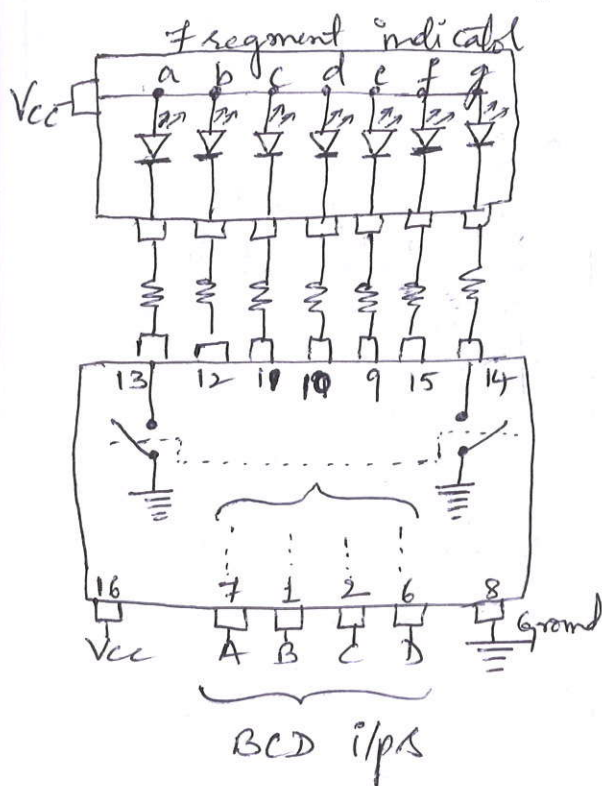
driver
s are
lates f
ches etc.

ss. &
-segment
decoder/
signal
sys

a
b
c
d

a

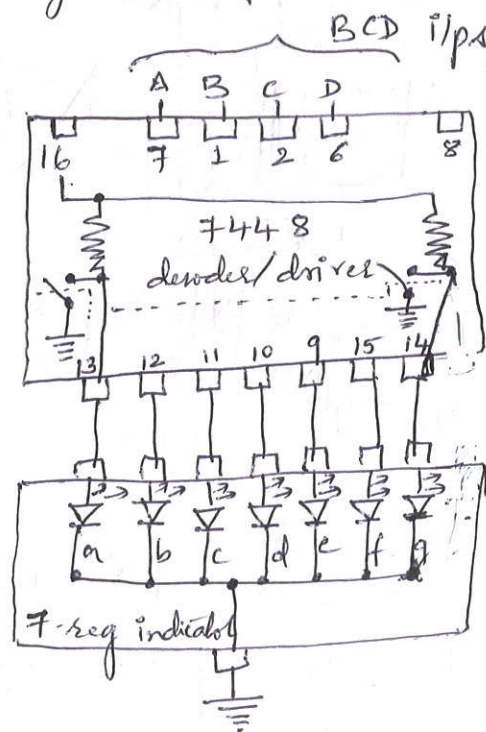
- Two types of decoder drivers, corresponding to the common-anode & common-cathode indicators.
- Each driver has 4 i/p pins (the BCD i/p) and 7-o/p pins (the a through g segments).



(a) 7446 - decoder driver

⇒ Logic chips inside the 7446 convert the BCD input to the required output.

For instance, if the BCD i/p is 0111, the internal logic of 7446 will force LEDs a, b, c to conduct. As a result, digit 7 will appear on 7-segment indicator.



(b) 7448 - decoder driver

⇒ Same with this as well.

⇒ 7446 requires external current-limiting resistors.

