

Modes of Heat Transfer

Convection

↳ Fluid

Conduction

↳ Solid
↳ Fluid

Radiation

↳ No medium.

Conduction: Conduction is the flow of heat in a substance due to exchange of energy between molecules having more energy and molecules having less energy.

⇒ Exchange of energy occurs because of interaction between molecules in microscopic scale.

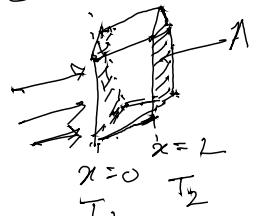
⇒ In solid heat transfer occurs due to lattice vibrations of the molecules and movement of free electrons.

⇒ In fluid, molecules collide each other at rest in an overall macroscopic sense.
(No bulk motion)

Thermal conductivity

$$K_{\text{solid}} > K_{\text{liquid}} > K_{\text{gas}}$$

⇒ Conduction is exchange of heat due to interaction of molecules. More compact the molecules are better the exchange of heat.



$$\left(\frac{dT}{dx}\right) = \frac{T_1 - T_2}{L - 0}$$

$$= \frac{T_2 - T_1}{L - 0}$$

$$Q = \frac{\Delta T}{\left(\frac{L}{KA}\right)} = \frac{\Delta T}{L/K A}$$

✓ $Q = \bigcirc K A \left(\frac{dT}{dx}\right)$

Rate of heat transfer
J/s or W

↳ Temp. gradient
↳ heat transfer area.
↳ Thermal conductivity of material (W/mK)

$$Q = K A \frac{\Delta T}{\Delta x} = Q = \frac{\Delta T}{\frac{L}{KA}}$$

Convection: transfer of energy from one region to another due to macroscopic (bulk) motion of fluid ~~mole~~ molecules is called convection.

$$\checkmark \checkmark \checkmark \quad \dot{Q} = \underbrace{(h A \Delta T)}_{\substack{\rightarrow \text{convective heat} \\ \text{transfer coeff.}}} \Rightarrow \dot{Q} = \frac{\Delta T}{R} ; R = \frac{1}{h A}$$

h depends on nature of fluid flow
thermodynamic properties of fluid.

$$\text{Nu} = \frac{h L}{K_f} = f(Re, Pr)$$

$$\Rightarrow \underbrace{h}_{\uparrow} = f(\underbrace{L, K, Re, Pr}_{\substack{\rightarrow \nu, \mu, D \\ \rightarrow c_p, \mu, K}})$$

$\Rightarrow$

Forced convection: When the fluid motion is caused by an external agency such as pump or blower, the situation is said to be forced convection.

Natural convection or free convection:

When the fluid motion occurs due to density variations caused by temperature differences, the convection is called natural convection.

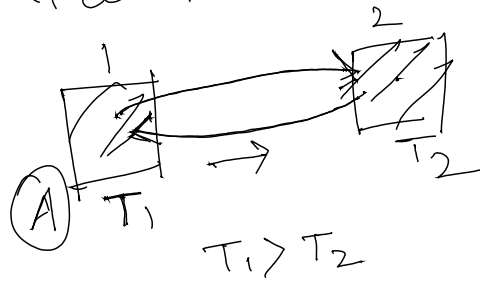
$$\text{Rate} = \frac{\text{Driving force } (\Delta T)}{\text{Resistance } (1/hA)}$$

Radiation: Transfer of heat by electromagnetic (thermal) radiation - from the surface of the body is called radiation.

⇒ All physical matter emits thermal radiation in the form of electromagnetic waves because of vibrational and rotational movement of the molecules.

⇒ Higher the temperature, higher the rate of emission.

⇒ The absorption of the emission occurs due to the presence of medium.



$$Q = A \epsilon (T_1^4 - T_2^4)$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$$

Stefan Boltzmann constant

ϵ = emissivity

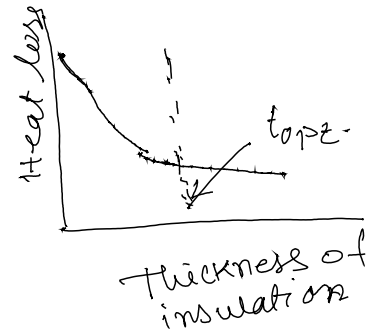
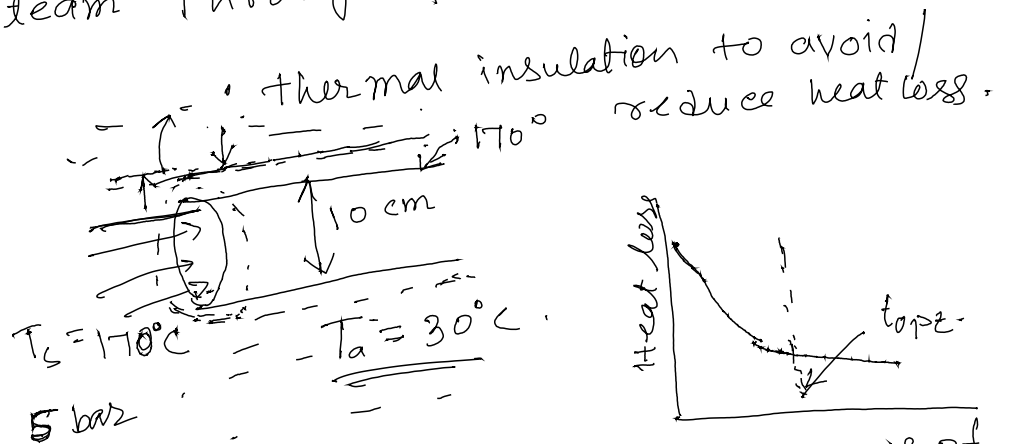
= 1 (for black body)

II

Heat transfer

Problems

1. Supply of steam through pipe from one point to another.



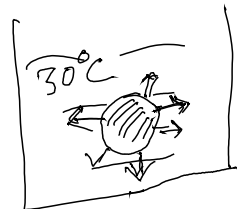
$\Rightarrow$ Determine the thickness of insulation to be used so that the heat loss rate is reduced below some specified value.

OR

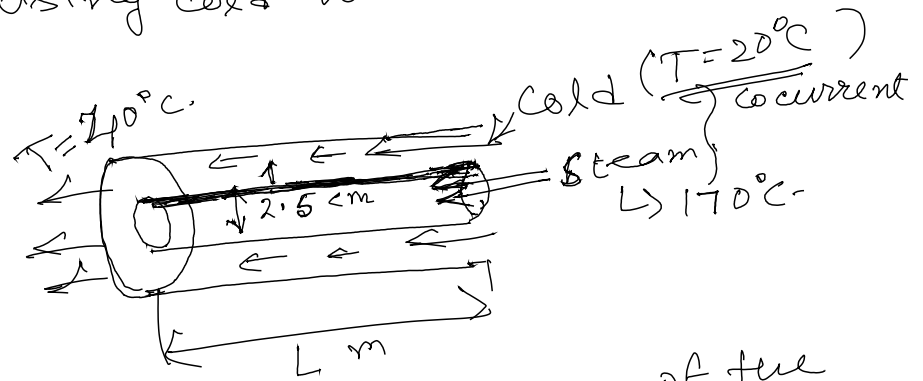
Determine the thickness of insulation so that the temp. of the outside surface of the insulation is less than some specified value.

$$Q = A \cdot (T_1^4 - T_2^4) \cdot (10^2)^4 \Rightarrow 10^8$$

$\downarrow$
 $5.67 \times 10^{-8} \times (10^8) \times ()$



II Condensation of steam in a double pipe heat exchanger using cold water

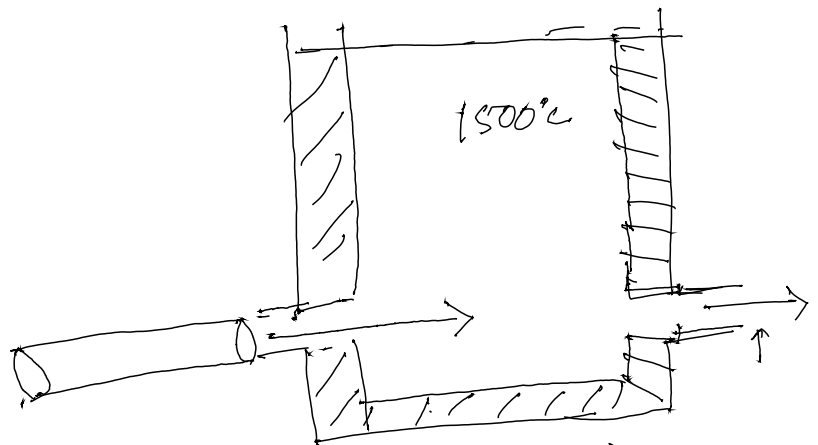


⇒ What will be the exit temperature of the water if the pipe length is 2m long
or

⇒ What must be the length of the pipe so that the exit temp. of water is 20°C.

III Heating of a steel rod in a furnace.

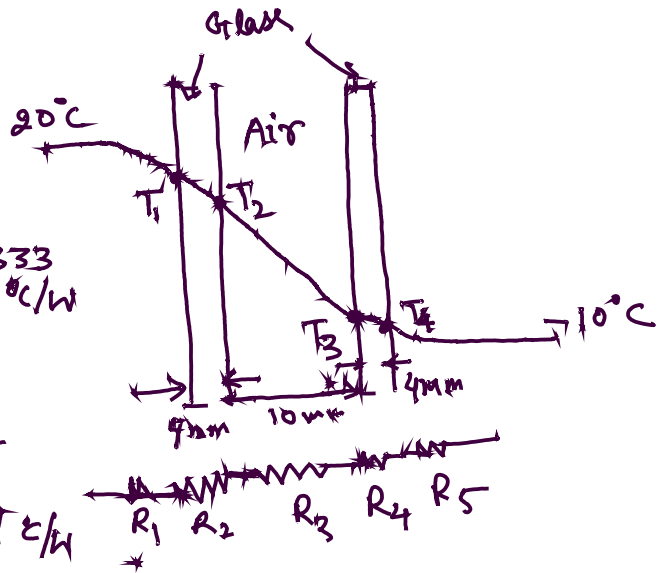
Steel rod temp = 40°C.
↓
600°C.



⇒ What should be the velocity of the rod in the furnace to reach correct temp?
or.

What should be the length of the furnace to reach the desired temperature?

Problem 3



$$R_1 = \frac{1}{h_1 A} = \frac{1}{10 \times 1.2} = 0.08333 \text{ } ^\circ\text{C/W}$$

$$R_2 = R_4 = \frac{x_1}{k_1 A} = \frac{0.004}{0.78 \times 1.2}$$

$$= 0.00427 \text{ } ^\circ\text{C/W}$$

$$R_3 = \frac{x_2}{k_2 A} = \frac{0.01}{0.026 \times 1.2} = 0.32059 \text{ } ^\circ\text{C/W}$$

R_1, R_5 = Convective Resistance

R_2, R_3, R_4 = Conductive Resistance

$$R_5 = \frac{1}{h_2 A} = \frac{1}{40 \times 1.2} = 0.02083 \text{ } ^\circ\text{C/W}$$

$$h_1 = 10 \text{ W/m}^2\text{K}$$

$$R_{\text{total}} = 0.4332$$

$$h_2 = 40 \text{ W/m}^2\text{K}$$

$$Q = \frac{(\Delta T)_{\text{total}}}{R_{\text{total}}} = \frac{30}{0.4332} = 69.2 \text{ W}$$

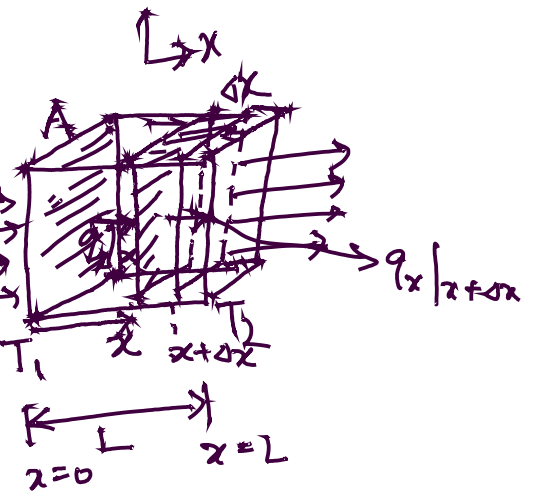
$$Q = \frac{(\Delta T)_1}{R_1}$$

$$69.2 = \frac{20 - T_1}{0.08333} \Rightarrow T_1 = 14.2^\circ\text{C} \gg -2.2^\circ\text{C}$$

Steady state unidirectional heat flow (constant k)

Single slab

Rate of Energy in - Rate of Energy out + generation = Accumulation



$$q_x|_x A - q_x|_{x+\Delta x} A = 0$$

$$\Rightarrow \frac{q_x|_x - q_x|_{x+\Delta x} - K C_1 A}{\Delta x} = 0 \quad \left[\begin{array}{l} \text{divide by constant term} \\ \frac{T_2 - T_1}{\Delta x} \\ \frac{f(x+\Delta x) - f(x)}{\Delta x} \end{array} \right]$$

$$\Rightarrow - \frac{d(q_x)}{dx} = 0$$

$$\Rightarrow - \frac{d}{dx} \left(-K \frac{dT}{dx} \right) = 0$$

$$\Rightarrow K \frac{d^2 T}{dx^2} = 0 \Rightarrow \boxed{\frac{d^2 T}{dx^2} = 0}$$

Boundary conditions
at $x=0$, $T = T_1$
 $x=L$, $T = T_2$

$$\frac{dT}{dx} = C_1$$

$$\Rightarrow T = C_1 x + C_2$$

$$\Rightarrow T_1 = C_1 \cdot 0 + C_2 \Rightarrow C_2 = T_1$$

$$T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$$

$q_x \Rightarrow$ heat flux at x .

$$q_x = -K \frac{dT}{dx}$$

$$K \neq K(T)$$

$$\boxed{K \neq K(x)}$$

$\rightarrow$ Isotropic

$$T = C_1 x + C_2$$

$$\Rightarrow \boxed{T = \left(\frac{T_2 - T_1}{L} \right) x + T_1} \quad \checkmark \Rightarrow T_2 = T_1 - \frac{(T_1 - T_2)}{L} x$$

$$\Rightarrow \cdot \quad Q = \cdot \cdot \cdot \cdot \cdot$$

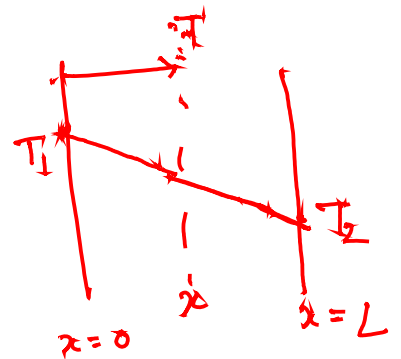
$$Q = q_x A = \cdot \cdot K \frac{dT}{dx} \cdot A$$

$$T_1 > T_2$$

$$= \cdot K C_1 A$$

$$Q = \cdot K A \left(\frac{T_2 - T_1}{L} \right)$$

$$\Rightarrow Q = K A \frac{(T_1 - T_2)}{L} = \frac{(T_1 - T_2) \leftarrow \text{Driving force}}{(L/K A) \leftarrow \text{Resistance}}$$



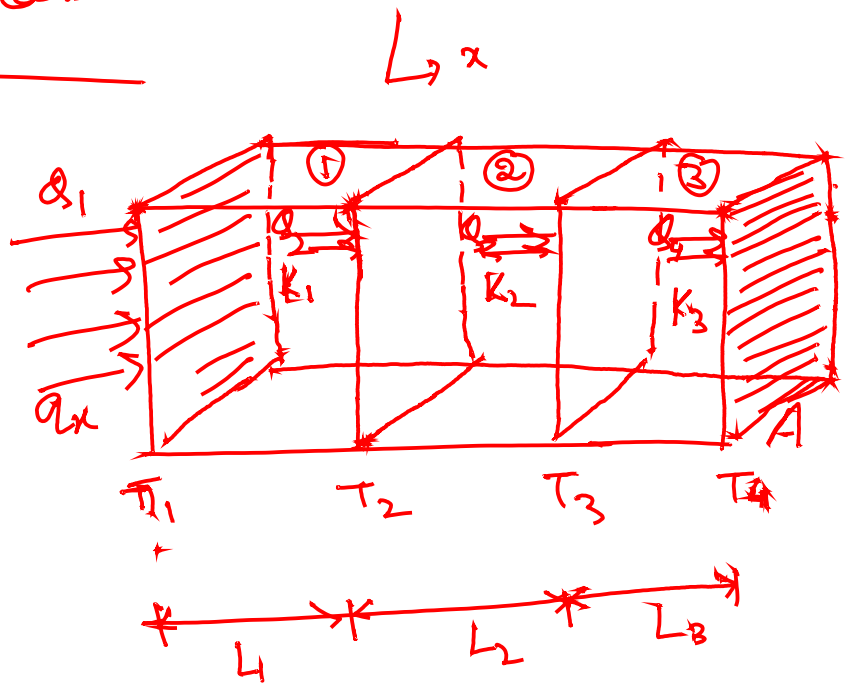
$$\text{Slope} = - \frac{(T_1 - T_2)}{L}$$

Multiple layer Slab

Steady state heat conduction

$$Q_1 = Q_2 = Q_3 = Q_4 = Q$$

No accumulation



Layer 1

$$Q = k_1 A \frac{(T_1 - T_2)}{L_1} \Rightarrow (T_1 - T_2) = \frac{Q L_1}{k_1 A} \quad \text{--- (1)}$$

Layer 2

$$Q = k_2 A \frac{(T_2 - T_3)}{L_2} \Rightarrow T_2 - T_3 = \frac{Q L_2}{k_2 A} \quad \text{--- (2)}$$

Layer 3 $\Rightarrow T_3 - T_4 = \frac{Q L_3}{k_3 A} \quad \text{--- (3)}$

eqnⁿ = (1) + (2) + (3).

$$(T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) = Q \left[\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} \right]$$

$$\Rightarrow (T_1 - T_4) = Q \left[\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} \right]$$

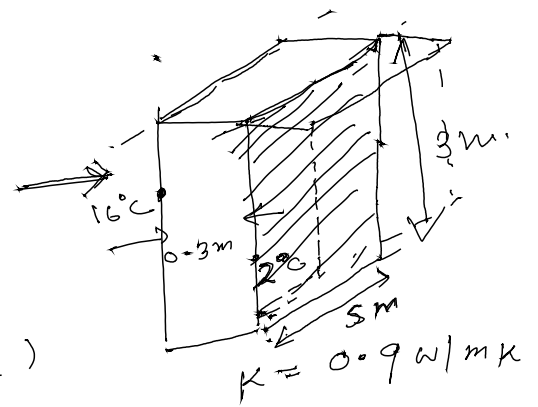
$$\Rightarrow Q = \frac{(T_1 - T_4)}{\left[\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} \right]} \quad \begin{matrix} \xrightarrow{\text{Driving force}} \\ \xleftarrow{\text{Total Resistance}} \end{matrix}$$

1.

$$Q = kA \frac{\Delta T}{\Delta x}$$

$$= 0.9 \times (5 \times 3) \times \frac{(16-2)}{0.3} \text{ J/s}$$

$$= 630 \text{ J/s}$$



$$Q = \frac{\Delta T}{R} \quad \left(\begin{array}{l} \text{Driving force} \\ \text{Resistance} \end{array} \right)$$

$$R = \frac{\left(\frac{\Delta x}{kA} \right)}{=} = \frac{0.3}{0.9 \times 15} = 0.02222 \frac{^\circ\text{C/W}}{\text{K/W}}$$

$$Q = \frac{630}{0.02222} \text{ W} = 630 \text{ W}$$

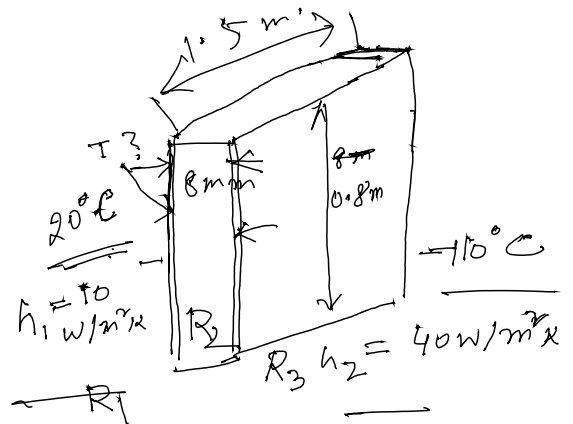
$$\checkmark Q = \frac{(\Delta T)_{\text{total}}}{R_{\text{total}}} = \frac{(\Delta T)_{\text{total}}}{R_1 + R_2 + R_3}$$

2) $k = 0.78 \text{ W/mK}$.

$Q \rightarrow$

$T_{\text{inner surface glass}} = ?$

$$\rightarrow 20 - (-10) = 30^\circ\text{C}$$



$$Q = \frac{(\Delta T)_{\text{total}}}{R_{\text{total}}} = \frac{(\Delta T)_1}{R_1} = \frac{(\Delta T)_2}{R_2}$$

$$R_1 = \frac{1}{h_1 A} = \frac{1}{10 \times (1.5 \times 0.8)} = 0.08333 \text{ } ^\circ\text{C/W}$$

$$R_2 = \frac{\Delta x}{kA} = \frac{0.008}{0.78 \times (1.5 \times 0.8)} = 0.00855 \text{ } ^\circ\text{C/W}$$

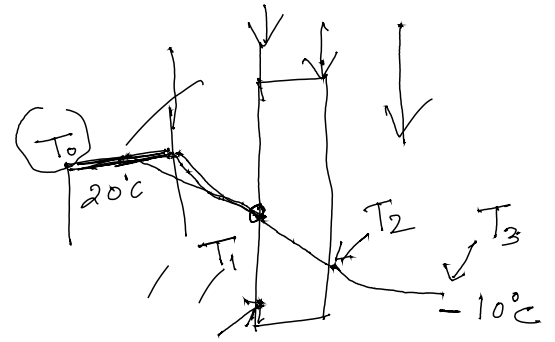
$$R_3 = \frac{1}{h_2 A} = \frac{1}{40 \times (1.5 \times 0.8)} = 0.02083 \text{ } ^\circ\text{C/W}$$

$$R_{\text{total}} = R_1 + R_2 + R_3 = 0.1127 \text{ } ^\circ\text{C/W}$$

$$Q = \frac{(\Delta T)_{\text{total}}}{R_{\text{total}}} = \frac{20 - (-10) ^\circ\text{C}}{0.1127 ^\circ\text{C/W}} = 266 \text{ W}$$

$$Q = \frac{(\Delta T)_{\text{total}}}{R_{\text{total}}} = \frac{(\Delta T)_1}{R_1}$$

$$= \frac{(T_0 - T_1) ^\circ\text{C}}{0.08333 ^\circ\text{C/W}}$$



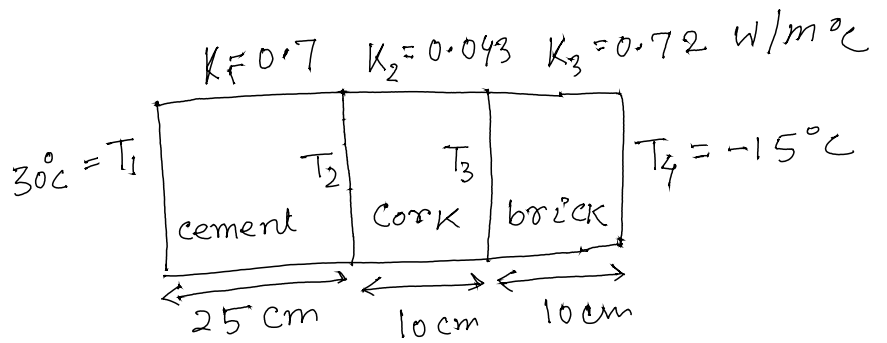
$$266 = \frac{20 - T_1}{0.08333}$$

$$\Rightarrow T_1 = -2.2 ^\circ\text{C} \quad \checkmark$$

Formation of fog or even frost on the inner surface.