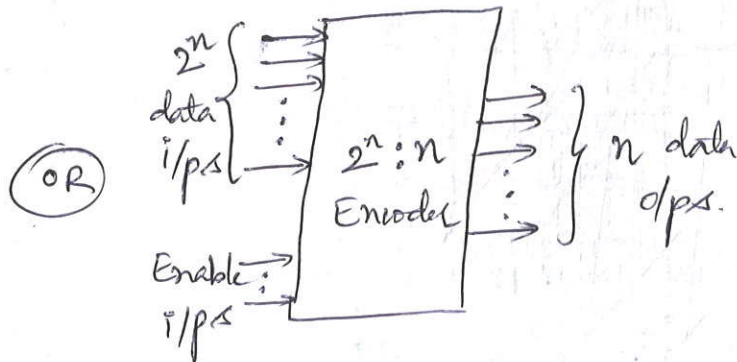
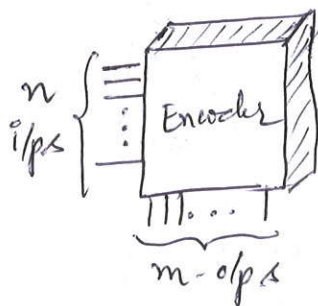
⇒ 7448 - ~~reqs~~ has its own current-limiting resistors on chip.

⇒ Switch-symbol is used to illustrate the operation of 7446 and 7448 in fig (a) & (b).

Encoders:

- An encoder is a digital circuit that performs the inverse operation of a decoder.
- An encoder converts an active i/p signal into a coded o/p signal.

General idea:



⇒ An encoder has 2^n (or fewer) i/p lines & n-o/p lines. In an encoder, the o/p lines generate the binary code corresponding to the i/p value.

Decimal to BCD Encoder: 74157 TTL.

→ 10 - i/p lines → The switches are push-button switches like those of pocket calcul.

→ When button-3 is pressed, C and D-OR gates have i/p's, therefore the o/p is

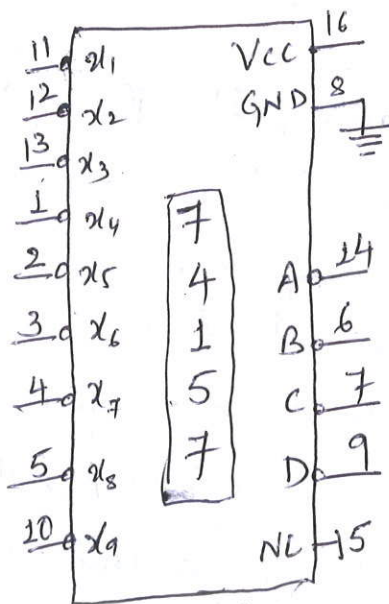
ABCD = 0011

→ For button 5, ABCD = 0101

→ For switch 9, ABCD = 1001

→ Decimal i/p x_1 to x_9 are connected to pins 1 to 5 and 10 to 13.

→ BCD o/p's A, B, C, D are



→ Pin 1

→ Pin

74157

Decimal value

0

1

2

3

4

5

6

7

8

9

Logic d



→ Pin 16 - VCC and pin-8 is grounded.

→ Pin 15 - NC - No connection - The pin is NOT used.

connections

74157 Truth table : Active-low-inputs
active-low-outputs

into

Decimal value	Inputs										Outputs			
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9		A 8	B 4	C 2	D 1
0	1	1	1	1	1	1	1	1	1		1	1	1	1
1	0													0
2		0											0	
3			0										0	0
4				0								0		
5					0							0		0
6						0						0	0	
7							0					0	0	0
8								0			0			
9									0		0			0

data
dps.

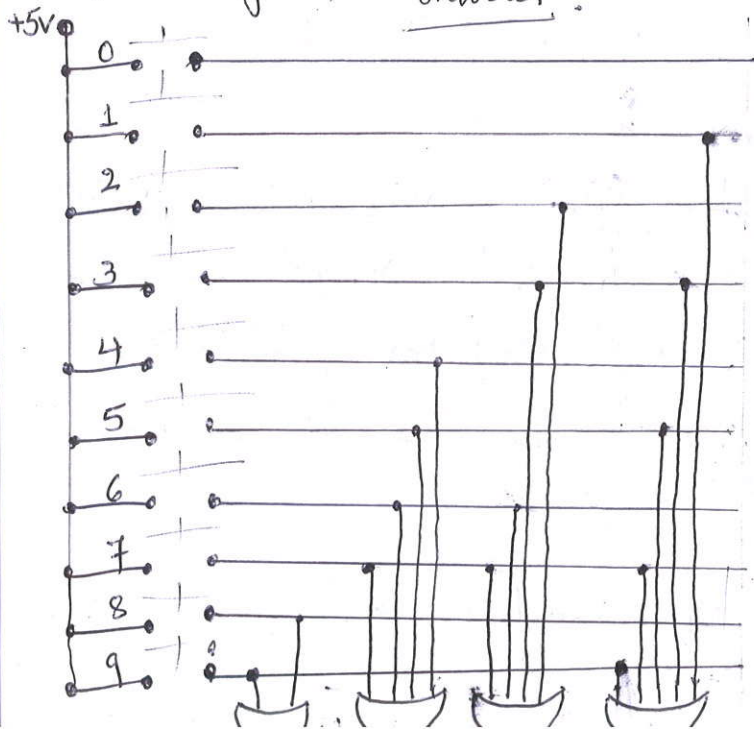
lines,
correspo

ntion

ocket calai.

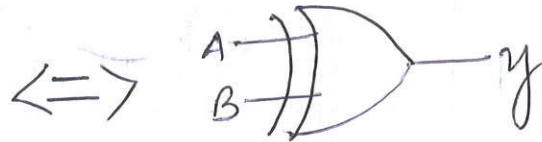
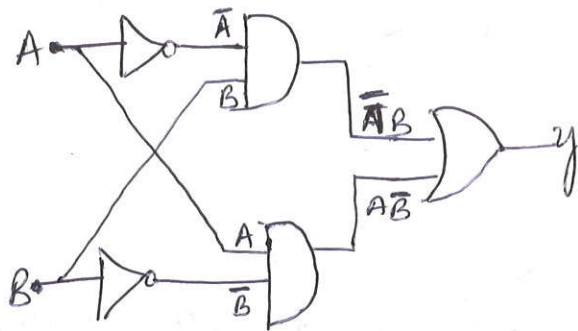
have

Logic diagram of Decimal to BCD encoder :

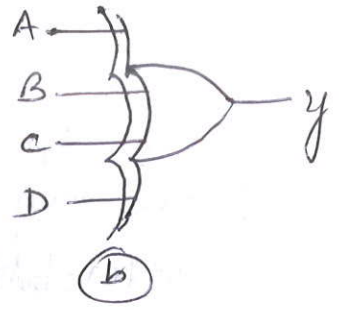
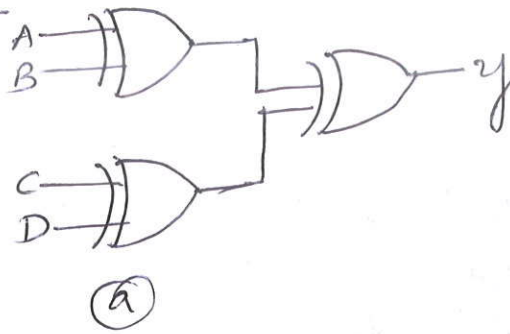


Incidentally, the 74147 is called a "priority encoder", because it gives priority to the higher-order input. From the TT, if all the i/p's from x_1 through x_9 are low, the highest of these x_9 , is encoded to get o/p 0110/LHHL. In other words, x_9 has priority over all

Four in



Four Inputs:



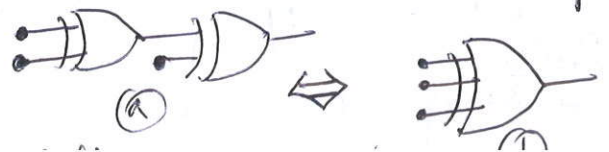
Four i/p's X-OR gate

Truth Table : When ABCD i/p's has an odd no of 1s, $o/p = 1$.