Heat conduction

Problem 1



Find 1. $Q/A \Rightarrow$

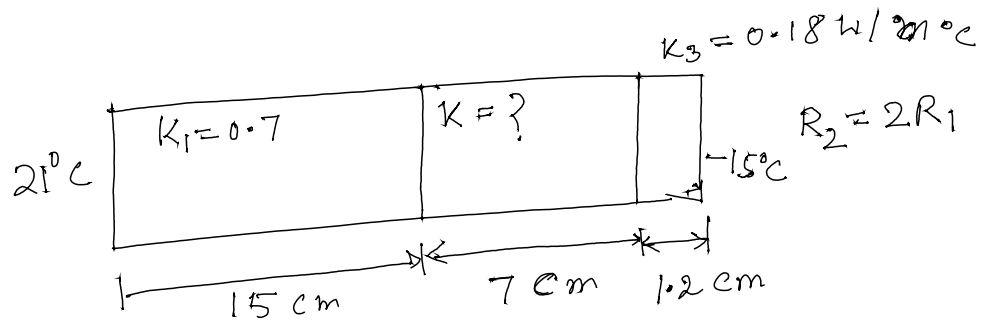
2. T_2

3. T_3

4. % of sharing of resistance

5. To ~~Decrease~~ 30% heat loss how much extra cork thickness need to add.

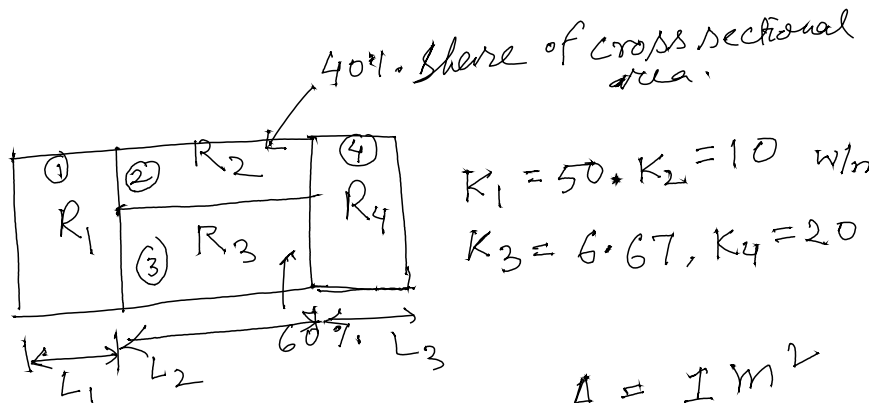
Problem 2



Find 1. $K_2 \Rightarrow$

2. Q/A

Problem 3



$L_1 = 5 \text{ cm}$

$L_2 = L_3 = 10 \text{ cm}$

$L_4 = 5 \text{ cm}$

Find R_1, R_2, R_3 and R_4

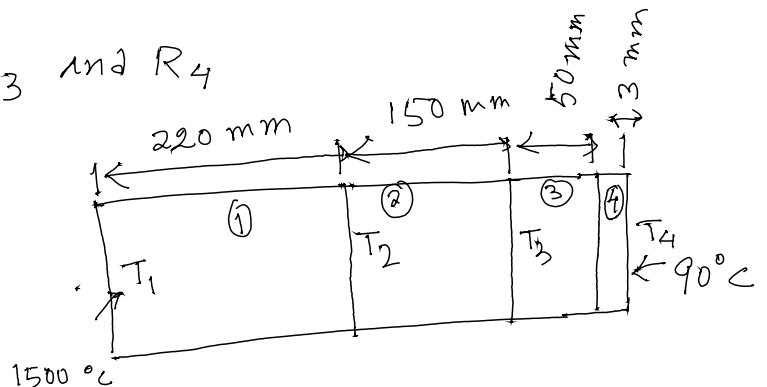
Problem 4

$K_1 = 4 \text{ KJ/m}^\circ\text{C}$

$K_2 = 2.8 \text{ KJ/m}^\circ\text{C}$

$K_3 = 0.24 \text{ KJ/m}^\circ\text{C}$

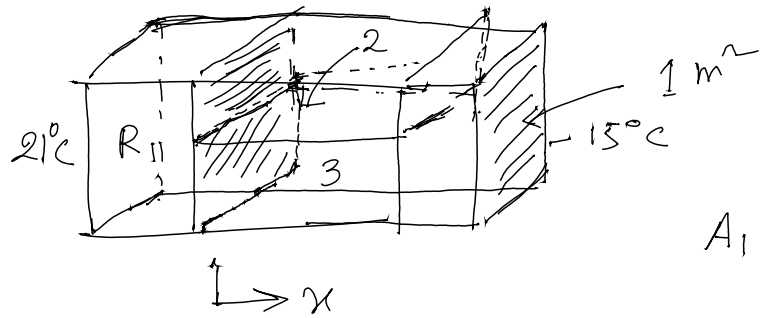
$K_4 = 240 \text{ KJ/m}^\circ\text{C}$



Find 1. Q/A

2. T_2, T_3

Problem 3



$$A_1 = 1 \text{ m}^2$$

$$R_1 = \frac{L_1}{K_1 A_1}$$

$$A_2 = 1 \times 0.4$$

$$R_2 = \frac{L_2}{K_2 A_2}$$

$$A_3 = 1 \times 0.6$$

$$R_3 = \frac{L_3}{K_3 A_3}$$

$$A_4 = 1 \text{ m}^2$$

$$R_4 = \frac{L_4}{K_4 A_4}$$

$$R_{\text{total}} = R_1 + R_4 + \left(\frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$= R_1 + R_4 + \frac{R_2 R_3}{R_2 + R_3}$$

$$R_1 = \frac{0.05}{50 \times 1} = 0.001$$

$$R_2 = \frac{0.1}{10 \times 0.4} = 0.025$$

$$R_3 = \frac{0.1}{6.67 \times 0.6} = 0.0249$$

$$R_4 = \frac{0.05}{20 \times 1} = 0.0025$$

$$R_{\text{total}} = 0.001 + 0.0025 + \frac{0.025 \times 0.0249}{0.025 + 0.0249}$$

$$= 0.0159$$

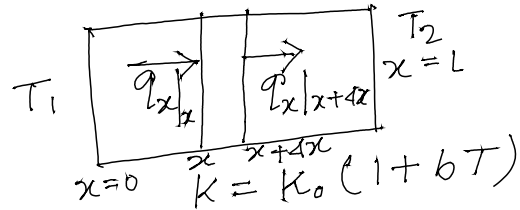
$$= 0.0159 \text{ } ^\circ\text{C/W}$$

Steady state, 1-D (x), No real heat generation

Single Slab of variable thermal conductivity with
layer Temperature.

$$k = k(T)$$

Energy balance in differential element of thickness Δx



Rate of Energy in - Rate of Heat out = 0

$A \Rightarrow$ cross section of area.

$$\Rightarrow q_x|_x A - q_x|_{x+\Delta x} A = 0$$

$q_x|_x \Rightarrow$ heat flux at x

Divide by a const. term ($A \Delta x$)
 $\Rightarrow$ and apply forward difference method

$q_x|_{x+\Delta x} \Rightarrow$ heat flux at $x+\Delta x$

$$\Rightarrow - \frac{d q_x}{d x} = 0$$

$$\Rightarrow \boxed{q_x = C_1} \checkmark \Rightarrow$$

$$q_x = -k \frac{dT}{dx} = -k_0(1+bT) \frac{dT}{dx}$$

$$\Rightarrow -k_0(1+bT) \frac{dT}{dx} = C_1$$

$$T_{\text{mean}} = \frac{T_2 + T_1}{2}$$

$$\Rightarrow k_0(1+bT) dT = - \frac{C_1}{k_0} dx$$

$$\Rightarrow k_0 \int_{T_1}^{T_2} (1+bT) dT = - C_1 \int_0^L dx$$

$$\Rightarrow k_0 \left[T + \frac{bT^2}{2} \right]_{T_1}^{T_2} = - C_1 L$$

$$\Rightarrow k_0 \left[(T_2 - T_1) + \frac{b}{2} (T_2^2 - T_1^2) \right] = - C_1 L$$

$$\Rightarrow k_0 (T_2 - T_1) \left[1 + \frac{b}{2} (T_2 + T_1) \right] = - C_1 L$$

$$\Rightarrow k_0(T_2 - T_1) K_m = - C_1 L$$

$$\Rightarrow C_1 = \frac{(T_1 - T_2) K_m}{L}$$

$$Q = q_x A = C_1 A = \frac{(T_1 - T_2) K_m A}{L}$$

$$\Rightarrow Q = \frac{(T_1 - T_2)}{(L/K_m A)}$$

$$K_0 (1 + bT) \frac{dT}{dx} = - C_1 dx$$

$$= - \frac{(T_1 - T_2) K_m}{L} dx$$

$$\Rightarrow K_0 \left[T + b \frac{T^2}{2} \right]_{T_1}^T = - \frac{(T_1 - T_2) K_m}{L} x$$

$$\Rightarrow K_0 \left[(T - T_1) + \frac{b}{2} (T^2 - T_1^2) \right] = - \frac{(T_1 - T_2) K_m}{L} x$$

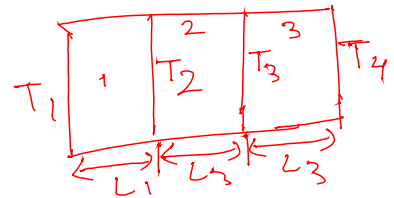
$$\Rightarrow K_0 (T - T_1) \left[1 + \frac{b}{2} (T + T_1) \right] = - \frac{(T_1 - T_2) K_m}{L} x$$

Exercice

✓

$Q =$

$$\frac{T_1 - T_4}{R_1 + R_2 + R_3}$$



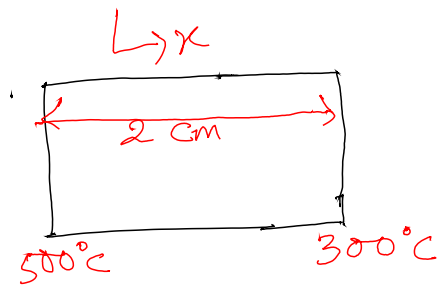
$$R_1 = \frac{L_1}{K_m A}$$

$$R_2 =$$

$$R_3 =$$

$$k \text{ at } 150^\circ\text{C} = 372 \text{ W/mK}$$

$$k = k_0 (1 + bT)$$



Find $b = ?$

$$3.63 \times 10^6$$

$$q = k_{\text{mean}} \frac{(500 - 300)}{2 \times 10^{-2}}$$

$$q_{\text{mean}} = 3.63 \text{ MW/m}^2$$

$$T_{\text{mean}} = \frac{500 + 300}{2} = 400^\circ\text{C}$$

k_{mean}

$$\Rightarrow k(\text{at } 400^\circ\text{C}) = 363$$

$$363 = k_0 (1 + b \times 400) \quad \text{--- (1)}$$

$$372 = k_0 (1 + b \times 150) \quad \text{--- (2)}$$

$$b = -1.7$$

$$\left(\frac{372}{363} \right) = \frac{1 + \frac{150}{400} b}{1 + \frac{150}{400} b} = 1.02479$$

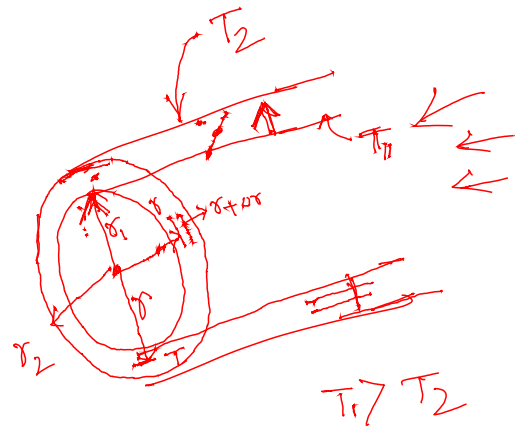
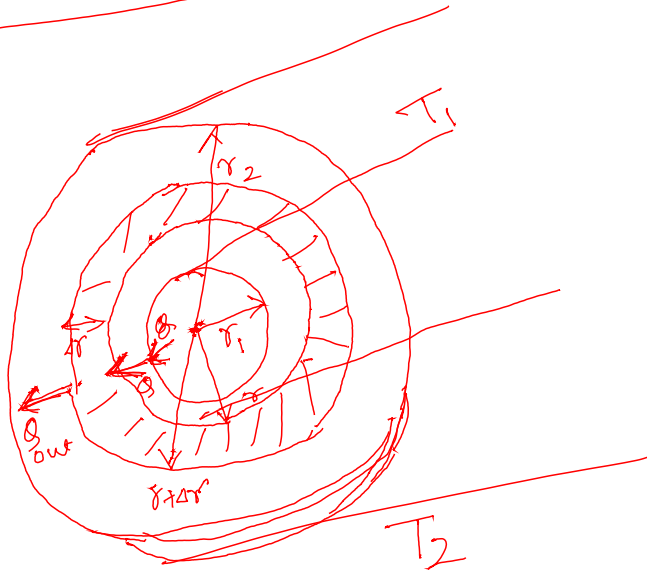
$$\Rightarrow 1 + 400b = 1.02479 +$$

$$\Rightarrow 1 + 150b = 1.02479 + 409.917b$$

$$\Rightarrow 259.917b = -0.02479$$

$$\Rightarrow b = -9.537 \times 10^{-5}$$

Heat transfer in cylinder/pipe



Energy balance in the differential element of thickness Δr (Volume $2\pi r \Delta r L$)
 Rate of heat
 In - out = 0.

$$\Rightarrow \left(2\pi r L q_r \right) \Big|_r - \left(2\pi (r+\Delta r) L q_{r+\Delta r} \right) \Big|_{r+\Delta r} = 0$$

Divide by $2\pi L \Delta r$

$$\Rightarrow \frac{(r q_r) \Big|_r - (r q_r) \Big|_{r+\Delta r}}{\Delta r} = 0$$

$$\Rightarrow \left[\frac{d(r q_r)}{dr} = 0 \right] \Rightarrow \left[\frac{d(r q_r)}{dr} = 0 \right]$$

$$\Rightarrow \boxed{r q_r = C_1}$$

at $r = r_1$, $T = T_1$
 $r = r_2$, $T = T_2$

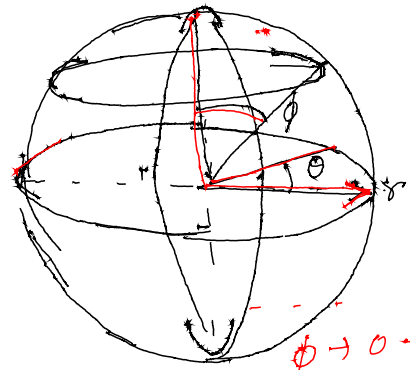
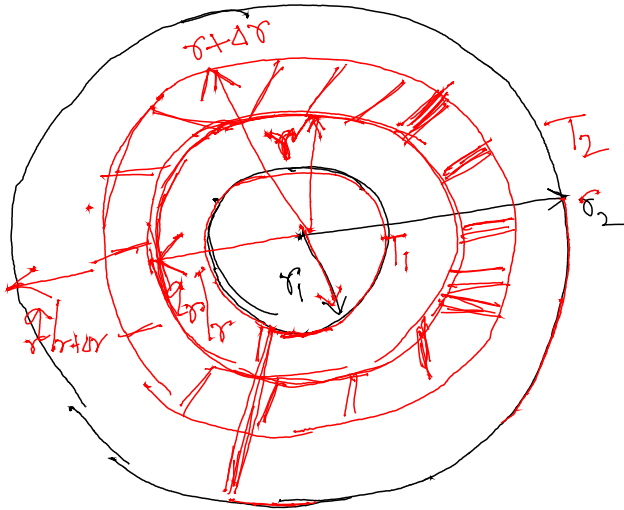
$$\boxed{Q = (2\pi r L q_r) = \frac{2\pi L K (T_1 - T_2)}{\ln r_2 / r_1} = \frac{(T_1 - T_2)}{\left(\frac{\ln(r_2 / r_1)}{2\pi L K} \right)}}$$

$$\boxed{T_1 - T(r) = (T_1 - T_2) \frac{\ln(r / r_1)}{\ln(r_2 / r_1)}}$$

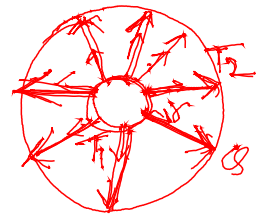
$$\frac{2\pi(r+\Delta r)L}{(2\pi r L) \Big|_{r+\Delta r}}$$

$$\left| \begin{array}{l} (r q_r) \Rightarrow f(r) \\ \frac{f(r+\Delta r) - f(r)}{\Delta r} \\ \Rightarrow \frac{df(r)}{dr} = 0 \end{array} \right|$$

Heat conduction in Sphere



$\phi \rightarrow 0 - 180^\circ$
 $\theta \rightarrow 0 - 360^\circ$



Cylinders

$$\bar{Q} = \frac{(T_1 - T_2)}{\left[\frac{\ln(r_2/r_1)}{2\pi L K} \right]}$$

if $r_2 \approx r_1$
 $r_2/r_1 \approx 1$

$\equiv$

Sphere

Volume of differential element of thickness $\Delta r \Rightarrow (4\pi r^2) \Delta r$

Heat transfer area at $r = 4\pi r^2$
 at $r + \Delta r = 4\pi (r + \Delta r)^2$

$T \neq T(\theta)$

$T \neq T(\phi)$

$T = T(r)$

$Q = Q(r)$

$\dot{P}$ = heat generation per unit volume

Heat flux at radial direction = q_r
 Rate of heat out + generation = Accum

Rate of heat in

$$\left(q_r \right)_{4\pi r^2} - \left(q_r \right)_{4\pi (r+\Delta r)^2} + \left(\dot{P} 4\pi r^2 \Delta r \right)$$

Divide by constant term $4\pi \Delta r$

$$= \frac{d}{dt} (Q m T)$$

$$= \frac{d}{dt} (Q T \frac{4\pi r^2 \Delta r \rho}{\Delta r})$$

$$\frac{(r^2 q_r)|_r - (r^2 q_r)|_{r+\Delta r}}{\Delta r} + r^2 \rho = \frac{d}{dt} (Q r^2 \rho T)$$

$$-\frac{d}{dr}(r^2 q_r) + r^2 \cancel{\rho} = C_p \rho \frac{d}{dt}(r^2 \cancel{T})$$

No ~~ge~~ heat generation, Steady state

$$-\frac{d}{dr}(r^2 q_r) = 0 \Rightarrow \frac{d}{dr}(r^2 q_r) = 0$$

$$\Rightarrow \boxed{r^2 q_r = C_1}$$

Cylinder $\Rightarrow r q_r = C_1$
Slab $\Rightarrow q_r = C_1$

$$q_r = -k \frac{dT}{dr}$$

Boundary conditions

$$\text{at } r=r_1, T=T_1$$

$$r=r_2, T=T_2$$

$$r^2 \frac{dT}{dr} = -\frac{C_1}{k}$$

$$\Rightarrow dT = -\frac{C_1}{k} \frac{dr}{r^2}$$

$$\Rightarrow \boxed{T = -\frac{C_1}{k} \left(-\frac{1}{r}\right) + C_2}$$

Apply B.C. Find C_1 & C_2

$$C_1 = ? \quad C_2 = ?$$

Substitute C_1 & C_2 to get T

$$T_1 - T = (T_1 - T_2) \left[\frac{\frac{1}{r_1} - \frac{1}{r}}{\frac{1}{r_1} - \frac{1}{r_2}} \right] \checkmark$$

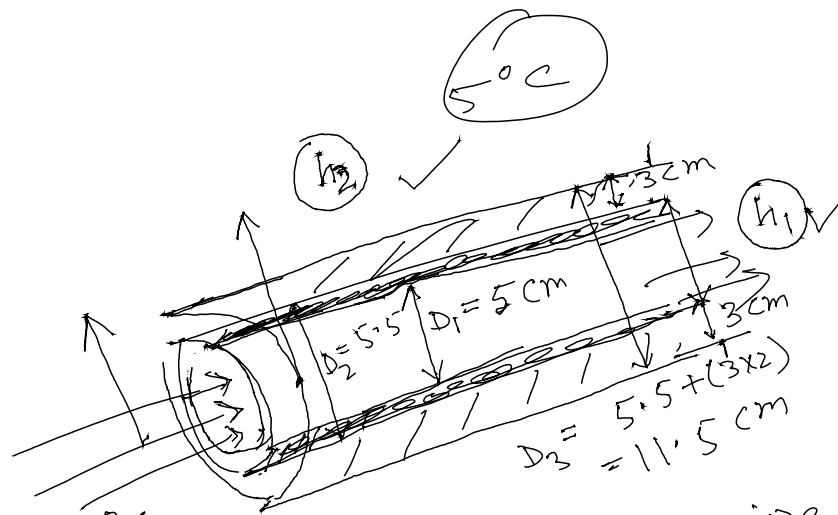
Substitute C_1 in $r^2 q_r = C_1$

Multiply 4π

$$Q = 4\pi r^2 q_r$$

$$= 4\pi C_1$$

$$Q = \frac{(T_1 - T_2)}{\frac{r_2 - r_1}{4\pi k r_1 r_2}} = \frac{(T_1 - T_2)}{\left[\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k} \right]} \quad \checkmark \text{ multilayer sphere } Q =$$



$$r_1 = D_1/2$$

$$r_2 = D_2/2$$

$$r_3 = D_3/2$$

- $R_1 \Rightarrow$ Convective resistance inside pipe
 $R_2 \Rightarrow$ Conductive resistance in pipe
 $R_3 \Rightarrow$ Conductive " in insulation
 $R_4 \Rightarrow$ Convective resistance outside pipe.

$$L = 1 \text{ m.}$$

$$Q = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$R_1 = \frac{1}{h_1 A_1} = \frac{1}{h_1 2\pi r_1 L} = \frac{1}{60 \times 2\pi (0.025)} = 0.106^\circ\text{C/W}$$

$$R_2 = \frac{\ln(r_2/r_1)}{2\pi L K_1} = \frac{\ln\left(\frac{2.75}{2.5}\right)}{2\pi (1) (80)} = 0.00019^\circ\text{C/W}$$

$$R_3 = \frac{\ln(r_3/r_2)}{2\pi L (K_2)} = \frac{\ln\left(\frac{5.75}{2.75}\right)}{2\pi (1) (0.05)} = 2.347^\circ\text{C/W}$$

$$R_4 = \frac{1}{h_2 A_2} = \frac{1}{(18) 2\pi r_3 L} = \frac{1}{18 \times 2\pi \times 0.0575} = 0.1537^\circ\text{C/W}$$

$$R_{\text{total}} = 2.61^\circ\text{C/W}$$

$$Q = \frac{320 - 5}{2.61} = 121 \text{ W}$$

$$Q = \frac{(\Delta T)_1}{R_1} = \frac{(\Delta T)_2}{R_2} = \frac{(\Delta T)_{total}}{R_{total}}$$

$$121 = \frac{(\Delta T)_{pipe}}{R_{pipe}} = \frac{(\Delta T)_{pipe}}{R_2} = \frac{(\Delta T)_{pipe}}{0.00019}$$

$$\Rightarrow (\Delta T)_{pipe} = 0.02^\circ C$$

$$121 = \frac{(\Delta T)_{insulation}}{R_{insulation}} = \frac{(\Delta T)_{insulation}}{R_3} \quad R_3 = 2.347$$

$$(\Delta T)_{insulation} = 284^\circ C \quad 2079 = m$$

$$333.7 \text{ KJ/kg}$$

R_1 = Convective Resistance inside tank

R_2 = conductive " in tank

R_3 = Convective Resistance outside tank

R_4 = Radiative " " "

$$Q = \frac{0-22}{0.00274} = -8029 \text{ W}$$

$$R_{total} = R_1 + R_2 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) = 0.00274^\circ C/W$$