Comment	A	B	C	D	y
Even	0	0	0	0	0
Odd	0	0	0	1	1
Odd	0	0	1	0	1
Even	0	0	1	1	0
Odd	0	1	0	0	1
Even	0	1	0	1	0
Even	0	1	1	0	0
Odd	0	1	1	1	1
Odd	1	0	0	0	1
Even	1	0	0	1	0
Even	1	0	1	0	0
Odd	1	0	1	1	1
Even	1	1	0	0	0
Odd	1	1	0	1	1
Odd	1	1	1	0	1
Even	1	1	1	1	0

First ABCD entry to produce an o/p 1 is 0001; it has odd no of 1s. The next ABCD entry to produce an o/p 1 is 0010; again an odd no of 1s. An o/p-1 also occurs for these ABCD i/p's 0100, 1000, 1011, 1101, and 1110, each having an odd no of 1s.

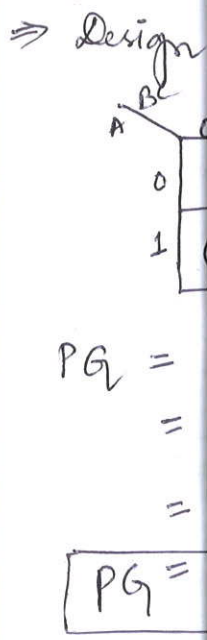
Any no of i/p's : Using 2-i/p's X-OR gates as building blocks, you can produce X-OR gates with any no



Parity Generators and checkers;

Soln:

- A parity-bit is used for the purpose of detecting errors during transmissⁿ of binary informⁿ. It is an extra-bit included with a binary message to make the no. of 1s either odd or even.
- The message, including the parity bit is transmitted & then checked at the receiving end for errors. An error is detected if the checked parity does NOT correspond with the one transmitted.
- Parity generator: Circuit that generates the parity bit in the transmitter.
- Parity checker: Circuit that checks the parity bit in the receiver.
- Even parity: n-bit input has an even no. of 1s.
eg: 110011 $\Rightarrow$ has even parity
∵ contains 4-1s.
- Odd parity: n-bit input has an odd no. of 1s.
eg: 110001 $\Rightarrow$ has odd parity.
∵ contain three -1s.



- Two examples:
1111 0000 1111 0011 $\Rightarrow$ Even parity
1111 0000 1111 0111 $\Rightarrow$ Odd parity

→ Table shows the 3-bit message with even parity and odd parity.
Problem:
→ Design an even-parity generator with 3-message bits

Solⁿ:

3-bit message			Odd parity bit	Even parity bit
A	B	C		
0	0	0	1	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

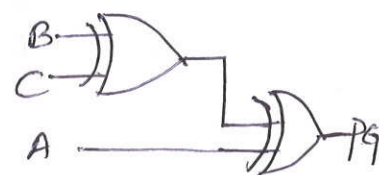
← parity generator table for even and odd parity

⇒ Design an even-parity generator with 3 message bits.

A	BC			
	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$\begin{aligned}
 P_G &= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC \\
 &= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC) \\
 &= \bar{A}(B \oplus C) + A(\overline{B \oplus C})
 \end{aligned}$$

$$P_G = A \oplus (B \oplus C)$$



no of 1s.

parity

4-1s.

2 1s.

-1s.

Even parity

Odd parity

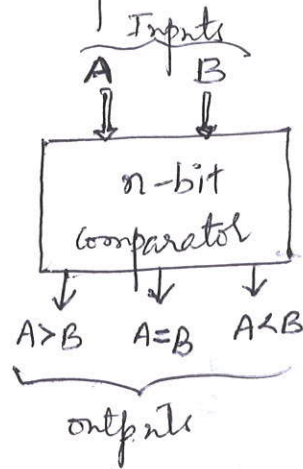
try

age bits

Magnitude Comparators:

Defⁿ: A comparator is a special combinational circuit designed primarily to compare the relative magnitude of 2-binary nos.