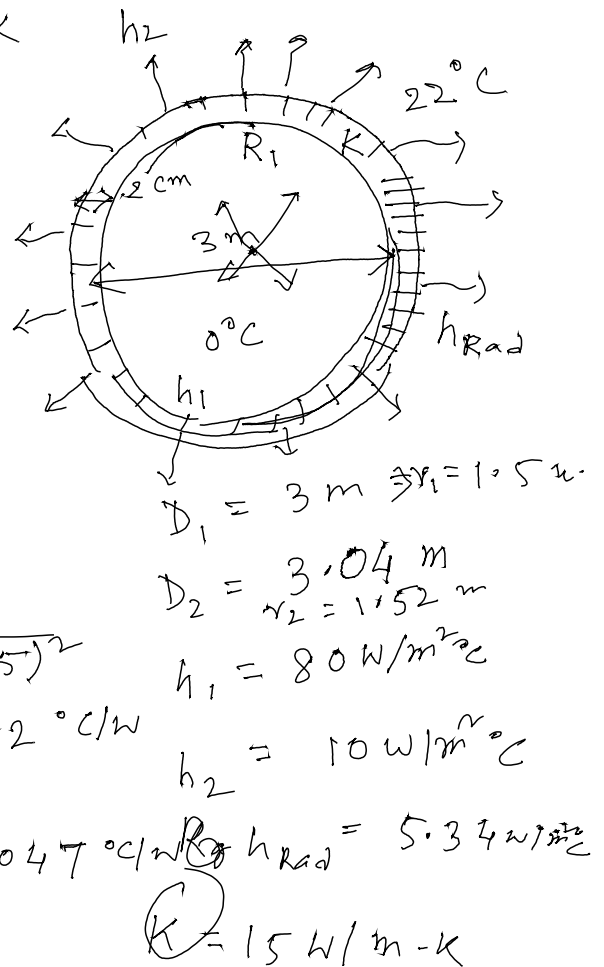
$$R_1 = \frac{1}{h_1 \times (4\pi r_1^2)} = \frac{1}{80 \times 4\pi (1.5)^2} = 0.000442^\circ C/W$$

$$R_2 = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi K} = 0.000047^\circ C/W$$

$$R_3 = \frac{1}{h_2 A_2} = \frac{1}{h_2 4\pi (1.52)^2} = 0.003452/W$$

$$R_4 = \frac{1}{h_{rad} A_2} = \frac{1}{h_{rad} 4\pi (1.52)^2} = 0.00646^\circ C/W$$

R_{total}



Heat loss from an insulated electric wire

$$G \quad r_1 = 1.5 \text{ mm}$$

$$r_2 = (1.5 + 2) \text{ mm} \\ = 3.5 \text{ mm}$$

$$L = 5 \text{ m}$$

$$k = 0.15 \text{ W/m}^\circ\text{C}$$

$$h = 12 \text{ W/m}^2\text{ }^\circ\text{C}$$

$$\dot{Q} = P = VA = 8 \times 10 = 80 \text{ W}$$

$$R_{\text{conduction insulation}} = \frac{\ln(r_2/r_1)}{2\pi k L} = \underline{\underline{0.18^\circ\text{C/W}}}$$

$$R_{\text{convection}} = \frac{1}{h 2\pi r_2 L} = \underline{\underline{0.76^\circ\text{C/W}}}$$

$$\dot{Q} = \frac{(T_1 - T_a)}{R_{\text{total}}} \Rightarrow T_1 - T_a = 75^\circ\text{C} \\ \Rightarrow T_1 = 105^\circ\text{C}$$

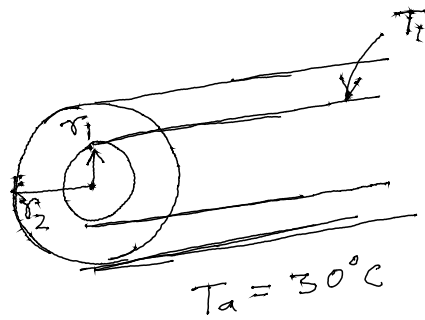
$$\text{thickness} \Rightarrow \underline{4 \text{ mm}}$$

$$r_2 = (1.5 + 4) = 5.5 \text{ mm}$$

$$R_{\text{conduction}} = \underline{\underline{0.275^\circ\text{C/W}}}$$

$$R_{\text{convection}} = \underline{\underline{0.48^\circ\text{C/W}}}$$

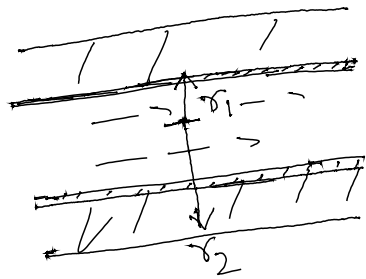
$$T_1 = 90.6^\circ\text{C}$$



Resistance of conduction (in the insulation)

Resistance of convection

SS, 1D (r-direction) heat flow



R-Conduction increases with insulation thickness

R-Convection decreases with insulation thickness

Aim is to find the critical insulation thickness for which total resistance is minimum.

$$R_{\text{conduction}} = \frac{\ln(r_2/r_1)}{2\pi L K}$$

$$R_{\text{con}} = \frac{1}{2\pi r_2 L h}$$

$$R_T = \frac{\ln(r_2/r_1)}{2\pi L K} + \frac{1}{2\pi \underline{r_2} L h}$$

$$R_T = f(r_2)$$

$$\frac{dR_T}{dr_2} = \frac{1}{2\pi L K} \cdot \frac{1}{(r_2/r_1)} \cdot \frac{1}{r_1} \cdot (1) + \frac{1}{2\pi L h} \cdot \left(-\frac{1}{r_2^2}\right) = 0$$

$$\Rightarrow \frac{1}{K r_2} - \frac{1}{h r_2^2} = 0$$

$$\Rightarrow \boxed{r_2 = \frac{K}{h}} \rightarrow \text{thermal conductivity of insulation thickness} = (r_2 - r_1)$$

$$\left. \frac{dR_T}{dr_2} \right|_{r_2 = K/h} = \frac{1}{2\pi K L} \left(-\frac{1}{r_2^2}\right) + \frac{1}{2\pi h L} 2 \frac{1}{r_2^3}$$

$$= \frac{1}{2\pi K L} \left(-\frac{h^2}{K^2}\right) + \frac{1}{2\pi h L} 2 \frac{h^3}{K^3}$$

$$= \frac{1}{\pi L} \frac{h^2}{K^3} \left[-\frac{1}{2} + 1\right] > 0$$

maximum
minimum

Critical Insulation

1)

$$D = 5.2 \text{ mm}$$

$$\text{PVC} \Rightarrow K = 0.43 \text{ W/m}^\circ\text{C}$$

$$T_w = 60^\circ\text{C}$$

$$h = 11.35 \text{ W/m}^2\text{ }^\circ\text{C}$$

Find critical insulation thickness.

$$r_{2cr} = 37.8 \text{ mm}$$

$$\text{Critical Ins. Thickness} = 35.2 \text{ mm}$$

2)

$$D = 15 \text{ cm}$$

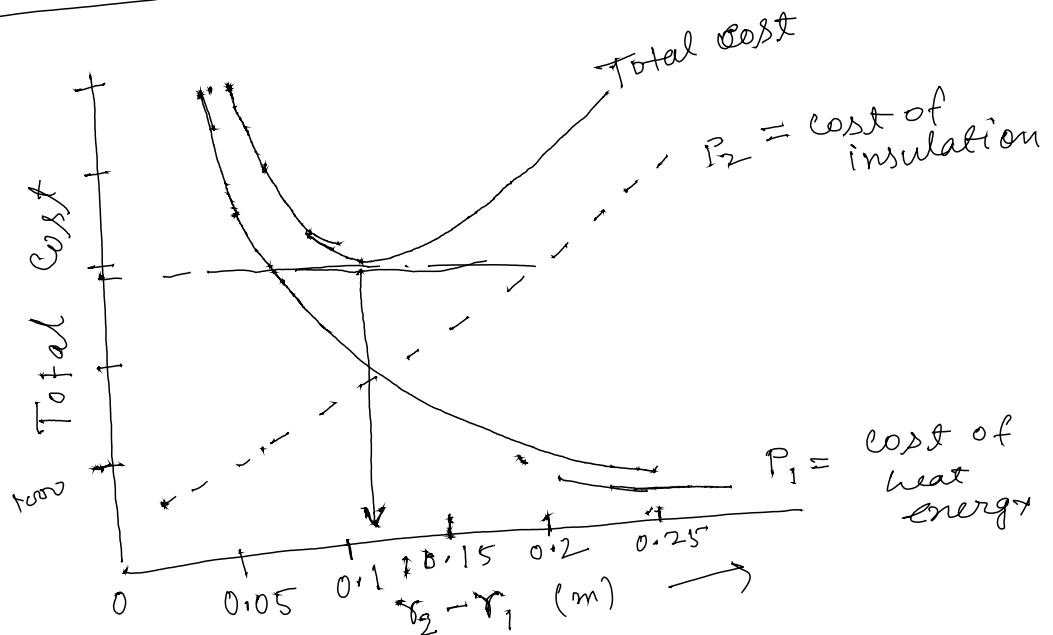
$$K = 0.035 \text{ W/m}^\circ\text{C}$$

$$h = 10 \text{ W/m}^2\text{ }^\circ\text{C}$$

Find critical insulation thickness

$$r_{2cr} = 3.5 \text{ mm}$$

Optimum insulation thickness



Thermal Insulation

$K \Rightarrow$ low

Properties of insulator

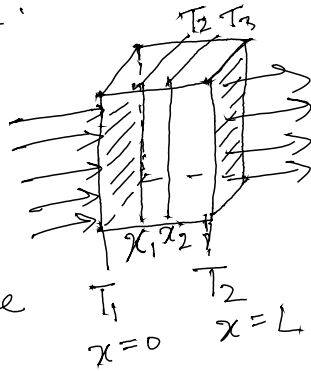
- 1) Low thermal conductivity
- 2) Chemically inert
- 3) Available in a form suitable for application on a surface
- 4) Light weight & cheap.
- 5) Fire proof
- 6) odorless
- 7) resist moisture absorption
- 8) Low density
- 9) Mechanically stable
- 10) porous structure

Types of thermal insulator

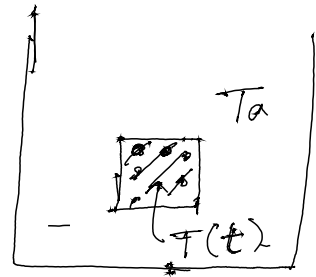
- 1) Fiber glass (Danger in handling, must use safety equipment) $R = 3.1$
- 2) Mineral wool (Not fire resistant) $R = 3.1$
- 3) Cellulose (Eco friendly, fire resistant) 3.7
- 4) Polyurethane foam (light weight, fire resistant) $R = 6.3$
- 5) Polystyrene (water proof, sound insulator, flammable $\Rightarrow$ require fire proof coating) $R = 6$

R -value $\Rightarrow$ Thermal resistance per unit
of barrier's exposed area.

$$\text{In} - \text{Out} + \text{gen} = \text{Acc.}$$



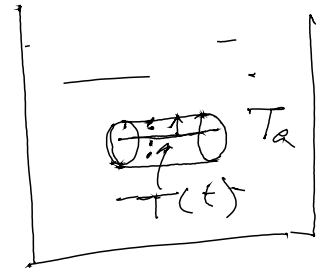
$(T_1 - T_2)$ very small
if $R_1 \Rightarrow$ Resistance
is very small



(L)

$$R = \frac{L}{KA} \quad (L) \downarrow K \uparrow$$

$L_c \Rightarrow$ characteristic length



$R \Rightarrow$ low Resistance
uniform temperature.



Rate of Energy in in the body $-$ Rate of Energy out from the body $+ \text{heat generation in the body} = \text{Accumulation of Energy in the body.}$

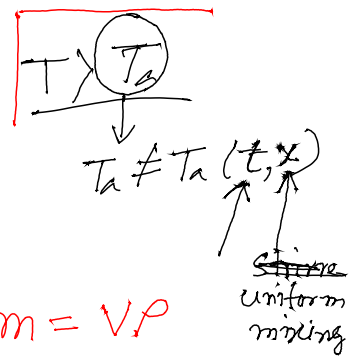
$$\text{at time } t \quad \text{O} - hA(T - T_a) + 0 = \frac{d}{dt}(mC_p T)$$

$$\Rightarrow V\rho C_p \frac{dT}{dt} = -hA(T - T_a)$$

$$\Rightarrow \int_{T_i}^{T_t} \frac{dT}{T - T_a} = - \frac{hA}{\rho V C_p} \int_0^t dt$$

$$\Rightarrow \frac{T_t - T_a}{T_i - T_a} = \exp\left(-\frac{hA}{\rho V C_p} t\right)$$

$$T_t = T_a + (T_i - T_a) \exp\left(-\frac{hA}{\rho V C_p} t\right)$$



$$m = V\rho$$

given initial cond.
at time $t = 0$

$$T = T_i$$

time $t = t$

$$T = T_t$$

$$\frac{hA}{\rho V c_p} t \Rightarrow \frac{h}{\rho c_p} \cdot \frac{A}{V} t$$

$$\frac{A}{V} = \frac{1}{L_c}$$

$$\Rightarrow \frac{h}{k} \cdot \left(\frac{\rho c_p}{k} \right) \frac{A}{V} t$$

$$\Rightarrow \frac{h}{k} \cdot \frac{1}{\infty} \cdot \frac{1}{L_c} t$$

$$\Rightarrow \left(\frac{h L_c}{k} \right) \frac{1}{\infty} \cdot \frac{1}{L_c} t$$

$$\Rightarrow (Bi) \cdot \left(\frac{\infty t}{L_c} \right)$$

$$(Bi) \cdot Fo$$

Dimensionless time
for a temperature
change to occur.

$$Bi \Rightarrow \frac{h L_c}{k} \Rightarrow \frac{L_c/k}{h/1} \rightarrow \begin{matrix} \text{Conductive Resistance} \\ \text{Convective Resistance} \end{matrix}$$

$Bi \Rightarrow \text{high} \Rightarrow \text{Conductive Resistance} \gg \text{Convective Resistance}$

✓ $Bi \Rightarrow \text{low} - \text{Conductive Resistance} < \text{Convective Resistance}$
Lumped Parameter Analysis.

$Bi < 0.1 \Rightarrow \text{no conduction resistance. (uniform temp of the body.)}$

$Bi > 0.1 \Rightarrow \text{cannot apply lumped system analysis}$

Find $L_c \Rightarrow$ for no uniform temp of the body

Sphere, cylinders & slab



$$D = 1 \text{ mm} = 0.001 \text{ m}$$

$$k = 35 \text{ W/mK}$$

$$\rho = 8500 \text{ kg/m}^3$$

$$C_p = 320 \text{ J/kgK}$$

$$h = 210 \text{ W/m}^2\text{K}$$

$$L_c = \frac{V}{A} = \frac{D}{6} = \frac{0.001}{6} = 1.67 \times 10^{-4} \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{210 \times 1.67 \times 10^{-4}}{35} = 0.001 < 0.1$$

uniform temperature in the thermocouple junction
Lumped parameters model.

$$\frac{T(t) - T_a}{T_i - T_a} = \exp(-bt)$$

Initial temp diff.

$$b = \frac{hA_s}{\rho C_p V}$$

$$b = 0.462 \text{ s}^{-1} = \left(\frac{h}{\rho C_p} \right) \left(\frac{A_s}{V} \right)$$

$$\frac{T(t) - T_a}{T_i - T_a} = 0.01$$

$$\left. \begin{array}{l} T_i = 0^\circ\text{C} \\ T_a = 100^\circ\text{C} \end{array} \right\} \Rightarrow \begin{array}{l} 100 \\ 30 \\ 100 \end{array}$$

$$\underline{T(t)} = 99 = 30 + \frac{70 \times 0.99}{0.99}$$

$$99 = 100$$

$$0 - 100$$

$$T_i = 30^\circ\text{C}, \quad T_a = 100^\circ\text{C}$$

$$\text{initial Diff.} = 100 - 30 = 70^\circ\text{C}$$

$$99\% \text{ of initial diff.} = 70 \times 0.99$$

$$T(t) = T_a + \text{change in temp}$$

$$= 30 + 70 \times 0.99 =$$

$$\frac{T(t) - T_a}{T_i - T_a} = \frac{30 + 70 \times 0.99 - 100}{30 - 100} = \frac{-1}{-70} = 0.01$$

$$= \frac{-70 + 70 \times 0.99}{-70} = 0.01$$

$$k = 401 \text{ W/mK}$$

$$D = 12 \text{ cm}$$

$$h = 15 \text{ W/m}^2\text{K}$$

$$L_c = \frac{D}{6} = 2 \times 10^{-2} \text{ m.}$$

$$Bi = \frac{15 \times 2 \times 10^{-2}}{401} = 0.00074 < 0.1$$

It is applicable.

Cylinder

$$h = 85 \text{ W/m}^2\text{K}$$

$$L_c = \frac{r}{2} = 0.125 \times 10^{-3} \text{ m.}$$

α = Thermal diffusivity

$$= \frac{k}{\rho c_p} = 5.3 \times 10^{-6} \text{ m}^2/\text{s}$$

$$K = 19.1 \text{ W/mK}$$

$$Bi = \frac{h L_c}{K} = \frac{85 \times 0.125 \times 10^{-3}}{19.1} = 5.56 \times 10^{-4} < 0.1$$

$$b = \frac{h}{\rho c_p} \cdot \frac{1}{L_c} = \frac{85}{3.6 \times 10^6} \cdot \frac{1}{0.125 \times 10^{-3}} = 0.1887$$

using thermal diffusivity

$$2) \rho c_p = \frac{k}{\alpha} = 3.6 \times 10^6$$

$$\frac{T_t - T_a}{T_i - T_a} = \exp(-bt) \Rightarrow t = 30 \text{ s}$$

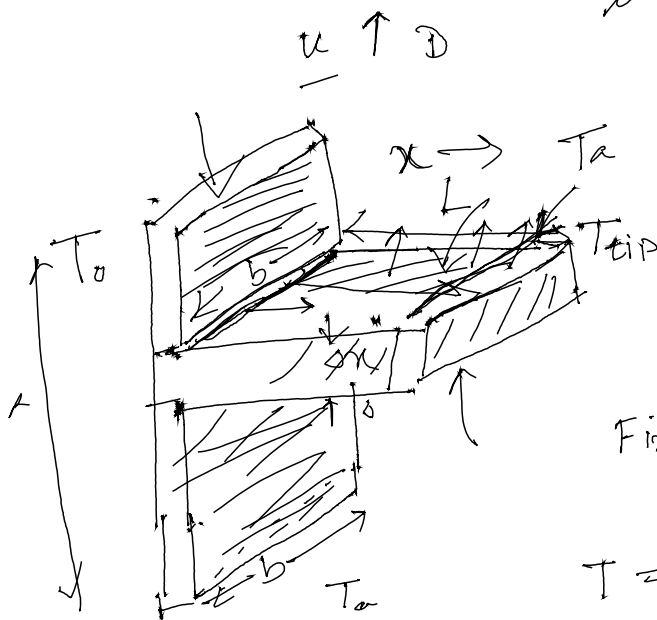
$$\left. \begin{array}{l} T_i = 30^\circ\text{C} \\ T_a = 600^\circ\text{C} \\ T(t) = 598^\circ\text{C} \\ t = ? \end{array} \right\}$$

Flow of heat from a fin/Extended Surface

$$\underline{\underline{Q = h A (\Delta T)}}$$

$$\frac{h L}{k} = Nu = C Re^m Pr^n$$

$$\Rightarrow (h) = \left(\frac{k}{L}\right) (C) \underbrace{Re^m}_{\frac{\rho U D}{\mu}} \underbrace{Pr^n}_{\frac{c_p \mu}{k}}$$



$$\text{Fin } A \approx 2 L b$$

$$T = T(x)$$

$$T_0 > T_{tip}$$

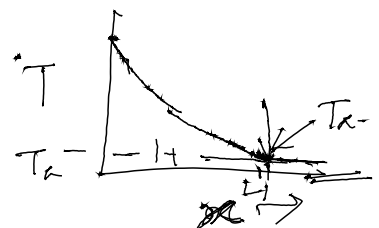
$$T = T(x) \quad T_{avg} = \frac{1}{L} \int_0^L T(x) dx$$

$$Q = h A (T_0 - T_a) \quad A(a \times b)$$

$$Q = \left[h (b(a - \cancel{L})) (T_0 - T_a) \right] + h (2 L b) (T_{avg} - T_a)$$

T high at $x=0$

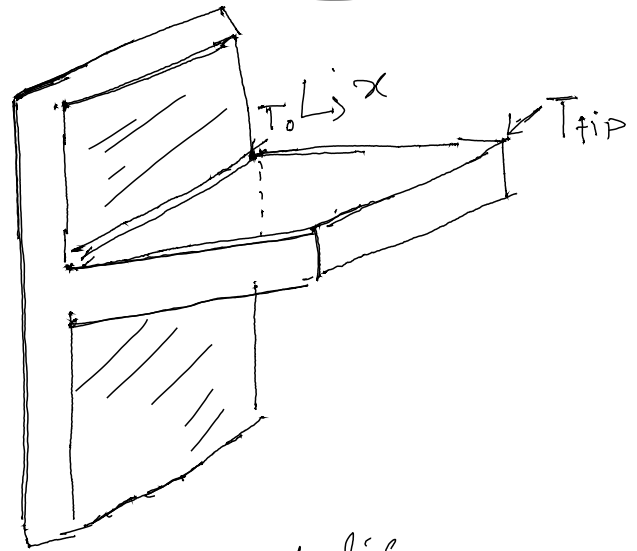
$x \uparrow \quad T \downarrow$



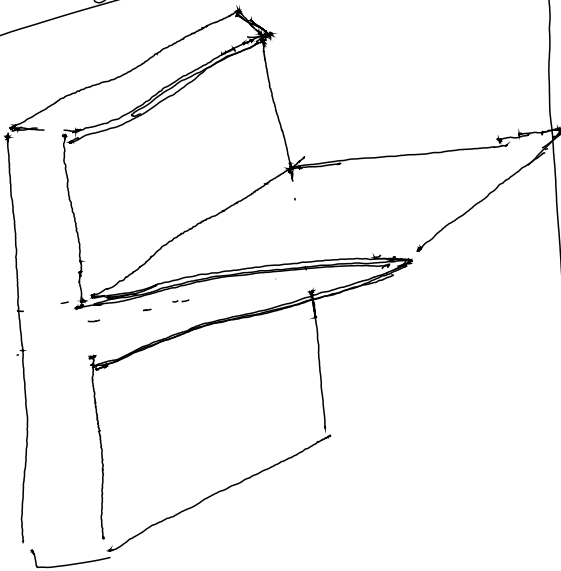
$L > L_{cr}$ [NO extra benefit]

Types of fin based on shape

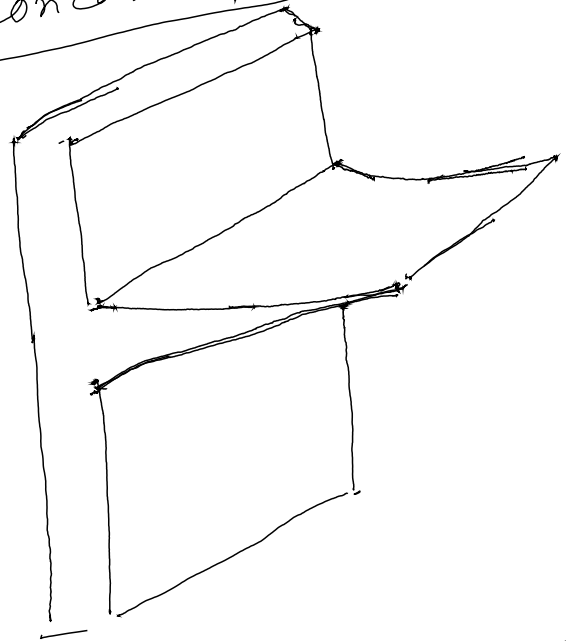
Rectangular



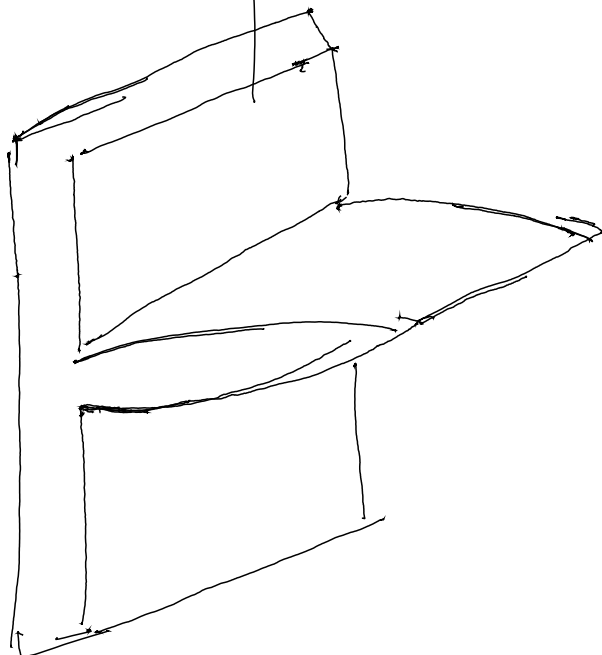
Triangular



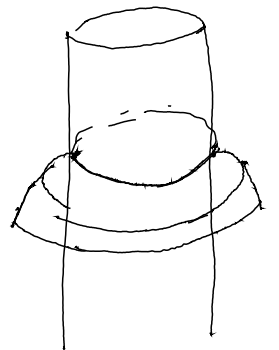
Concave parabolic



Parabolic Convex



Radial profile



$$T - T_{\infty} = C_1 e^{-mx} + C_2 e^{mx}$$

1st BC:

$$\Rightarrow \bar{T}_0 = C_1 e^{-mx \cdot 0} + C_2 e^{mx \cdot 0}$$

$$\boxed{\bar{T}_0 = C_1 + C_2} \Rightarrow \bar{T}_0 = C_1 + 0 = C_1$$

2nd BC:

$$\Rightarrow 0 = C_1 e^{-mx \cdot \infty} + C_2 e^{mx \cdot \infty}$$

$$\Rightarrow 0 = 0 + C_2 e^{m \cdot \infty}$$

$$\Rightarrow \boxed{C_2 e^{m \cdot \infty} = 0}$$

$$\underline{m \neq 0}$$

$$\boxed{\text{if } C_2 = 0}$$

$$C_2 = 0$$

$$\bar{T} = C_1 e^{-mx} + C_2 e^{mx}$$

$$\bar{T} = \bar{T}_0 e^{-mx} + 0$$

$$\Rightarrow \boxed{\frac{\bar{T}}{\bar{T}_0} = e^{-mx}}$$

@ $x=0$

$$T = T_0 \Rightarrow \bar{T} = \bar{T}_0$$

$$\bar{T} = T - T_{\infty}$$

$$\Rightarrow x=0, T=T_0 \Rightarrow \bar{T}_0 - T_{\infty} = \bar{T}_0$$

$$\Rightarrow \bar{T} = T_0 - T_{\infty} = \bar{T}_1$$

$$\bar{T} = T - T_{\infty}$$

$$\bar{T}_1 = T_1 - T_{\infty}$$

$$\bar{T}_2 = T_2 - T_{\infty}$$

$$\bar{T}_0 = T_0 - T_{\infty}$$

$$x \rightarrow \infty$$

$$T = T_{\infty}$$

$$\Rightarrow \bar{T} = T - T_{\infty} = T_{\infty} - T_{\infty} = 0$$

Case II

$$A_1 = \bar{T}_0$$

$$A_2 = -\bar{T}_0 \tanh(mL)$$

$$= -\bar{T}_0 \frac{\sinh(mL)}{\cosh(mL)}$$

$$\bar{T} = \underbrace{\bar{T}_0}_{\tilde{A}_1} \cosh(mx) + \underbrace{\left(-\bar{T}_0 \frac{\sinh(mL)}{\cosh(mL)} \right)}_{A_2} \sinh(mx)$$

$$\Rightarrow \frac{\bar{T}}{\bar{T}_0} = \frac{\cosh(mx) \cosh(mL) - \sinh(mx) \sinh(mL)}{\cosh(mL)}$$

$$\frac{\sinh(a) \cosh(b) - \cosh(a) \sinh(b)}{\cosh(a-b)}$$

$$\Rightarrow \cosh(a) \cosh(b) - \sinh(a) \sinh(b)$$

$$\Rightarrow \frac{\bar{T} - \bar{T}_0}{\bar{T}_0} = \frac{\cosh(m(L-x))}{\cosh(mL)}$$

$$\Rightarrow \frac{T - T_0}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

Case 1

Infinite long fin

$$\frac{\bar{T}}{\bar{T}_0} = e^{-mx}$$

Case 2

$$\frac{\bar{T}}{\bar{T}_0} = \frac{\cosh[m(L-x)] + \frac{h}{mk} \sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mk} \sinh(mL)}$$

Finite fin length.

Case 2 (Adiabatic fin tip)

$$\frac{\bar{T}}{\bar{T}_0} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\bar{T} = T - T_\infty$$

$$\bar{T}_0 = T_0 - T_\infty$$

I Rate of heat transfer

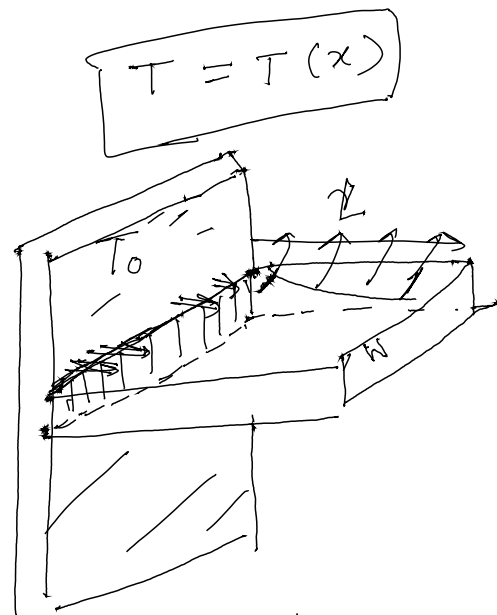
$$\dot{Q} = -KA \left. \frac{d\bar{T}}{dx} \right|_{x=0}$$

$$= -KbW \left. \frac{d\bar{T}}{dx} \right|_{x=0}$$

$$= -KbW \bar{T}_0 (-m) e^{-mx} \Big|_{x=0}$$

$$= KbW \bar{T}_0 m (1)$$

$$= bW m K (T_0 - T_\infty)$$



$$A = bW$$

II

$$\dot{Q} = -KA \left. \frac{dT}{dx} \right|_{x=0}$$

$$= -KbW \bar{T}_0 (-m) \frac{\sinh[m(L-x)]}{\cosh(mL)} \Big|_{x=0}$$

$$= bW K \bar{T}_0 m \frac{\sinh(mL)}{\cosh(mL)}$$

$$= bW K \bar{T}_0 m \tanh(mL)$$

$$\bar{T} = \bar{T}_0 e^{-mx}$$

$$\Rightarrow \frac{d\bar{T}}{dx} = \bar{T}_0 (-m) e^{-mx}$$

$$\bar{T} = \bar{T}_0 \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

$$\frac{d\bar{T}}{dx} = \bar{T}_0 (-m) \frac{\sinh[m(L-x)]}{\cosh(mL)}$$

III

$$\dot{Q} = \frac{A}{mK} bW m K \bar{T}_0 \left[\frac{\sinh(mL) + \frac{h}{mk} \cosh(mL)}{\cosh(mL) + \frac{h}{mk} \sinh(mL)} \right]$$

$$\frac{d(\cosh(ax))}{dx} = a \sinh(ax)$$

Efficiency,

$\eta = \frac{\text{Actual rate of heat transfer in a fin}}{\text{calculated rate of heat transfer if the temp. of the fin remains uniform at the base temp., } T_0}$

$$= \frac{Q_{\text{fin}}}{Q_{\text{max}}} \quad Q_{\text{max}} = h(PL + bW)(T_0 - T_{\infty})$$

case 1

$$\eta = \frac{bWmk(T_0/T_{\infty})}{hPL(T_0/T_{\infty})} \quad \left| \begin{array}{l} PL \gg bW \\ (PL + bW) \approx PL \end{array} \right.$$

$$= \left(\frac{bWk}{Ph} \right) \cdot \frac{m}{L}$$

$$= \frac{1}{m^r} \cdot \frac{m}{L}$$

$$= \frac{1}{mL}$$

$$\frac{bWk}{Ph} = m^r$$

case 2

$$\eta = \frac{KbWm \tanh(mL)(T_0/T_{\infty})}{hPL(T_0/T_{\infty})} \quad \text{Adiabatic}$$

$$= \left(\frac{bWk}{Ph} \right) \frac{m}{L} \tanh(mL)$$

$$= \frac{\tanh(mL)}{mL}$$

case 3

$$\eta \approx \frac{1}{mL}$$

$$\eta = > 90\% \quad < 60\%$$

Fin effectiveness

The performance of the fin is judged on the basis of enhancement in heat transfer rate relative to that of no fin case

$$\epsilon_{fin} = \frac{Q_{fin}}{Q_{no\ fin}}$$

$$= \frac{\text{Rate of heat transfer from fin of base area } (A_b)}{\text{Rate of heat transfer from ~~fin~~ surface of surface area } (A_b)}$$

$\epsilon = 1$, Addition of fin ~~does~~ not enhance the rate of heat transfer

$\epsilon > 1$ Enhances the rate of heat transfer

$\epsilon < 1$, fin acts as insulation.

$$\epsilon_{fin} = \frac{Q_{fin}}{Q_{no\ fin}} = \frac{\eta_{fin} \times \cancel{h} \times A_{fin} (T_o - T_\infty)}{A_b \cancel{h} (T_o - T_\infty)}$$

$$\checkmark = \eta_{fin} \left[\frac{A_{fin}}{A_b} \right] = \eta_{fin} \frac{PL}{bK}$$

Design

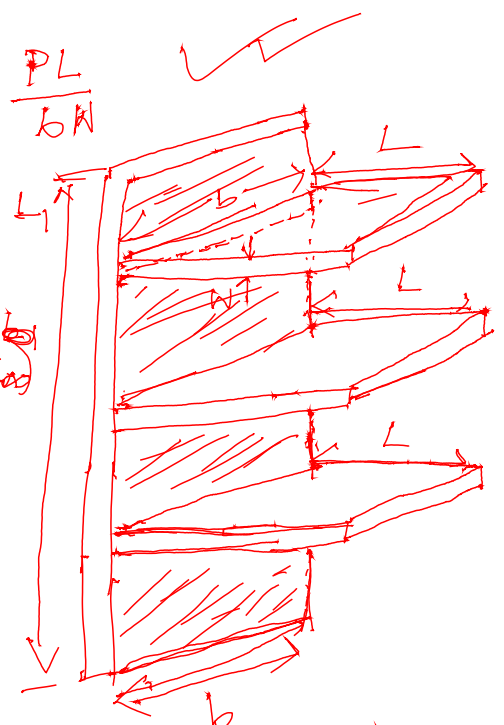
For n fin

overall

ϵ_{fin}

$$= \frac{[3 \eta_{fin} (h) PL (T_o - T_\infty)] + A_e h (T_o - T_\infty)}{(L_1 b) h (T_o - T_\infty)}$$

✓



$$A_e = (L_1 b - 3b^2)$$

①

Insulate fin tip

cylindrical fin
 $K = 32 \text{ W/m}^\circ\text{C}$
 $d = 12 \text{ mm}$

$T_b = 95^\circ\text{C}$

$T_\infty = 60^\circ\text{C}$

$h = 60^\circ\text{C} \ 55 \text{ W/m}^\circ\text{C}$

$L = 60 \text{ mm}$

$\eta = ?$

②

Rectangular fin

Insulated fin tip

$L = 50 \text{ mm}$ Thickness = 5 mm

width (b) = 100 mm

$K = 210 \text{ KJ/m}^\circ\text{C}$

$h = 42 \text{ KJ/m}^\circ\text{C}$ $T_b = 80^\circ\text{C}$, $T_\infty = 20^\circ\text{C}$

$\eta = ?$

③

Infinite long fin

(Pin fin) cylindrical fin

$Q = ?$

$d = 60 \text{ mm}$

$h = 6.5 \text{ W/m}^\circ\text{C}$

$K = 50 \text{ W/m}^\circ\text{C}$

pin fin

④

case $d_1 = x$

Q_1

case $d_2 = 3x$

Q_2

$\frac{Q_2}{Q_1} = ?$

$$\textcircled{1} \quad m =$$

$$m L \eta = 62.16 \%$$

$$\eta = \frac{Q_{\text{insulated fin tip}}}{Q_{\text{fin tip}}} = 2.7 \text{ W}$$

$$T_{\text{tip}} = T(x=L) = 75.75^\circ\text{C}.$$

$\textcircled{2}$

$$Q_{\text{fin}} = 24.75 \text{ KW}$$

$$T_{\text{HP}} = 74.2^\circ\text{C}.$$

$\textcircled{3}$

$$Q_{\text{fin}} = 24.96 \text{ W}.$$

$\textcircled{4}$

$$\frac{Q_2}{Q_1} = 5.2$$

$$m = \sqrt{\frac{hP}{KA_c}} = 12.99$$

$$mL = 0.0519$$

$$\eta_{\text{insulated fin tip}} = \frac{\tanh(mL)}{mL} = 0.919$$

$$A_c = (0.002)^2 m^2$$

$$P = 4(0.002) m$$

$$h = 20 W/m^2K$$

$$K = 237 W/mK$$

$$L = 0.04 m$$

$$A_{fin} = \eta PL = \eta \times 0.008 \times 0.04$$

$$= 0.00032 \eta$$

(n)

T_b = base temp

T_a = env. temp

$$Q_{fin} = \eta_{fin} \times A_{fin} \times h \times (T_b - T_a)$$

$$= 0.919 \times 0.00032 \eta \times 20 \times (85 - 25)$$

$$= 0.3529 \eta \text{ (W)}$$

$$A_{unfin} = (0.15 \times 0.2) - \eta \times (0.002)^2$$

$$= 0.03 - 0.000004 \eta$$

$$Q_{unfin} = A_{unfin} \times h \times (T_b - T_a)$$

$$= 36 - 0.0048 \eta$$

$$Q_{total, fin} = Q_{fin} + Q_{unfin}$$

$$A_{no fin} = 0.15 \times 0.2$$

$$Q_{no fin} = A_{no fin} \times h \times (T_b - T_a)$$

$$= 36 \text{ (W) } \checkmark$$

$$\epsilon = \frac{Q_{total, fin}}{Q_{no fin}} = 3$$

$$\eta = ? \quad \epsilon = \frac{0.3529 \eta + 36 - 0.0048 \eta}{36} = 3$$

$$\eta = 207$$

Convective heat transfer

$Q = h A (T_s - T_a)$ [Newton's law of cooling]

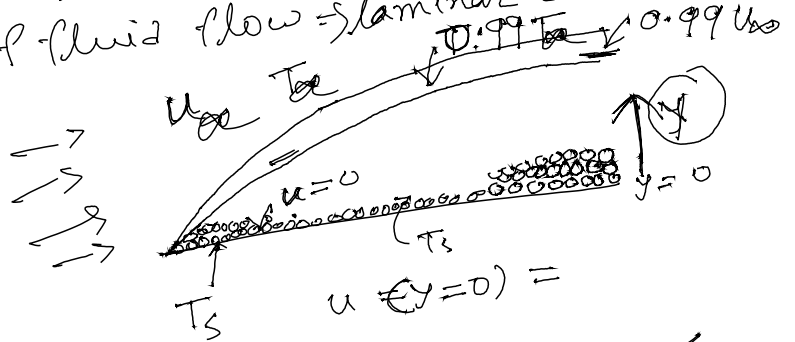
h = convective heat transfer coefficient

Depends on fluid μ , K , ρ , C_p , velocity

roughness of surface

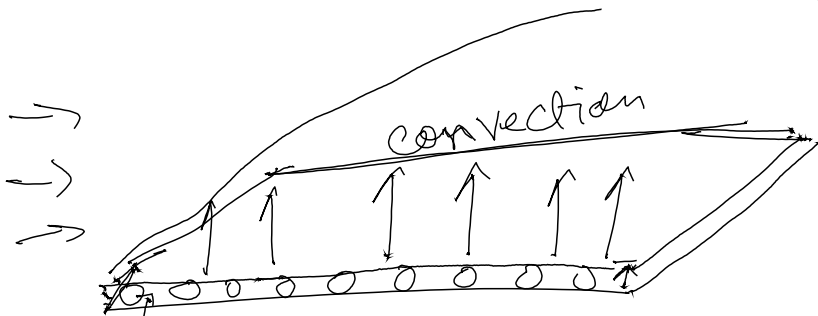
type of fluid flow = laminar or turbulent

(h)



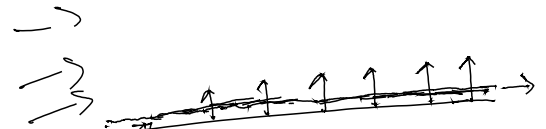
$T = T(y)$ ✓

✓ $\left. \frac{dT}{dy} \right|_{y=0} \Rightarrow$ Temp. grad at $y=0$



First layer of fluid

↳ heat transfer by conduction



First fluid layer
↳ heat transfer by conduction

Heat transfer by conduction in the first layer of fluid

$\Rightarrow -K A \left. \frac{dT}{dy} \right|_{y=0} = h A (T_s - T_a)$ ✓

$T_a \Rightarrow$ bulk fluid temp.

(h) =

$\frac{-K \left. \frac{dT}{dy} \right|_{y=0}}{(T_s - T_a)}$

$T = T(y)$ ✓

$$\frac{T(y) - T_a}{(T_s - T_a)} = e^{-ay}$$

$$\begin{aligned} T_a &= 20^\circ\text{C} \\ T_s &= 160^\circ\text{C} \end{aligned} \left. \vphantom{\begin{aligned} T_a &= 20^\circ\text{C} \\ T_s &= 160^\circ\text{C} \end{aligned}} \right\} 90^\circ\text{C}$$

$$a = 3200 \text{ m}^{-1}$$

$$\frac{dT(y)}{dy} = (T_s - T_a)(-a) e^{-ay}$$

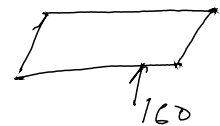
$$K_{90} = 0.03024 \text{ W/mK}$$

$$\left. \frac{dT(y)}{dy} \right|_{y=0} = (T_s - T_a)(-a)(1)$$

$$K = \frac{0.03024}{20}$$

$$= (160 - 20) \times (-3200)$$

$$= -4.48 \times 10^5 \text{ C/m} \checkmark \Rightarrow$$



convective heat transfer coeff.

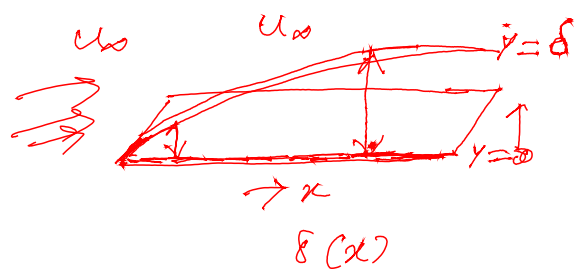
$$\begin{aligned} T_{\text{mean}} &= 90^\circ\text{C} \\ K_{90} &= \end{aligned}$$

$$h = \frac{-K_{\text{fluid}} \left. \frac{dT(y)}{dy} \right|_{y=0}}{(T_s - T_a)} = 96.8 \text{ W/m}^2\text{K}$$

classification of fluid flow

- viscous versus inviscid Region of flow
- Internal versus external flow
- Compressible versus incompressible flow
- Laminar versus turbulent flow
- Natural vs forced flow
- Steady vs unsteady flow
- 1D, 2D or 3D flow
- Velocity boundary layer
- Wall shear stress

$$\tau_w = -\mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$



$$\tau_w = C_f \frac{\rho V^2}{2}$$

$\downarrow$
 or
 skin friction coefficient

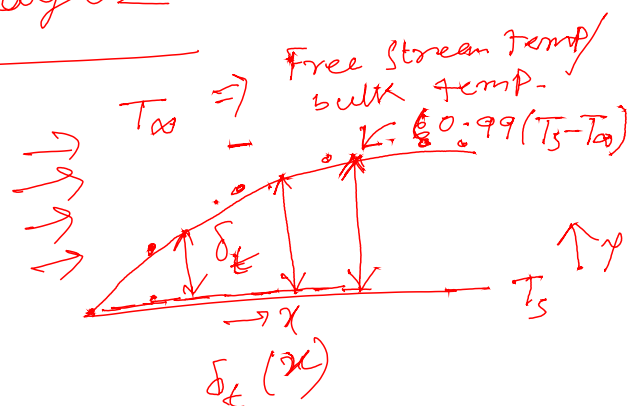
$$F_f = A_s \tau_w = C_f A_s \frac{\rho V^2}{2}$$

Thermal boundary layers

Prandtl number

$$\Rightarrow Pr = \frac{C_p \mu}{K}$$

$$= \frac{\mu / \rho}{K / \rho c} = \frac{\nu}{\alpha}$$



$$\underline{Pr} = \frac{C_p \mu}{k} = \frac{\mu/\rho}{\frac{k}{\rho C_p}} = \frac{V}{\alpha} \begin{matrix} \nearrow \text{momentum diff.} \\ \searrow \text{thermal diffusivity} \end{matrix}$$

Unit of all diffusivity $\Rightarrow \text{m}^2/\text{s}$
 $D_{AB} \Rightarrow \text{mass/molar diffusivity}$

$Pr > 1$ Heavy oils 100,000

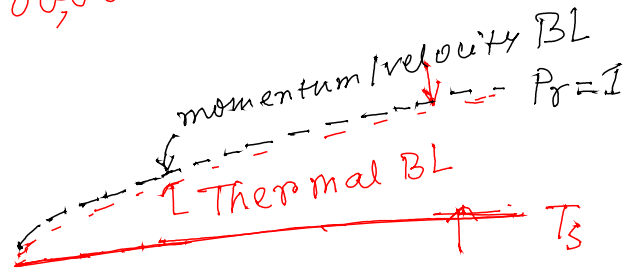
$Pr < 1$

$\hookrightarrow$ liquid metal

$k \uparrow \quad C_p \downarrow$

$Pr \approx 0.01$

$\delta_t \Rightarrow \text{very small}$



Water $\rightarrow Pr = 10 \Rightarrow \underline{\text{Plot both BL}}$
 Air $\rightarrow Pr = 1$

$$\boxed{Pr = \frac{V}{\alpha} = \frac{\delta_v}{\delta_t}}$$

$$Re = \frac{\rho D V}{\mu} \Big|_{\text{pipe flow}} = \frac{\rho V L_c}{\mu}$$

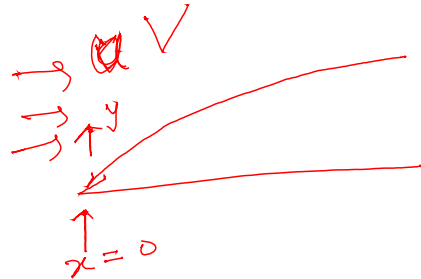
$= \frac{\text{Inertia force}}{\text{Viscous force}}$