Block diagⁿ:



It receives two n -bit nos A and B as inputs and the outputs are $A > B$, $A = B$ and $A < B$. Depending upon the relative magnitude of the 2 nos, one of the o/p's will be high.

① $\Rightarrow$ Design 1-bit comparator using basic gates.

Solⁿ: Consider two 1-bit no A and B .

Inputs		Outputs		
A	B	$Y_{A>B}$	$Y_{A=B}$	$Y_{A<B}$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

$Y_{A>B}$:- k-map.

A \ B	0	1
0	0	0
1	1	0

$$Y_{A>B} = A\bar{B}$$

$Y_{A=B}$ - k-map:

A \ B	0	1
0	1	0
1	0	1

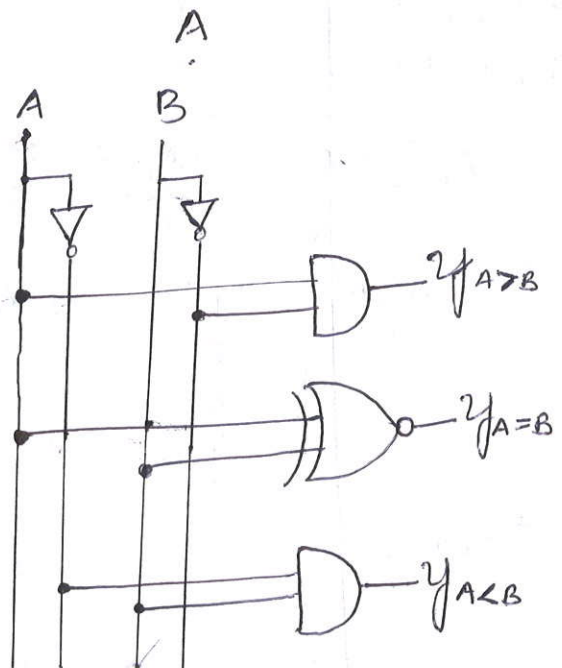
$$Y_{A=B} = \bar{A}\bar{B} + AB = \overline{A \oplus B}$$

$$Y_{A=B} = A \odot B$$

$Y_{A<B}$ - k-map:

A \ B	0	1
0	0	1
1	0	0

$$Y_{A<B} = \bar{A}B$$



Logic-diagram

② Design 2-bit comparator using gates.

Solⁿ:

Inputs				Outputs		
A_1	A_0	B_1	B_0	$A > B$	$A = B$	$A < B$
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

K-map Simplification:

(i) $A > B$

$A_1 A_0 \backslash B_1 B_0$	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	1	1	0	1
10	1	1	0	0

$$\therefore A > B = A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0$$

(ii) $A < B$

$A_1 A_0 \backslash B_1 B_0$	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

$$\therefore (A < B) = \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0$$

(ii) $A = B$

$A_1 A_0 \backslash B_1 B_0$	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	0	0	1	0
10	0	0	0	1