Solution of convection equations for a flat plate

2D

Continuity eqn

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



Momentum eqn

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

↑
momentum diff

Energy eqn

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

Boundary condition

at $x=0$, $\underline{u(0,y) = V}$; $T(0,y) = T_\infty$ ← free velocity

at $y=0$, $u(x,0) = 0$, $v(x,0) = 0$,
 $T(x,0) = T_s$

$y \rightarrow \infty$, $u(x,\infty) = V$; $T(x,\infty) = T_\infty$

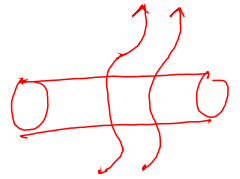
Stream function $\psi(x,y)$

$$\tau_w = 0.332 V \sqrt{\frac{\rho \mu V}{x}} = 0.332 \frac{\rho V^2}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau_w}{\frac{\rho V^2}{2}} = 0.664 Re_x^{-1/2}$$

$$\delta = \frac{4.91}{\sqrt{\frac{V}{\nu x}}} = \frac{4.91 x}{\sqrt{Re_x}}$$

momentum diff.



$$\left. \frac{\partial T}{\partial y} \right|_{y=0} = 0.332 Pr^{1/3} (T_w - T_s) \sqrt{\frac{V}{\nu x}}$$

momentum diff.

$$-k \left. \frac{\partial T}{\partial y} \right|_{y=0} = \frac{h_x (T_s - T_w)}{1}$$

h_x = local convective H.T. coeff.

$$h_x =$$

$$= 0.332 Pr^{1/3} k \sqrt{\frac{V}{\nu x}}$$

Diff.

$$Nu_x = \frac{h_x x}{k} = 0.332 Pr^{1/3} Re_x^{1/2} \quad Pr > 0.6$$

$$\delta_t = \frac{\delta}{Pr^{1/3}} = \frac{4.91 x}{Pr^{1/3} \sqrt{Re_x}}$$

$$Nu_L = 2 Nu_x$$

$$h_L = \frac{1}{L} \int_0^L h_x dx$$

$$C_{f,x} \frac{Re_L}{2} = Nu_x \quad Pr = 1 \quad \text{Reynolds analogy}$$

$$St = \frac{Nu}{Re Pr}$$

$$\frac{C_{f,x}}{2} = St_x Pr^{2/3} = J_H$$

$J_H \Rightarrow$ Colburn j factor

Chilton-Colburn analogy $0.6 < Pr < 60$

$$C_{p, oil} = 2.05 \text{ kJ/kg}^\circ\text{C}$$

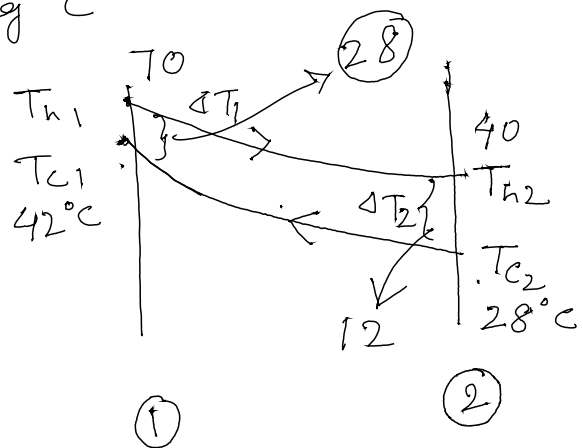
counter current

$$C_{p, water} = 4.17 \text{ kJ/kg}^\circ\text{C}$$

$$\dot{m}_c = ?$$

$$A = 3 \text{ m}^2$$

$$U = ?$$



$$\dot{m}_c C_{p,c} (T_{c1} - T_{c2}) = \dot{m}_h C_{p,h} (T_{h1} - T_{h2})$$

$$\Rightarrow \dot{m}_c \times 4.17 (42 - 28) = \frac{10000 \times 2.05 (70 - 40)}{1}$$

$$\Rightarrow \dot{m}_c = 1053 \text{ kg/h}$$

$$Q = U A (\Delta T_m)$$

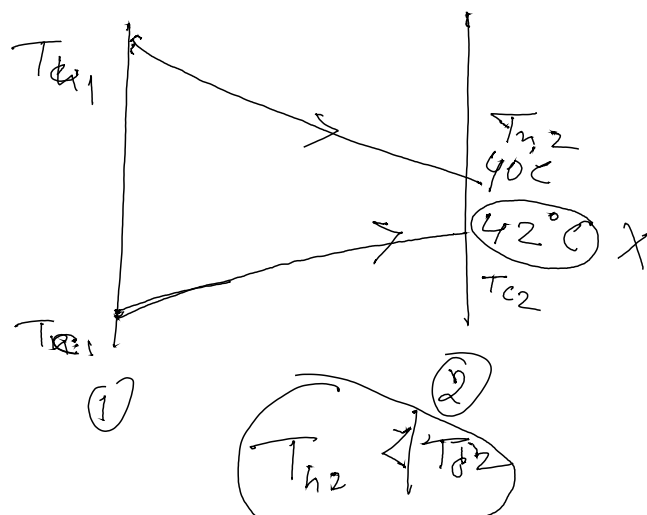
$$61500 \text{ (kJ/h)} = U \times 3 \times 18.9$$

$$\Delta T_m = \frac{28 - 12}{\ln(28/12)} = 18.9^\circ\text{C}$$

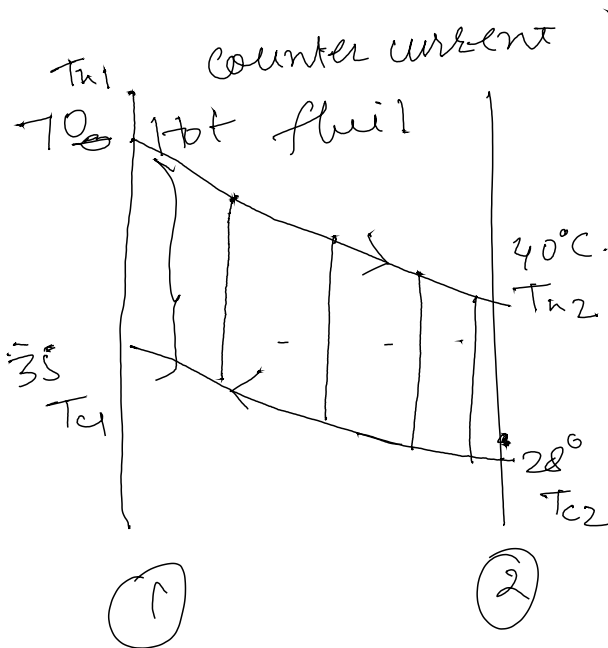
$$U = \frac{Q}{A (\Delta T_m)}$$

$$U = 300 \text{ W/m}^2\text{}^\circ\text{C}$$

co-current

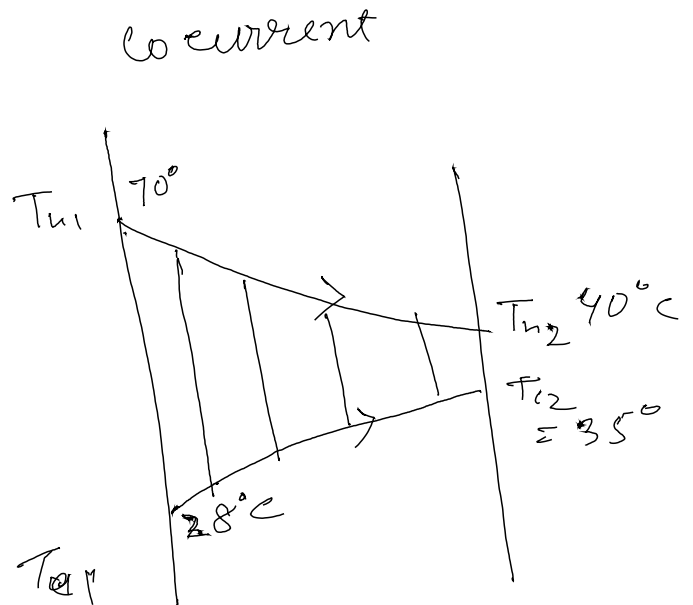


Hot fluid in 70°C out 40°C
 Cold fluid in 28°C out 35°C .



$$\Delta T_m = 21.486^{\circ}\text{C}$$

$$Q = UA(\Delta T_m)$$



$$(\Delta T_m) = 17.38^{\circ}\text{C}$$

$$Q = UA(\Delta T_m)$$

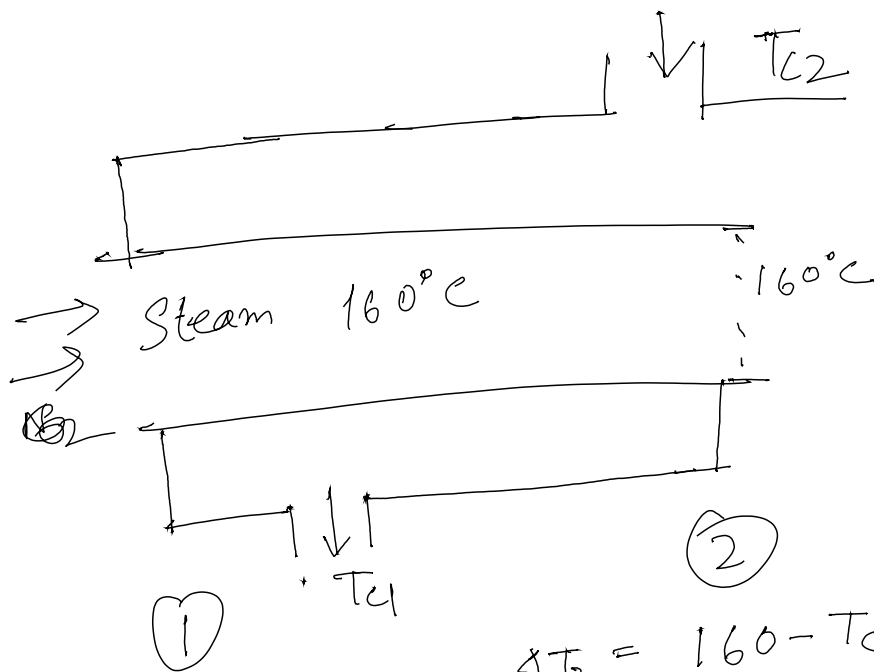
$$T_h = 160^{\circ}\text{C} \quad (5.7 \text{ bar})$$

$$\dot{m}_c = 2000 \text{ kg/h} \quad \left| \text{Heat duty} = 38,700 \frac{\text{Kcal}}{\text{h}} \right.$$

$$C_{p_{N_2}} = 0.239 \text{ Kcal/kg}^{\circ}\text{C}$$

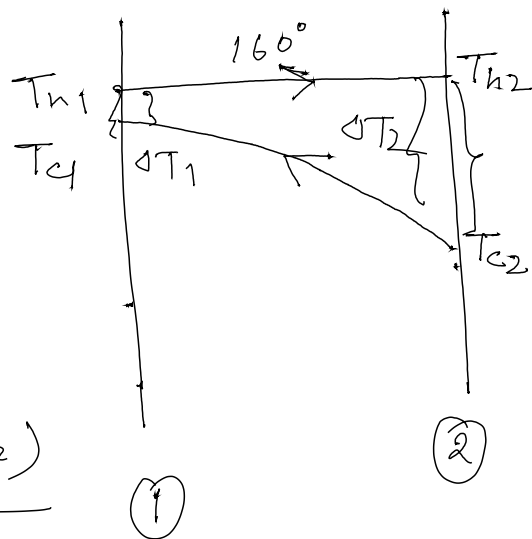
$$A = 10 \text{ m}^2, \quad U = 44 \text{ Kcal/h m}^2^{\circ}\text{C}$$

$$T_{c1} \text{ \& } T_{c2}$$



$$\Delta T_1 = 160 - T_{c1}$$

$$\Delta T_2 = 160 - T_{c2}$$



$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln \left(\frac{\Delta T_1}{\Delta T_2} \right)}$$

$$\Rightarrow = \frac{(160 - T_{c1}) - (160 - T_{c2})}{\ln \left(\frac{160 - T_{c1}}{160 - T_{c2}} \right)}$$

$$= \frac{(T_{c2} - T_{c1})}{\ln \left(\frac{160 - T_{c1}}{160 - T_{c2}} \right)} = \frac{f \cdot (T_{c1} - T_{c2})}{f \ln \left(\frac{160 - T_{c2}}{160 - T_{c1}} \right)}$$

$$38700 = \frac{UA}{\ln \left(\frac{160 - T_{c2}}{160 - T_{c1}} \right)} \quad (1)$$

$$38700 = m_c c_{pc} (T_{c1} - T_{c2})$$

$$= 2000 \times 0.239 (T_{c1} - T_{c2})$$

$$- 2700 = -$$

$$T_{c1} - T_{c2} = 81 \quad \text{--- (1) } \Rightarrow T_{c1} = 81 + T_{c2}$$

$$87.95 = \frac{81}{\ln \left(\frac{160 - T_{c2}}{160 - T_{c1}} \right)} \quad \text{--- (2)}$$

$$\Rightarrow T_{c2} = 25^\circ\text{C} \quad \checkmark$$

$$T_{c1} = 106^\circ\text{C} \quad \checkmark$$

$$d_i = 3.5 \text{ cm}$$

(thickness 3.6 mm)

$$V = 1.8 \text{ m/s}$$

$$L = 15 \text{ m}$$

$$T_{ht} = 110^\circ\text{C}$$

Insulation thickness = 2 cm.

$$(k_i = 0.12 \text{ W/m}^\circ\text{C})$$

$$T_a = 20^\circ\text{C}$$

$$h_o = 10 \text{ W/m}^2\text{C}$$

calculate $T_{h2} = ?$

Tube side

$$C_{p\text{water}} = 4.23 \text{ kJ/kg}^\circ\text{C}$$

$$\mu = 2.55 \times 10^{-4} \text{ kg/m s}$$

$$\rho = 950 \text{ kg/m}^3, \quad K_{\text{water}} = 0.685 \text{ W/m}^\circ\text{C}$$

$$Nu = 0.023 Re^{0.8} (Pr)^{0.3} \text{ for cooling}$$

$$Re = \frac{\rho d_i \dot{V}}{\mu} = 234700$$

$$Pr = \frac{c_p \mu}{k} = 1.57$$

$$Nu = 5216 = \frac{h d_i}{k}$$

$$h_i = 10,208 \text{ W/m}^2\text{e}$$

$$\frac{1}{U_1} = \frac{1}{h_1} + \frac{r_1}{k_p} \ln(r_2/r_1) + \frac{r_1}{k_i} \ln(r_3/r_2) + \frac{r_1}{r_3} \frac{1}{h_2}$$

$k_p = 43 \text{ W/m}^2\text{e}$

$$r_1 = \frac{35}{2} \text{ cm} = 0.0175 \text{ m}$$

$$r_2 = 0.0175 + 0.0036 \text{ m} = 0.0211 \text{ m}$$

$$r_3 = 0.0211 + 0.02 \text{ m} = 0.0411 \text{ m}$$

$$\frac{1}{U_1} = 0.14$$

$$U_1 = 7.15 \text{ W/m}^2\text{e}$$

$$Q = m_A c_{pA} (T_{h1} - T_{h2}) = U_1 A_1 (\Delta T_m)$$

$$\Delta T_1 = 110 - 20 = 90^\circ\text{C}$$

$$\Delta T_2 = (T_{h2} - 20)^\circ\text{C}$$

$$\dot{m} = \rho \left(\frac{\pi d_i^2}{4} \right) V$$

$$T_{h2} = 109.8^\circ\text{C}$$

4

$$d_i = 41 \text{ mm}$$

$$\dot{m}_c = 1 \text{ kg/s}$$

$$T_{c1} = 28^\circ\text{C}$$

$$V_g = 10 \text{ m/s}$$

$$L = 20 \text{ m}$$

$$\text{Pipe thickness} = 3.7 \text{ mm}$$

Data water at 28°C

$$\rho = 996 \text{ kg/m}^3$$

$$C_p = 1 \text{ kcal/kg}^\circ\text{C}$$

$$\mu = 8.6 \times 10^{-4} \text{ kg/ms}$$

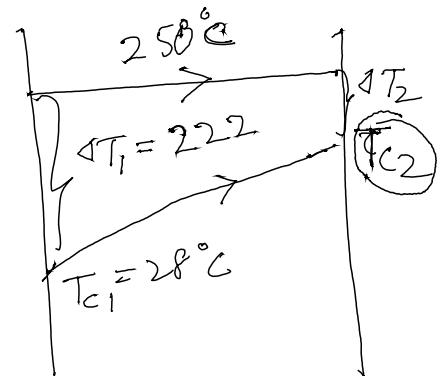
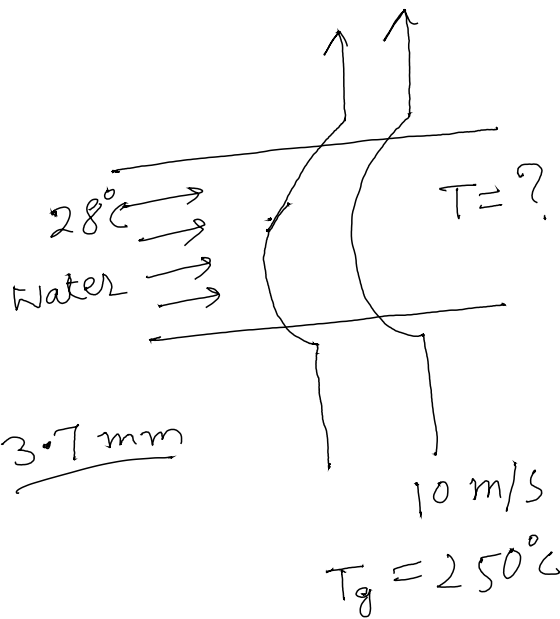
$$K_{\text{water}} = 0.528 \text{ kcal/hr m}^\circ\text{C}$$

For flue gas at air temp.

$$\rho = 0.891 \text{ kg/m}^3$$

$$C_p = 0.243 \text{ kcal/hr m}^\circ\text{C}$$

$$Pr = 0.69$$



$$\Delta T_2 = 250 - T_{c2}$$

$$Q = \dot{m}_c C_{pc} (\overset{?}{T_{c2}} - T_{c1}) = \dot{Q} = U_2 A_2 (\Delta T_m)$$

$$\frac{1}{U_2} = \frac{\gamma_2}{\gamma_1} \frac{1}{h_1} + \frac{1}{h_2}$$

$$d_2 = (41 + 3.7) \text{ mm}$$

$$A_2 = \pi d_2 L$$

$$\frac{h_1 d_i}{k} = Nu = 0.023 (Re)^{0.8} (Pr)^{0.4}$$

$$Re = \frac{\rho d_i V}{\mu}$$

$$= \frac{996 \times 0.041 \times 0.761}{8.6 \times 10^{-4}}$$

$$= 36100$$

$$V = \left(\frac{\dot{m}}{\rho} \right) \cdot \frac{1}{\left(\frac{\pi}{4} d_i^2 \right)}$$

$$= 0.761 \text{ m/s}$$

$$d_i = 0.041 \text{ m}$$

$$Pr = \frac{c_p \mu}{k} = \frac{1 \times 8.6 \times 10^{-4}}{0.528} = 5.86$$

$$Nu = 206$$

$$h_1 = Nu \times \frac{k}{d_i} = 2653 \text{ kcal/h m}^2 \text{ } ^\circ\text{C}$$

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr} \right)^{1/4} \right]^{1/4}} \left[1 + \left(\frac{Re}{2.82 \times 10^5} \right)^{5/8} \right]^{4/5}$$

$$\mu = 2.33 \times 10^{-5} \text{ kg/m s}$$

$$Re = \frac{\rho V d_2}{\mu} = 18,500$$

$$K_a = 0.0292 \text{ kcal/h m}^2 \text{ } ^\circ\text{C}$$

$$Pr = 0.69$$

$$Nu = 75 \Rightarrow h_2 = Nu \times \frac{K_a}{d_2} = 45.2 \text{ kcal/h m}^2 \text{ } ^\circ\text{C}$$

$$\frac{1}{U_2} = \frac{d_2}{d_1} \frac{1}{h_1} + \frac{1}{h_2} \Rightarrow 44.3 \text{ kcal/h m}^2 \text{ } ^\circ\text{C}$$

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln \left(\frac{\Delta T_1}{\Delta T_2} \right)} = \frac{222 - (250 - T_{c2})}{\ln \left(\frac{222}{250 - T_{c2}} \right)}$$

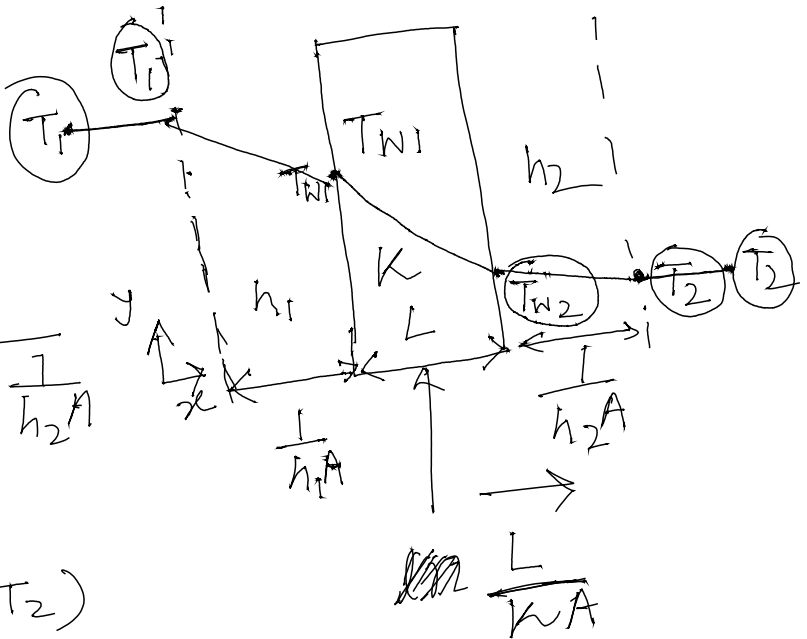
$$= \frac{T_{c2} - 28}{\ln \left(\frac{222}{250 - T_{c2}} \right)}$$

$$m_c C_{pc} (T_{c2} - 28) = U_2 A_2 (\Delta T_m)$$

$$\Rightarrow (3600 \frac{\text{kg}}{\text{h}}) (1 \frac{\text{Kcal}}{\text{kg}^\circ\text{C}}) (T_{c2} - 28) = 44.3 \frac{\text{Kcal}}{\text{h m}^2^\circ\text{C}} \times \frac{(\pi D_2 L) \times \frac{T_{c2} - 28}{\ln(\frac{222}{250 - T_{c2}})}}{(\text{m}^2)}$$

$$\Rightarrow \underline{T_{c2} = 36^\circ\text{C}}$$

Overall heat transfer coefficient



$$Q = \frac{T_1 - T_2}{\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}}$$

$$Q = UA (T_1 - T_2)$$

$$= U_1 A_1 (T_1 - T_2)$$

$$= U_2 A_2 (T_1 - T_2)$$

$$T_1 > T_2$$

$A = \text{constant wrt } x\text{-direction}$
heat flow

$$\Rightarrow Q = \frac{T_1 - T_2}{(1/UA)}$$

$$\frac{1}{UA} = \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}$$

$$\Rightarrow \frac{1}{UA} = \frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}$$

$$\Rightarrow U = \frac{1}{\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}}$$

con

$$Q = \frac{(T_1 - T_2)}{\frac{1}{L 2\pi r_1 h_1} + \frac{\ln(r_2/r_1)}{2\pi L K} + \frac{1}{2\pi r_2 h_2}}$$

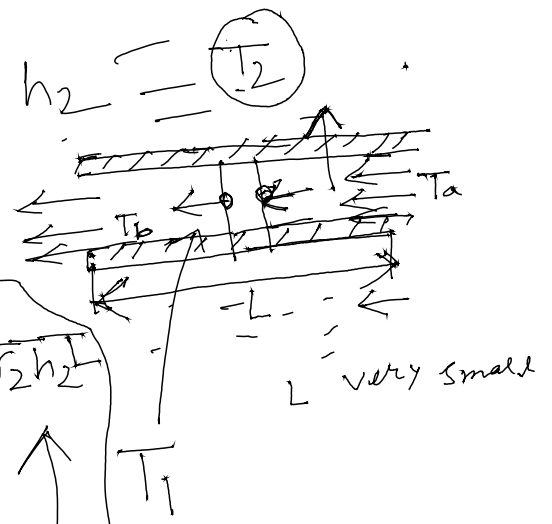


$$Q = U_1 (2\pi r_1 L) (T_1 - T_2)$$

$$Q = U_2 (2\pi r_2 L) (T_1 - T_2)$$

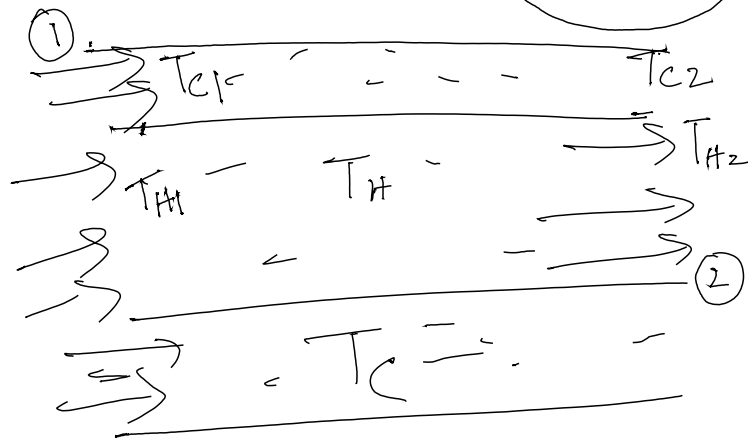
$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{A_1 \ln(r_2/r_1)}{2\pi L K} + \frac{r_1}{r_2} \frac{1}{h_2}}$$

$$U_2 = \frac{1}{\left(\frac{r_2}{r_1}\right) \frac{1}{h_1} + \frac{A_2 \ln(r_2/r_1)}{2\pi L K} + \frac{1}{h_2}}$$



$T_1 > T_2$

U_1 = overall H.T. Coeff. based on inside surface.
 U_2 = based on outside surface



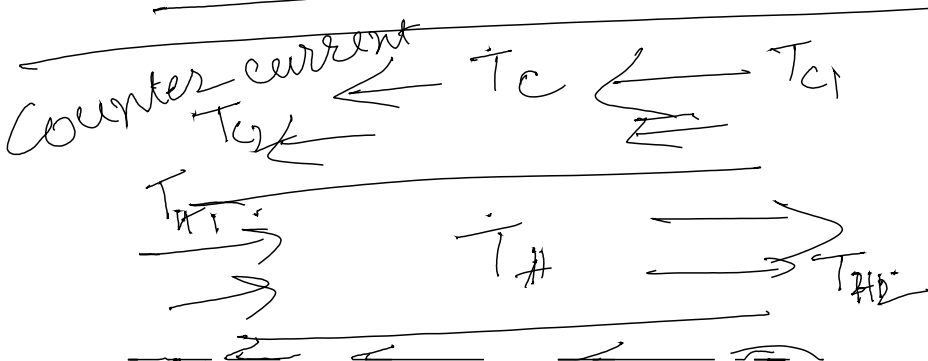
case 1

Co current



$$\Delta T_1 = T_{H1} - T_{C1}$$

$$\Delta T_2 = T_{H2} - T_{C2}$$

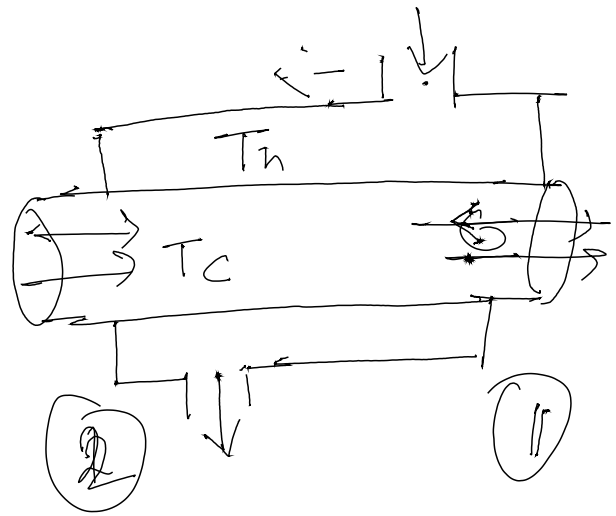
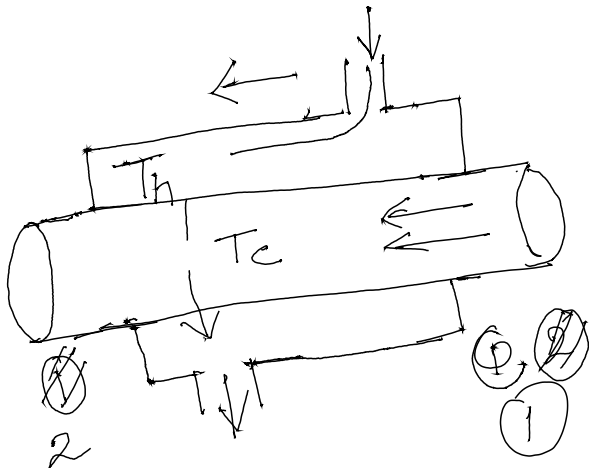


$$\Delta T_1 = T_{H1} - T_{C2}$$

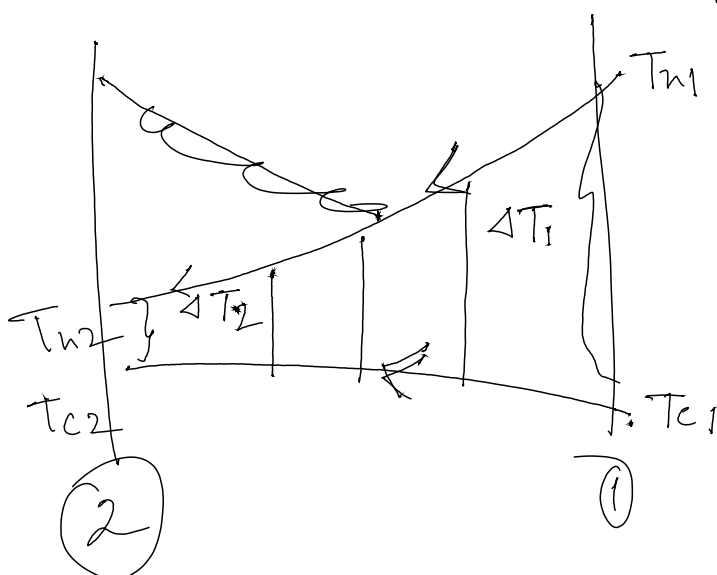
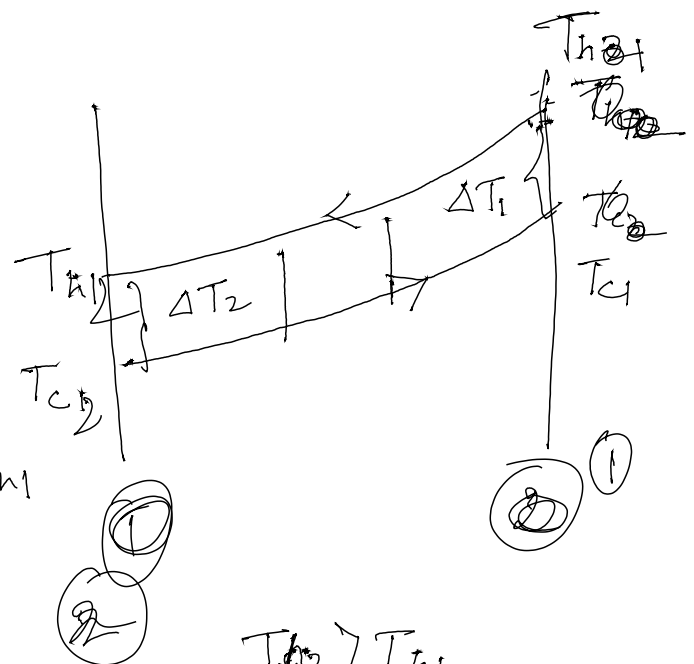
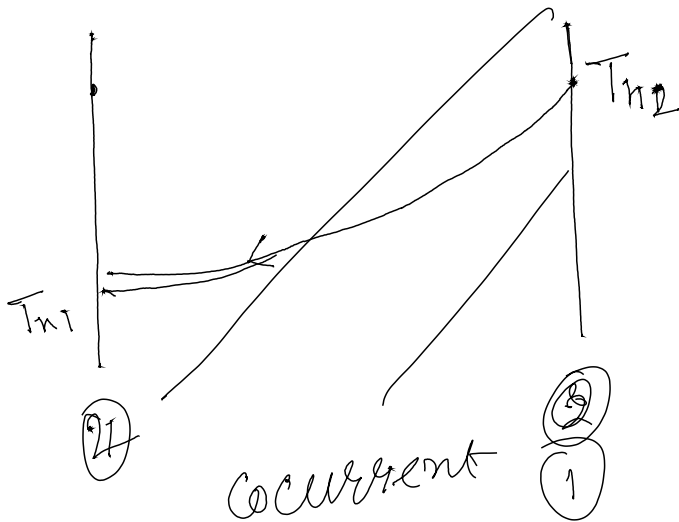
$$\Delta T_2 = T_{H2} - T_{C1}$$

$$\Delta T_m \rightarrow A_1 \Delta T_m$$

Heat transfer with a variable driving force (variable ΔT) for cocurrent and counter current operation



counter current



$$\begin{aligned} T_{c2} &> T_{h1} \\ T_{h1} &> T_{h2} \\ T_{c1} &> T_{c2} \end{aligned}$$

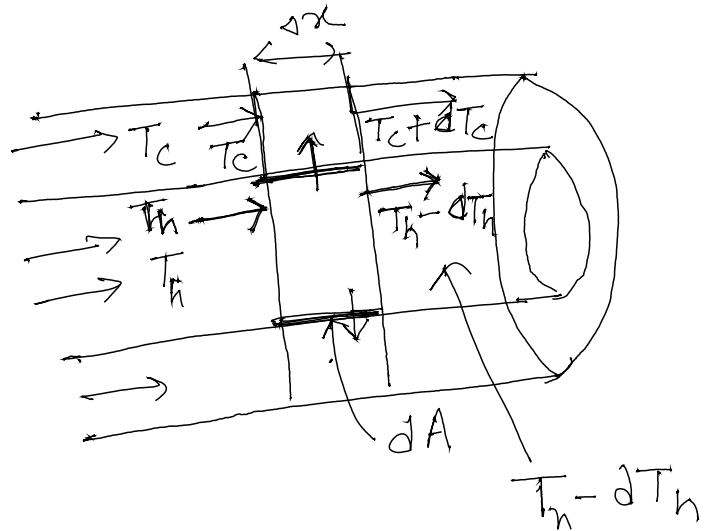
$$\begin{aligned} T_{c2} &> T_{c1} \\ T_{h1} &> T_{h2} \end{aligned}$$

$$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_m = \frac{(\Delta T_1 - \Delta T_2)}{\ln(\Delta T_1 / \Delta T_2)} = \frac{\Delta T_2 - \Delta T_1}{\ln(\frac{\Delta T_2}{\Delta T_1})}$$

$$dQ = U dA (T_h - T_c)$$

$$dQ = U dA (\Delta T)$$



$$dQ = -\dot{m}_h C_{ph} dT_h$$

$$dQ = \dot{m}_c C_{pc} dT_c$$

$$\Delta T = T_h - T_c$$

$$\Rightarrow T_h = \Delta T + T_c$$

$$dQ = -\dot{m}_h C_{ph} [d(T_c + \Delta T)]$$

$$\Rightarrow dQ = -\dot{m}_h C_{ph} dT_c - \dot{m}_h C_{ph} d(\Delta T)$$

$T_c \Rightarrow$ cold fluid temp. at diff. element
 $T_h \Rightarrow$ hot fluid temp at diff. element

$\Delta T \Rightarrow$ Temp. diff. betⁿ hot & cold fluid at diff. element.

$$\Rightarrow dQ = -\dot{m}_h C_{ph} \left[\frac{dQ}{\dot{m}_c C_{pc}} \right] - \dot{m}_h C_{ph} d(\Delta T)$$

$$\Rightarrow dQ + \dot{m}_h C_{ph} \frac{dQ}{\dot{m}_c C_{pc}} = -\dot{m}_h C_{ph} d(\Delta T)$$

$$\Rightarrow dQ \left[1 + \frac{\dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} \right] = -\dot{m}_h C_{ph} d(\Delta T)$$

$$\Rightarrow dQ \left[\frac{\dot{m}_c C_{pc} + \dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} \right] = -\dot{m}_h C_{ph} d(\Delta T)$$

$\dot{m}_c C_{pc} = a$
 $\dot{m}_h C_{ph} = b$

$$dQ = - \left(\frac{a b}{a+b} \right) d(\Delta T)$$

$$\downarrow$$

$$U dA \Delta T = - \left(\frac{a b}{a+b} \right) d(\Delta T)$$

$$U dA = - \frac{a b}{(a+b)} \frac{d(\Delta T)}{\Delta T}$$

$$\Rightarrow \int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = + \left[\frac{(a+b)}{a b} \right] \int_0^A U dA$$

At end (1) $A=0$ and $\Delta T = T_{h1} - T_{c1} = \Delta T_1$

at end (2) $A=A$ and $\Delta T = T_{h2} - T_{c2} = \Delta T_2$

$$\Rightarrow \ln \left(\frac{\Delta T_1}{\Delta T_2} \right) = \left(\frac{a+b}{a b} \right) U A$$

$$Q = \dot{m}_c C_{pc} (T_{c2} - T_{c1}) \Rightarrow \frac{Q}{\dot{m}_c C_{pc}} = T_{c2} - T_{c1} \quad (+)$$

$$Q = \dot{m}_h C_{ph} (T_{h1} - T_{h2}) \Rightarrow \frac{Q}{\dot{m}_h C_{ph}} = T_{h1} - T_{h2}$$

$$\Rightarrow Q \left[\frac{1}{\dot{m}_c C_{pc}} + \frac{1}{\dot{m}_h C_{ph}} \right] = T_{h1} - T_{h2} + T_{c2} - T_{c1}$$

$$\Rightarrow Q \left[\frac{a+b}{a b} \right] = (T_{h1} - T_{c1}) - (T_{h2} - T_{c2})$$

$$Q = \left(\frac{a \cdot b}{a+b} \right) (\Delta T_1 - \Delta T_2)$$

$$\Rightarrow Q = \left[\frac{UA}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \right] (\Delta T_1 - \Delta T_2)$$

$$\Rightarrow Q = UA \left[\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} \right]$$

$$\Rightarrow Q = UA (\Delta T_m)$$

$\Delta T_m \Rightarrow$ log mean Temperature difference.

Applicable for both concurrent & counter current operation.

✓
 T_{h1}
✓
 $\dot{m}_h C_{ph}$

✓
 T_{h2}
✓
 $\dot{m}_h C_{ph}$

✓
 T_{c1}

$T_{c2} = ?$

$$\dot{m}_h C_{ph} (T_{h1} - T_{h2}) = \dot{m}_c C_{pc} (T_{c2} - T_{c1})$$

Heat flux in Nucleate boiling

$$q_s = \mu_L L_v \left[\frac{g(P_L - P_V)}{a} \right]^{\frac{1}{2}} \left[\frac{C_{pf} T_e}{C_{sf} L_v (Pr_L)^n} \right]^3$$

μ_L = liquid viscosity

L_v = Enthalpy of vaporization of liquid

T_e = Excess temp. ($T_s - T^{sat}$)
 $\hookrightarrow$ surface temp. of wire

$(Pr_L) \Rightarrow$ Prandtl number of liquid

$\eta = 1$ for water

≈ 1.7 for organic liquid

Find rate of boiling.
 $\frac{q_s}{L_v} = 0.205 \checkmark$
 $\text{kg/m}^2 \text{s}$

C_{sf} = constant (depends on the surface characteristics)

a = surface tension

$P_L \Rightarrow$ liquid density

$P_V \Rightarrow$ vapor density

$C_{pf} \Rightarrow$ sp. heat liquid.

$a = 0.095 \text{ N/m}$
 $Pr_L = 1.76$
 $P_V = 0.5955 \text{ kg/m}^3$
 $C_{sf} = 0.013, \eta = 1$
 $q_s = ? \quad q = q_s \times \frac{\pi d^2}{4}$

critical heat flux (max heat flux)

$$q_{\max} = 0.149 L_v P_V \left[\frac{a g (P_L - P_V)}{(P_V)^2} \right]^{\frac{1}{4}}$$

$= 1259 \text{ kW/m}^2 \quad q_s = 462.5 \text{ kW/m}^2$

$Q = ? \quad Q = q \times A \quad C_{pf} = 4.22 \text{ kJ/kg}^\circ\text{C}$

$d_i = 0.35 \text{ m}$

$T_s = 115^\circ$

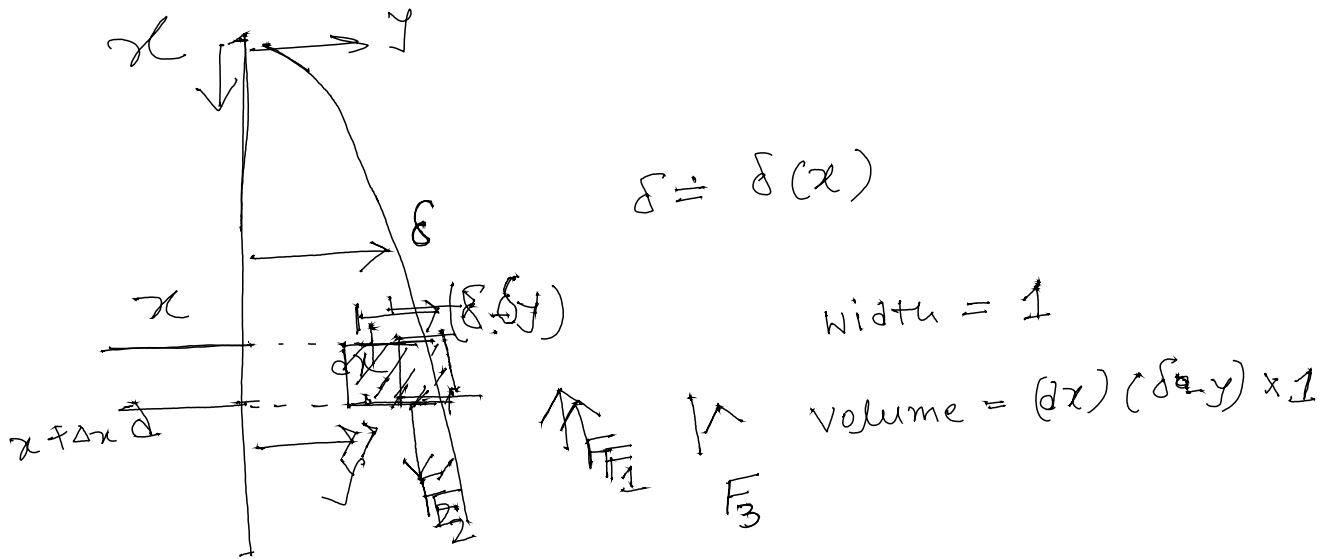
$T_e = T_s - T^{sat}$

$\mu_L = 2.79 \times 10^{-4} \text{ N/m}^2 \text{s}$

$= 115 - 100 = 15^\circ\text{C}$

$L_v = 2257 \text{ kJ/kg} \quad P_L = 958 \text{ kg/m}^3$

Condensation



Shear force = F_1

gravity force = F_2

Buoyancy force = F_3

1. Saturated vapor

2. Film condensation $\Rightarrow$ fully developed

3. Conduction is the only

heat transfer mode in the condensate film

Apply force balance on liquid element
of a differential volume $(dx)(\delta - y) \cdot 1$

$$F_1 = F_2 - F_3$$

$$\mu \frac{du}{dy} (dx)(1) = \rho_L [(\delta-y) dx \cdot 1] - \rho_V [(\delta-y) dx \cdot 1]$$

$$\Rightarrow \mu \frac{du}{dy} = (\rho_L - \rho_V) g (\delta - y)$$

$$\Rightarrow \int_0^u du = \frac{(\rho_L - \rho_V) g}{\mu} \int_{y=0}^y (\delta - y) dy$$

$$\Rightarrow u = \frac{(\rho_L - \rho_V) g}{\mu} \left[\delta y - \frac{y^2}{2} \right]_0^y$$

$$\Rightarrow u = \frac{(\rho_L - \rho_V) g}{\mu} \left[\delta y - \frac{y^2}{2} \right] \checkmark$$

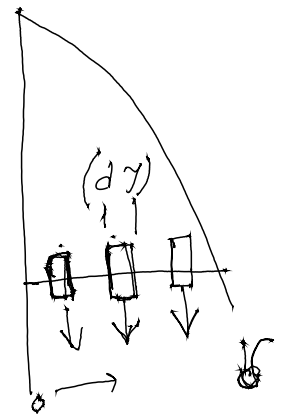
$$\text{mass flow } \dot{m} = \rho_L \int_0^\delta u(y) [(dy) 1]$$

$$\dot{m} = \rho$$

$$\dot{m} = \rho_L \int_0^\delta \frac{(\rho_L - \rho_V) g}{\mu} \left[\delta y - \frac{y^2}{2} \right] dy$$

$$= \frac{\rho_L (\rho_L - \rho_V) g}{\mu} \left[\delta \frac{y^2}{2} - \frac{y^3}{6} \right]_0^\delta$$

$$= \frac{\rho_L (\rho_L - \rho_V) g}{\mu} \frac{\delta^3}{3} = \frac{\rho_L^2 g \delta^3}{3 \mu} \left[\rho_L - \rho_V \approx \rho_L \right]$$



The rate of increase of mass flow rate
in x -direction

$$\dot{m} = \dot{m}(\delta)$$

$$\begin{aligned} \frac{d\dot{m}}{dx} &= \frac{d\dot{m}}{d\delta} \cdot \frac{d\delta}{dx} \\ &= \frac{\rho_v^2 g}{3\mu} \frac{d(\delta^3)}{d\delta} \frac{d\delta}{dx} \\ &= \frac{\rho_v^2 g}{3\mu} 3\delta^2 \frac{d\delta}{dx} \end{aligned}$$

— (a)

Energy balance

λ = latent heat of
condensation

$$K(T_v - T_w)$$

$$\frac{K(T_v - T_w)}{\delta} [dx \cdot 1] = (d\dot{m}) \lambda$$

$$\frac{d\dot{m}}{dx} = \frac{K(T_v - T_w)}{\delta \lambda} \quad \text{— (b)}$$

~~equates~~ equate (1) & (2).