$$\therefore (A = B) = A_1 A_0 B_1 B_0$$

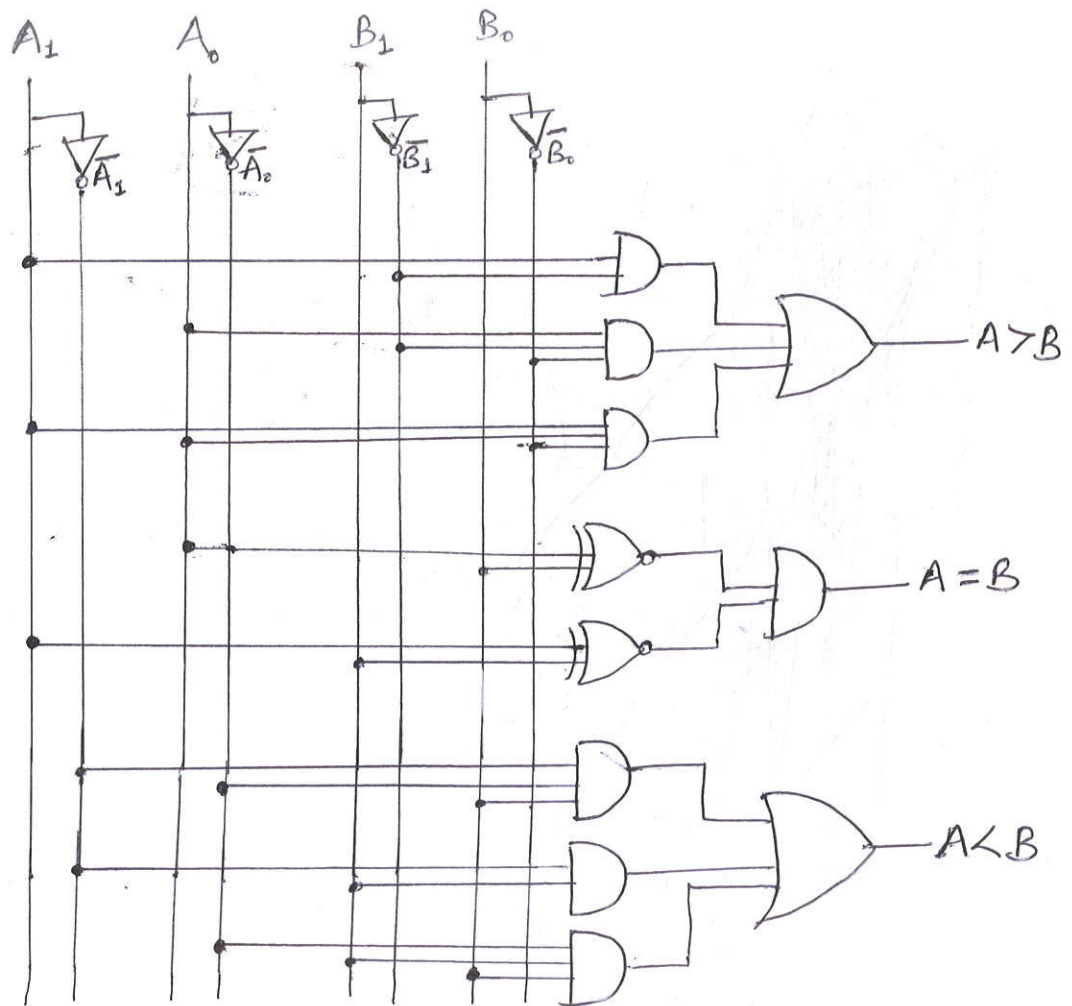
$$\Rightarrow \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0$$

$$= \bar{A}_1 \cdot \bar{B}_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (A_0 B_0 + \bar{A}_0 \bar{B}_0)$$

$$= \bar{A}_1 \bar{B}_1 (\overline{A_0 \oplus B_0}) + A_1 B_1 (\overline{A_0 \oplus B_0})$$

$$= (\overline{A_0 \oplus B_0}) (\bar{A}_1 \bar{B}_1 + A_1 B_1) = (\overline{A_0 \oplus B_0}) (\overline{A_1 \oplus B_1})$$

Logic Diagram of 2-bit comparator



		1	0
1	1	1	0
0	1	1	0
0	0	0	0

$$\bar{A}_1 B_1 +$$

$$\bar{A}_0 B_1 B_0$$

$$B_1 B_0 +$$

$$B_1 \bar{B}_0$$

$$+ (A_0 B_0 +$$

$$\bar{A}_0 \bar{B}_0)$$

$$\bar{B}_0)$$

$$(\oplus B_0), (A_1 \oplus B_1)$$