$$\frac{\rho_v^2 g \delta^2}{\mu} \frac{d\delta}{dx} = \frac{K(T_v - T_w)}{\delta \lambda}$$

$$\Rightarrow \int_0^\delta \delta^3 d\delta = \frac{K(T_v - T_w) \mu}{\lambda \rho_v^2 g} \int_0^x dx$$

$$\Rightarrow \frac{\delta^4}{4} = \frac{K(T_v - T_w) \mu}{\lambda \rho_v^2 g} x$$

$$\delta = \left[\frac{4 K (T_v - T_w) \mu}{\rho^2 g l} x \right]^{1/4}$$

$$\frac{K (T_v - T_w) (dx \cdot 1)}{(\delta)} = \underline{h_x} (T_v - T_w) (dx \cdot 1)$$

conductive heat transfer

$$h_x = \frac{K}{(\delta)}$$

$$h_x = \frac{K}{\left[\frac{\rho^2 g l}{4 K (T_v - T_w) \mu} x \right]^{1/4}}$$

$$= \frac{K^3 \rho^2 g l}{4 (T_v - T_w) \mu} x^{-1/4}$$

$[h_x \propto x^{-1/4}]$

$$\bar{h} = \frac{1}{L} \int_0^L h_x dx$$

$$= \left(\frac{4}{3} \right) h_x \Big|_0^L$$

$$= \frac{1}{L} \int_0^L \left(\frac{K^3 \rho^2 g l}{4 (T_v - T_w) \mu} \right)^{1/4} x^{-1/4} dx$$

$$x^{3/4} \Rightarrow \frac{K}{x^{1/4}}$$

$$\Rightarrow \frac{L}{L^{1/4}}$$

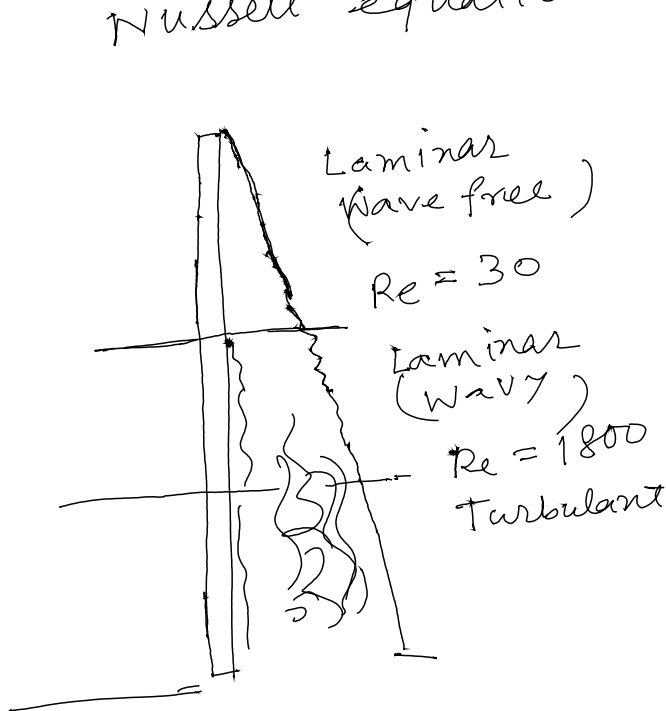
$$\frac{x^{-1/4+1}}{(-1/4+1)}$$

$$\frac{x^{3/4}}{3/4} = \frac{4}{3} \frac{x^{3/4}}{x^{1/4}}$$

=

$$\bar{h} = 0.943 \left[\frac{k^3 \rho^2 g}{\mu L (T_v - T_w)} \right]^{1/4}$$

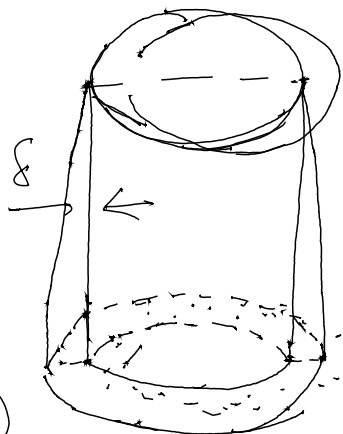
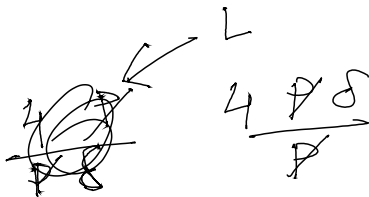
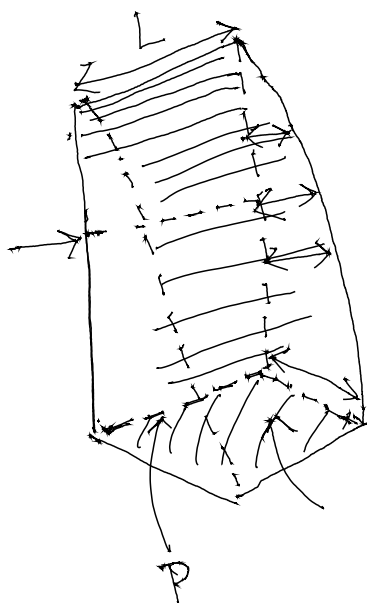
Nusselt equation for condensation



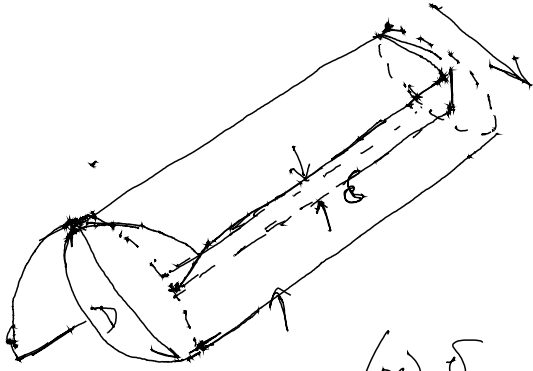
$$Re = \frac{\rho \overset{\textcircled{D_H}}{L_c} V}{\mu}$$

$D_H \Rightarrow$ hydraulic diameter

$$D_H = \frac{4 \text{ (wetted perimeter)} \times \text{Flow area}}{\text{Perimeter}} = 4\delta$$



$$D_H = \frac{4 (2\pi r \times \delta)}{(2\pi r)} = 4\delta$$



$$D_H = \frac{4 (2\pi r) \delta}{(2\pi r)} = 4\delta$$

$$Re = \frac{4 \rho V \delta}{\mu} = \frac{4 [(\rho \delta) V L]}{\mu L} = \frac{4 \dot{m}}{\rho \mu} \checkmark$$

$$Q = h A_s (T_v - T_w) = \dot{m} \Delta T \Rightarrow \dot{m} = \frac{Q}{\Delta T}$$

$$Re = \frac{4 Q}{\rho \mu \Delta T} = \frac{4 A_s (T_v - T_w) h}{\rho \mu \Delta T} = \frac{h A_s (T_v - T_w)}{\Delta T}$$

$$h = 0.943 \left[\frac{k^3 \rho^2 g \Delta T}{\mu L (T_v - T_w)} \right]^{1/4}$$

$$\begin{aligned}
 Re &= \left. \frac{4m}{(1) \mu_x} \right|_{x=L} \\
 &= 4 \cdot \frac{g (1) \rho_x^2 \delta^3}{3 \mu_x} \cdot \frac{1}{\mu_x} \Big|_{x=L} \\
 &= \frac{4}{3} \frac{g \rho_x^2 (\delta^3)}{\mu_x} \Big|_{x=L}
 \end{aligned}$$

$$\delta = \frac{K_x}{h} \Big|_{x=L}$$

$$Re|_{x=L} = \frac{4}{3} g \left(\frac{\rho_x^2}{\mu_x} \right) \left(\frac{K_x}{h} \Big|_{x=L} \right)^3$$

$$\Rightarrow \bar{h} = \frac{4}{3} h_x \Big|_{x=L}$$

$$\Rightarrow h_x \Big|_{x=L} = \frac{3}{4} \bar{h}$$

$$\Rightarrow Re|_{x=L} = \frac{4}{3} g \left(\frac{\rho_x}{\mu_x} \right)^2 \left(\frac{K_x}{\frac{3}{4} \bar{h}} \right)^3$$

$$\frac{\mu_x}{\rho_x}$$

$$\Rightarrow (Re|_{x=L})^{1/3} = \left(\frac{4}{3} \right)^{1/3} (g)^{1/3} \left(\frac{\rho_x}{\mu_x} \right)^{2/3} \frac{K_x}{\bar{h}}$$

$$\Rightarrow \bar{h} = \left(\frac{4}{3} \right)^{4/3} \left(\frac{g}{\mu_x^2} \right)^{1/3} K_x \cdot Re_L^{-1/3}$$

$$\bar{h} = 1.4675 \left(\frac{g}{\gamma_L^2} \right)^{1/3} K_L Re_L^{-1/3}$$

$$\Rightarrow \frac{\bar{h}}{K_L} \left[\frac{\gamma_L^2}{g} \right]^{1/3} = 1.4675 Re_L^{-1/3} \quad \checkmark \checkmark$$

$$\Rightarrow \boxed{Nu' = 1.4675 Re_L^{-1/3}}$$

$$\begin{aligned} Q &= h [(1) L] (T_v - T_w) \\ &= \dot{m} \lambda \end{aligned}$$

$$\dot{m} = \frac{Q}{\lambda}$$

$$\lambda' = \lambda + 0.68 C_{pL} (T^{sat} - T_b) + C_{pV} (T_v - T^{sat})$$

$$Re < 30$$

$$\rho_v \ll \rho_L$$

$$\dot{m} = \frac{Q}{\lambda'}$$

$$\begin{aligned} & T_v < T^{sat} \\ & \lambda' = \lambda + C_{pV} (T_v - T^{sat}) \\ & \text{If } T_v \neq T^{sat} \\ & T \end{aligned}$$

Wavy laminar

$T_s = T_w = \text{Surface}^{\text{plate}} \text{ temp.}$

Kutateladze (1963)

$$\bar{h} = \frac{Re_L k_L}{1.08 Re_L^{1.22} - 5.2} \left(\frac{g}{\nu_L^2} \right)^{1/3} \quad 30 < Re < 1800$$

$$\bar{h}_{\text{Wavy laminar}} = 0.8 Re_L^{0.11} \bar{h}_{\text{laminar}}$$

T_{sat} if vapor available at saturation 0.82

$$Re = \left[4.81 + \frac{3.7 L k_L (T_v - T_w)}{\mu_L} \left(\frac{g}{\nu_L^2} \right)^{1/3} \right]$$

or μ_L' (modified)

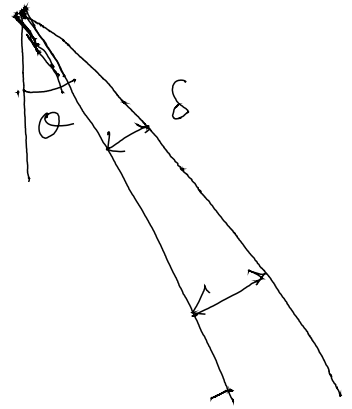
Turbulent flow

$$\bar{h} = \frac{Re_L k_L}{8750 + 58 Pr^{0.5} (Re^{0.75} - 253)} \left(\frac{g}{\nu_L^2} \right)^{1/3} \quad Re > 1800$$

$$Re = \left[\frac{0.0692 k_L Pr^{0.5} (T_v - T_w)}{\mu_L} \left(\frac{g}{\nu_L^2} \right)^{1/3} - 151 Pr^{0.5} + 253 \right]^{4/3}$$

Inclined plate

$$\bar{h}_{inc} = \bar{h}_{ver} \times (\cos \theta)^{1/4}$$



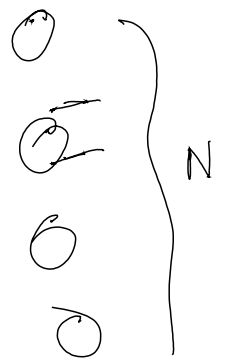
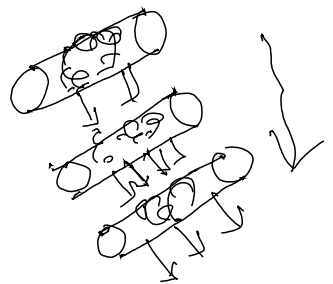
Horizontal tubes

$$\bar{h} = 0.729 \left[\frac{g \rho_L^2 \Delta K_L^3}{\mu_L (T_v - T_w) D} \right]^{1/4}$$

$$Q = \bar{h} (A)_{1, tube} (T_v - T_w)$$

Vertical tube

$$\frac{\bar{h}_{ver}}{\bar{h}_{horizontal}} = 1.29 \left(\frac{D}{L} \right)^{1/4}$$



$$\text{If } L = 1.29^4 D = 2.77 D$$

$$\text{then } \bar{h}_{ver} = \bar{h}_{horizontal}$$

Horizontal tube banks

$$\bar{h} = 0.729 \left[\frac{g \rho_L^2 \Delta K_L^3}{\mu_L (T_v - T_w) N D} \right]^{1/4}$$

$$= \frac{1}{N^{1/4}} \cdot \bar{h}_{horizontal, (1 tube)}$$

$$= \frac{1}{N^{1/4}} \bar{h} (A) (T_v - T_w)$$

$$Q = \bar{h} (A) (T_v - T_w) N_{tube}$$

condensation

$$T_v = T_{sat}$$

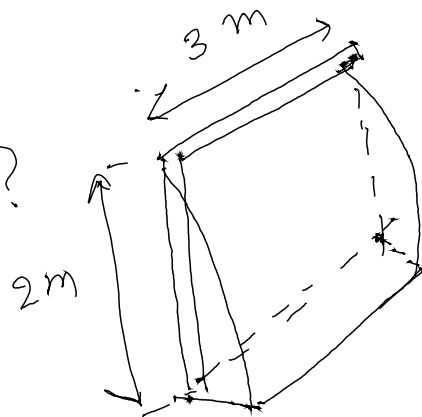
$$L = 2 \text{ m}$$

$$b = 3 \text{ m}$$

$$T_w = 80^\circ \text{C}$$

$$Q = ?$$

$$\dot{m} = ?$$



water at $T_{sat} (100^\circ \text{C})$

$$\lambda = \frac{2257 \times 10^3 \text{ J/kg}}{1}$$

$$\rho_v = 0.6 \text{ kg/m}^3$$

$$T_f = \text{Film temp} = \frac{T_{sat} + T_w}{2} = 90^\circ \text{C}$$

Liquid properties at T_f

$$\rho_l = 965.3 \text{ kg/m}^3$$

$$\mu_l = 0.315 \times 10^{-3} \text{ kg/ms}$$

$$C_{p,l} = 4206 \text{ J/kg K}$$

$$K_l = 0.675 \text{ W/m K}$$

$$v_l = \frac{\mu_l}{\rho_l} =$$

$$\lambda' = \frac{\lambda + 0.68 C_{p,l} (T_s - T_v)}{1} = 2314 \times 10^3 \text{ J/kg} + C_{p,v} (T_v - T_{sat})$$

$$Q = \bar{h} A (T_{sat} - T_w)$$

$$Re = \frac{4g}{3 v_l^2} \left(\frac{K_l}{\bar{h} \frac{3}{4}} \right)^3 \checkmark =$$

$$\bar{h} = 0.943 \left[\frac{g \rho_l \lambda' K_l^3}{\mu_l (T_{sat} - T_w) L} \right]^{\frac{1}{4}}$$

$$= 7944.65 (44.95)$$

30	1800
laminar	Wavy laminar
	Turbulent

$$Re > 30$$

$$Re = \left[4.81 + \frac{3.7 L k_f (T_{sat} - T_w)}{\mu_f d'} \left(\frac{g}{\gamma_f} \right)^{1/3} \right]^{0.82}$$

$$= 1287$$

$$\bar{h} = \frac{Re k_f}{1.08 Re^{1.22} - 5.2} \left(\frac{g}{\gamma_f} \right)^{1/3}$$

$$= 5850 \text{ W/m}^2 \text{ K}$$

$$Q = h A (T_{sat} - T_w)$$

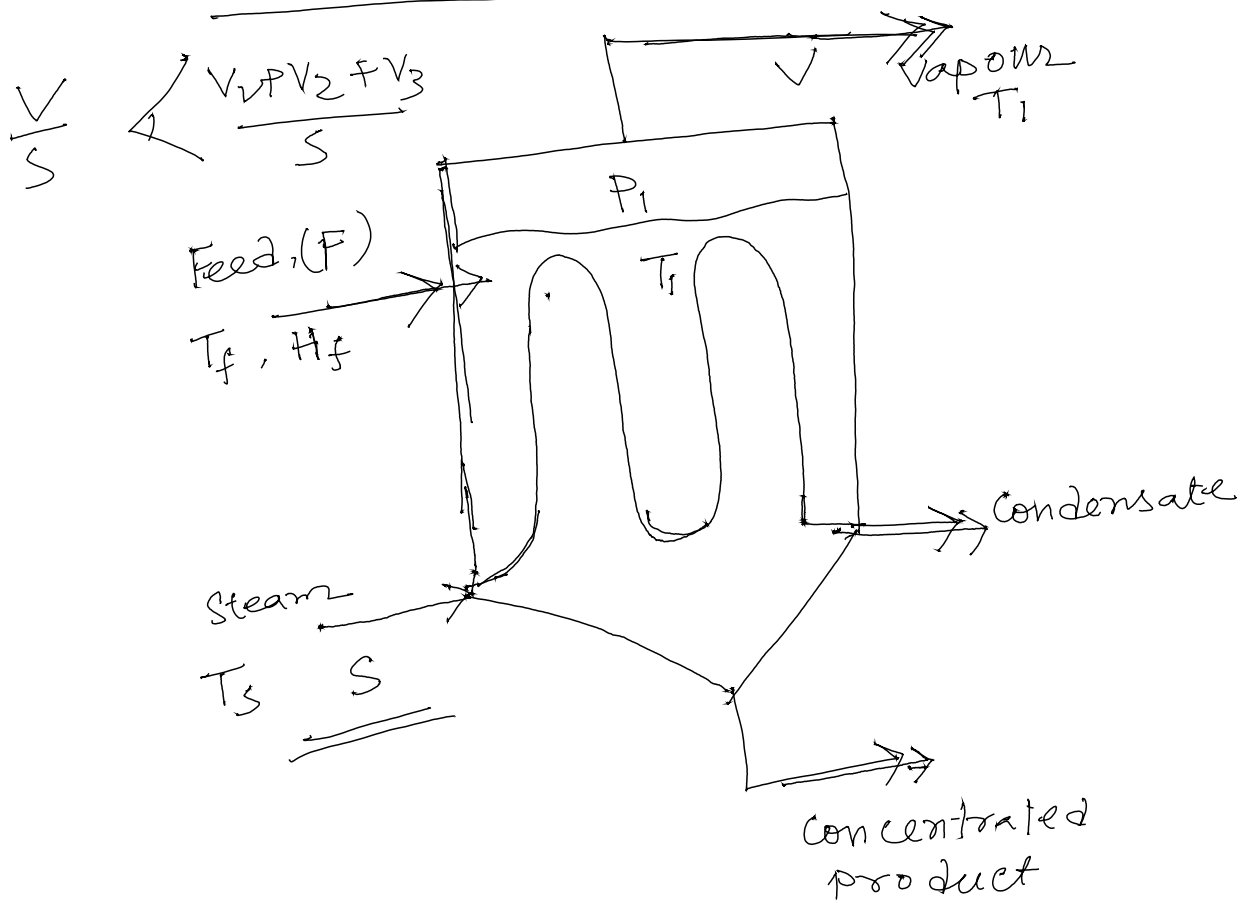
$$= 5850 \times 6 \times (100 - 80)$$

$$= 7.02 \times 10^5 \text{ W}$$

$$Q = \dot{m} h' \Rightarrow \dot{m} = \frac{Q}{h'} = \frac{7.02 \times 10^5}{2314 \times 10^3}$$

$$\dot{m} = 0.303 \text{ kg/s}$$

Evaporation and Evaporators

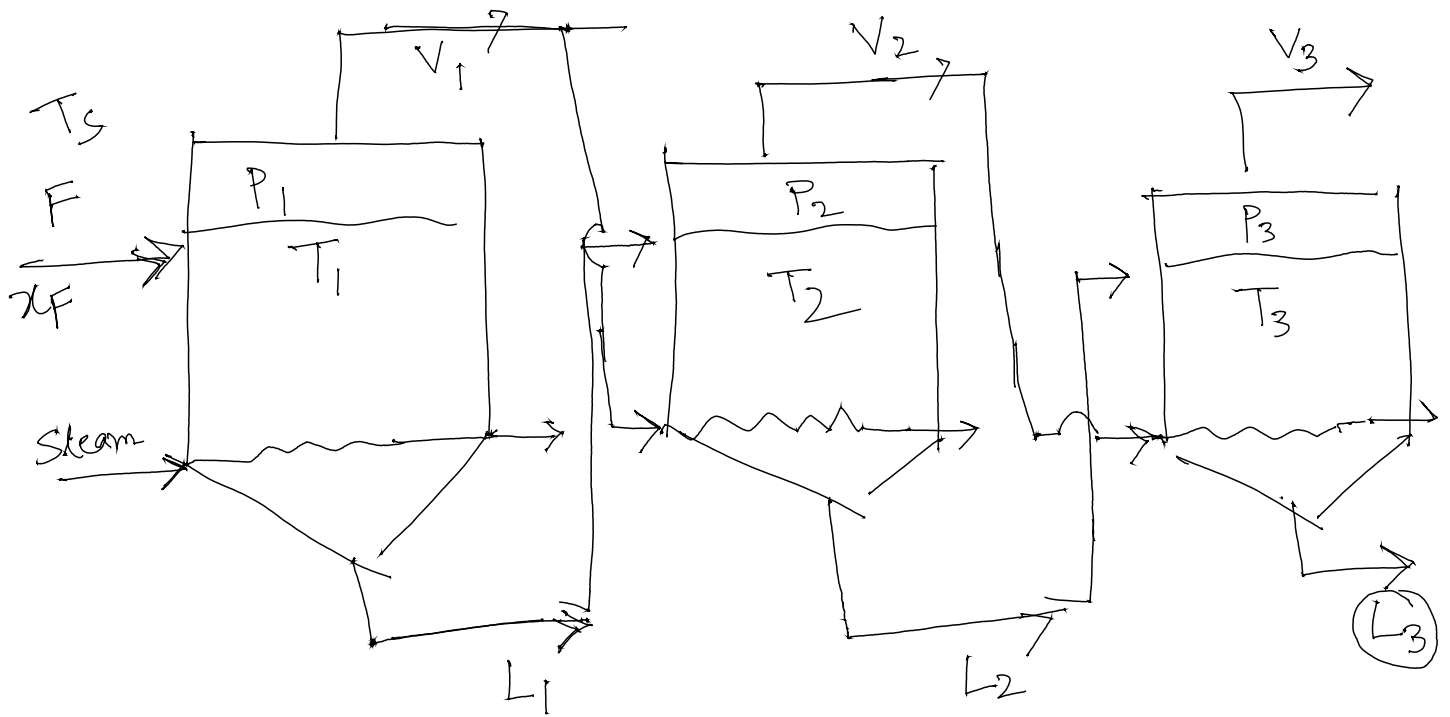


In chemical Engineering terminology evaporation means removal of a part of the solvent from a non-volatile solute by vaporization.

- The objective of vaporization is to concentrate the solution.

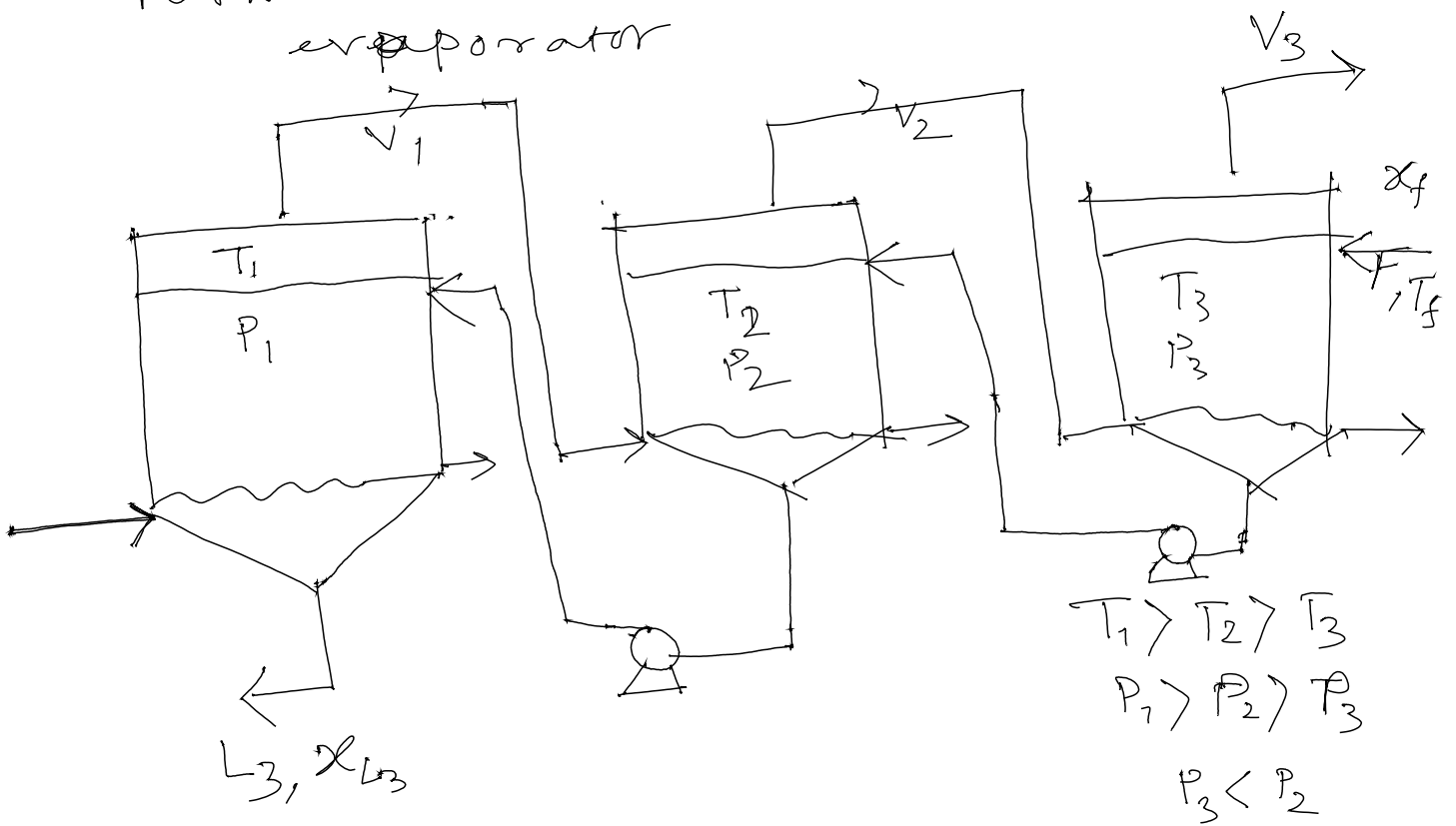
- Examples: concentration of cane sugar juice,

Single effect



$$P_1 > P_2 > P_3$$

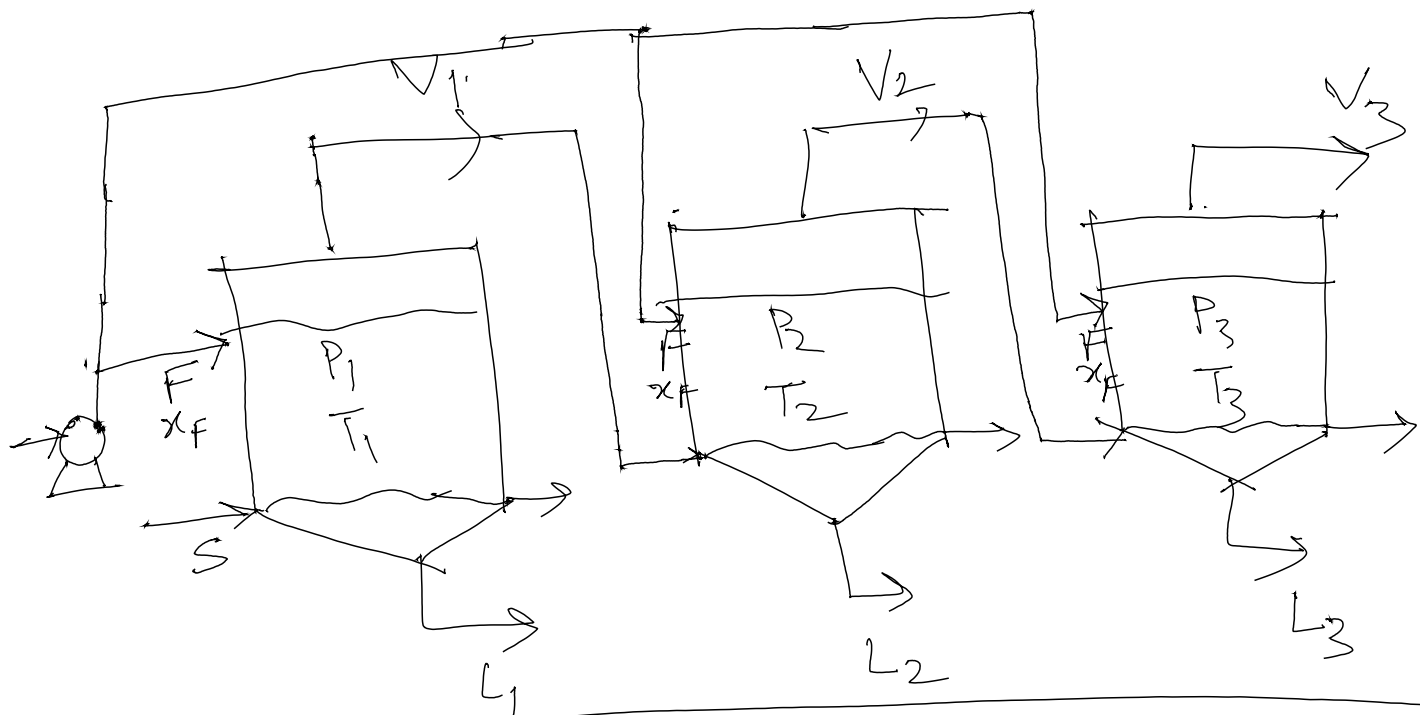
Forward feed multiple effect evaporator



$$T_1 > T_2 > T_3$$

$$P_1 > P_2 > P_3$$

$$P_3 < P_2$$



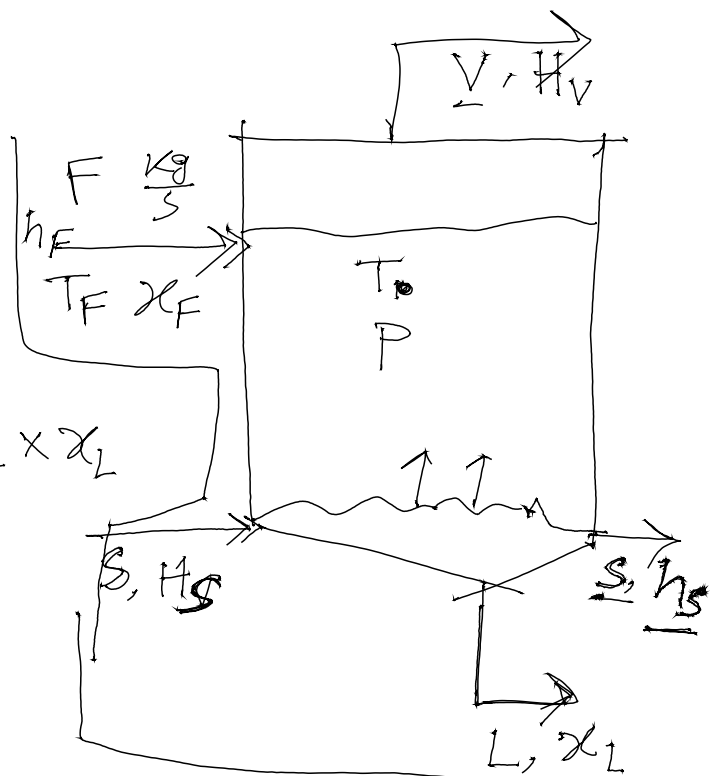
Material balance

$$F = L + V \quad \checkmark$$

Solute balance

$$F x_F = V x_V + L x_L$$

$$\Rightarrow F x_F = L x_L$$



Energy balance

$$F h_F + S H_S = L h_L + V H_V + S h_S$$

$$\Rightarrow F h_F + S (H_S - h_S) = L h_L + V H_V$$

$$\Rightarrow F h_F + S \Delta T = L h_L + V H_V$$

$$Q = U A (T_S - T) = S \Delta T$$

↗ area of heat transfer

Problem of BPE

$$F = 4536 \text{ kg/h} \checkmark$$

$$x_F = 20\% \checkmark$$

(solution of NaOH in water)

$$x_L = 50\% \checkmark$$

Saturated steam pressure

$$172.4 \text{ kPa} \rightleftharpoons T_s = 115.6^\circ\text{C}$$

$$U = 1560 \text{ W/m}^2\text{K}$$

$$S = ? \checkmark$$

$$\text{steam economy} = ? \checkmark$$

$$A = ?$$

$$\text{Water BP at } 11.7 \text{ kPa} \Rightarrow 48.9^\circ\text{C} \checkmark$$

use Dühring's chart BP for 50% conc. solⁿ ??
89.5°C ✓

Data from conc. vs. Enthalpy chart

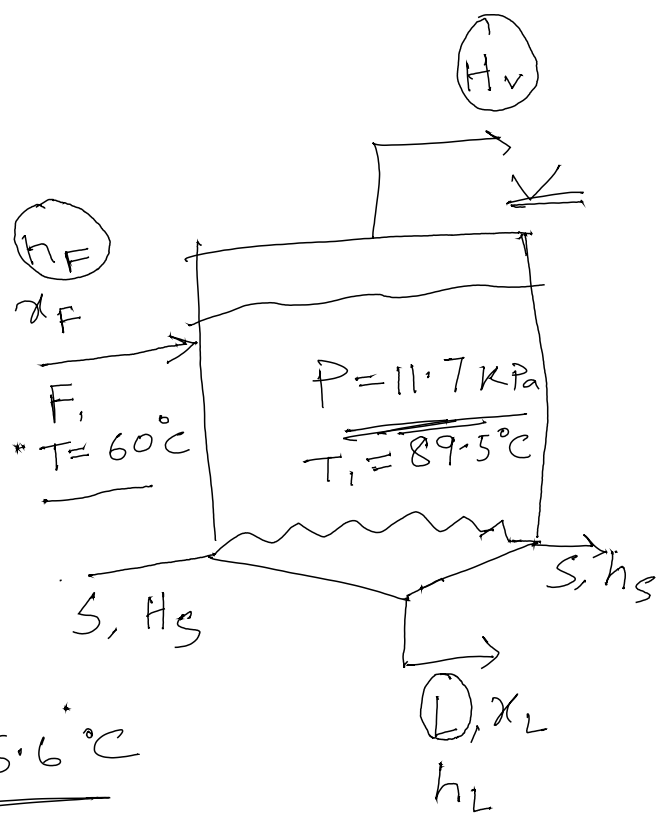
Enthalpy of 20% concentrated NaOH solⁿ at 60°C,

$$h_F = 214 \text{ kJ/kg of sol}^n \checkmark \checkmark$$

Enthalpy of 50% concentrated NaOH solⁿ at 60°C

$$89.5^\circ\text{C} \quad h_L = 505 \text{ kJ/kg}$$

Steam table $\Rightarrow 89.5^\circ\text{C}$ & 11.7 kPa, Enthalpy of superheated vapor, $H_v = 2667 \text{ kJ/kg}$ ✓



$$\text{latent heat of steam at } 172.4 \text{ kPa} = 2214 \text{ kJ/kg} \checkmark$$

$$F = L + V = 4536$$

$$F x_F = L x_L$$

$$\Rightarrow 4536 \times 0.2 = L \times 0.5$$

$$\Rightarrow \underline{L = 1814.4 \text{ kg/h}}$$

$$V = F - L$$

$$= 2722 \text{ kg/h}$$

$$\underline{\underline{2721.6 \text{ kg/h.}}}$$

Energy balance $\rightarrow (H_s = h_s)$

$$F h_F + S 1 = L h_L + V H_V$$

$$\Rightarrow \checkmark \quad \underline{4535} \times 214 + S (2214) = \underline{1814.4} \times 505 + \underline{2722} \times 2667 \checkmark$$

$$\Rightarrow \underline{\underline{S = 3255 \text{ kg steam/hr.}}} \checkmark \checkmark *$$

$$Q = \underline{S 1} = \underline{\underline{2002 \text{ KW}}} \checkmark \checkmark$$

$$Q = \underline{U A} (T_s - T) \rightarrow 115.6^\circ\text{C}$$

$\downarrow$ Evaporator temperature $\rightarrow 89.5^\circ\text{C}$

$$\Rightarrow \underline{\underline{A = 49.2 \text{ m}^2}} \checkmark$$

$$\text{Steam Economy} = \frac{V}{S} = \frac{2722}{3255} = \underline{\underline{0.836}}$$

Radiation heat transfer

Radiation heat transfer is the transfer of heat by electro-magnetic radiation.

$$\lambda = 0.1 \text{ to } 100 \mu\text{m} \quad \checkmark$$

↳ Thermal radiation.

$$\lambda = \frac{c}{\nu}$$

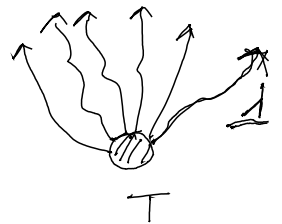
$c \rightarrow$ speed of light
 $\nu \rightarrow$ wave frequency

$$E = h \nu$$

$h \rightarrow$ plank constant.

The energy radiated at any wave length is known as monochromatic emissive power.

As temperature increases a large fraction of radiation is emitted at lower wave length.



Solar radiation $\Rightarrow$ most of the energy emitted at wave length ~~less~~ below $3 \mu\text{m}$

Black body

$$\Rightarrow F = L + V$$

$$F x_F = L x_L$$

$$F h_F + S \lambda = L h_L + V H_v$$

$$h_F = C_{pF} (T_f - T)$$

$$h_L = C_{pL} (T - T) = 0$$

$$\Rightarrow F C_{pF} (T_f - T) + S \lambda = L h_L + V H_v$$

Evaporator & temp. and concentrated product temp. are same.
for dilute soln.

$$F = 9072 \text{ kg/h}$$

$$T_F = 311 \text{ K}$$

$$x_F = 1 \% \text{ wt Salt}$$

$$x_L = 1.5 \text{ wt \%}$$

$$U = 1704 \text{ W/m}^2 \text{K}$$

$$P_1 = 101.325 \text{ kPa}$$

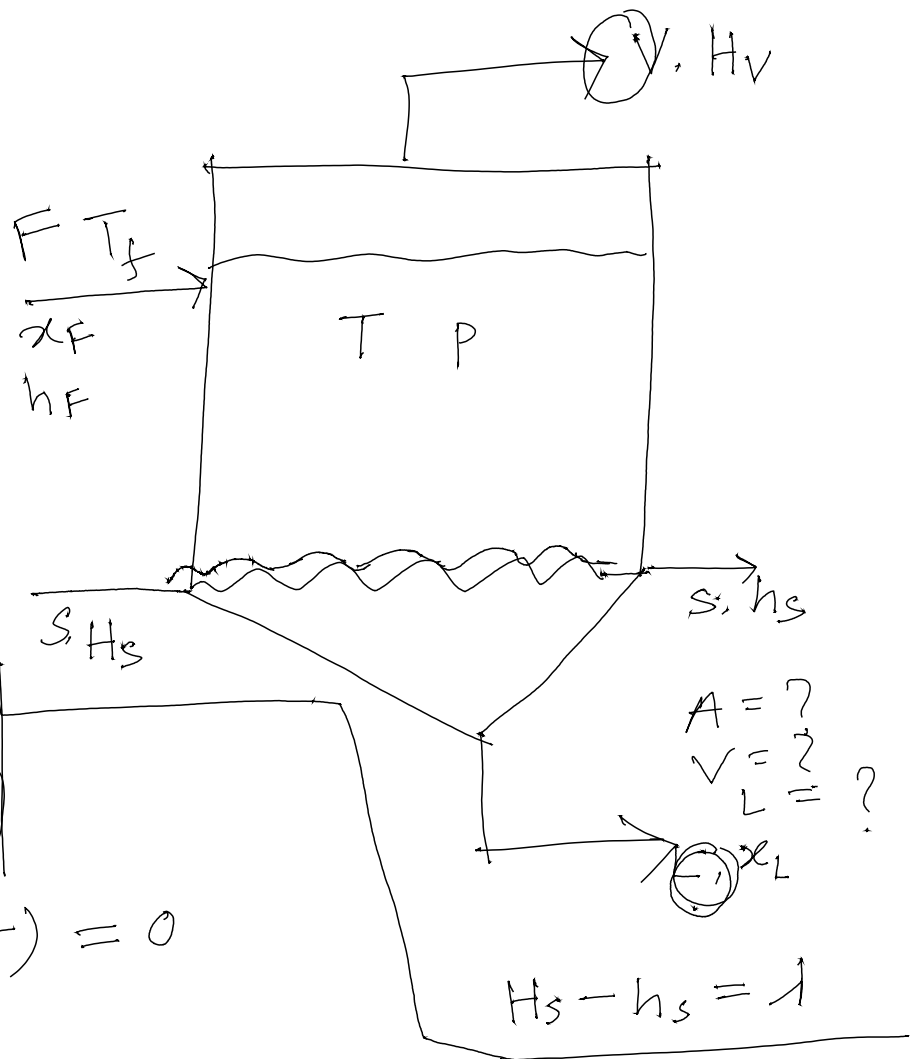
$$C_{pF} = 4.14 \text{ kJ/kg K}$$

$$\text{Saturated Steam supplied at } 143.375 \text{ kPa}$$

$$\text{Latent heat of water at } 373.2 \text{ K}$$

$$H_v = 2257 \text{ kJ/kg} \checkmark$$

$$\text{Latent heat of steam } \lambda = 2230 \text{ kJ/kg}$$



$$F = 9072 \text{ kg/h.}$$

$$x_F = 0.01$$

$$9072 = L + V$$

$$x_L = 0.015$$

$$F x_F = L x_L$$

$$\Rightarrow 9072 \times 0.01 = L \times 0.015$$

$$\Rightarrow L = 6048 \text{ kg/h}$$

$$V = F - L = 3024 \text{ kg/h}$$

$$F C_{PF} (T_F - T) + S \lambda = \frac{L C_{PL} (T - T)}{+ 3024 \times 2257}$$

$$\Rightarrow 9072 \times 4.14 (311 - 373.2) + S (2230) = 3024 \times 2257$$

$$\Rightarrow S = 4108 \text{ kg steam/h.}$$

$$Q = S \lambda = 4108 \times 2230 \text{ kJ/hr}$$

$$= 2544 \text{ kW}$$

$$= 2544 \times 10^3 \text{ W}$$

$$Q = U A (T_s - T)$$

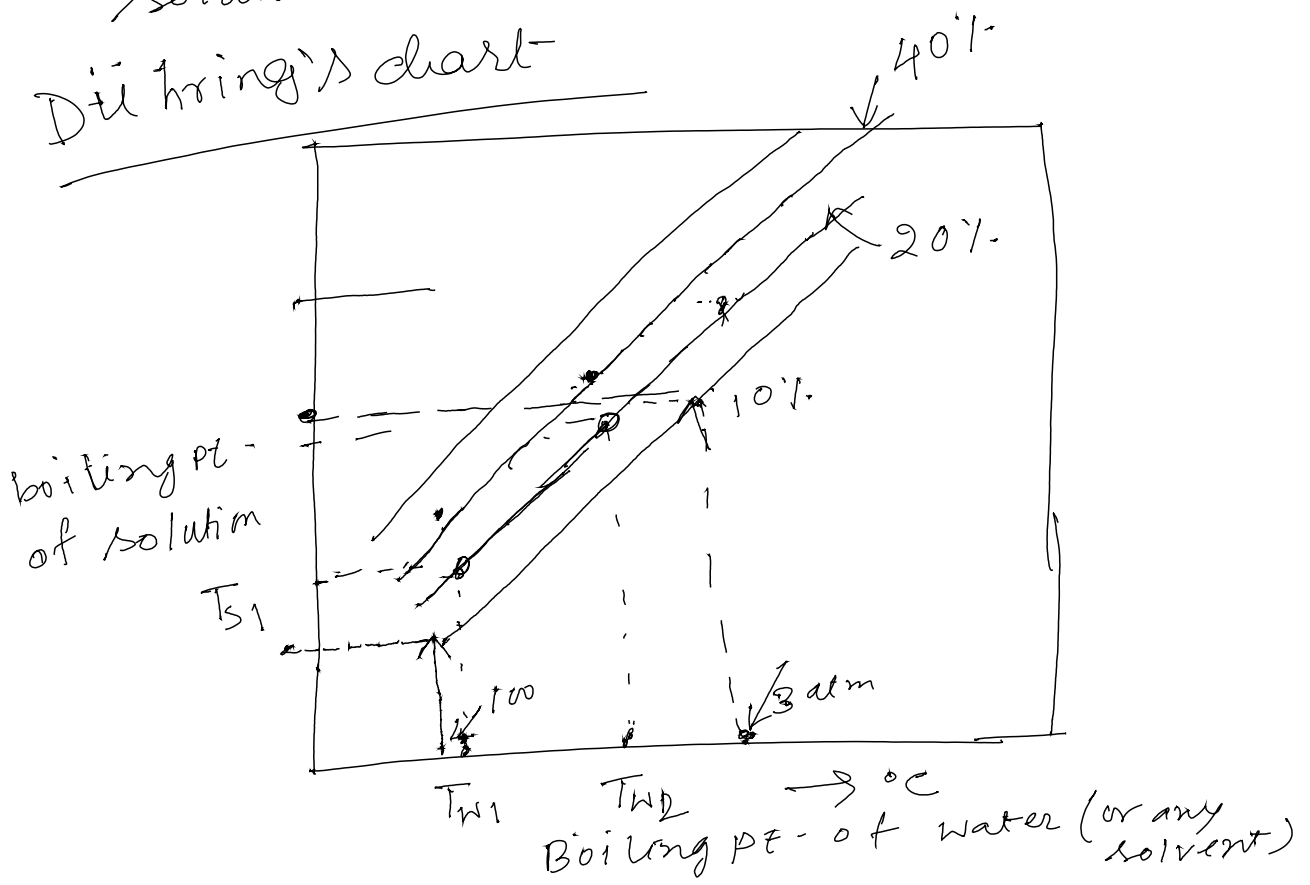
$$A = \frac{Q}{U (383.2 - 373.2)} = 149.3 \text{ m}^2$$

$$\text{Steam economy} = \frac{V}{S} = \frac{3024}{4108} = 0.736$$

$$\text{capacity} = \text{kg of vap. evaporated/time} = V$$

T_b concentrated solution $>$ T_b pure solvent

Dühring's chart



20% concentrated soln.

1 atm.

boiling point pure water = 100°C . (T_{w1})
 " " of soln = $100 + (\Delta T) = (T_s)_1$

2 atm

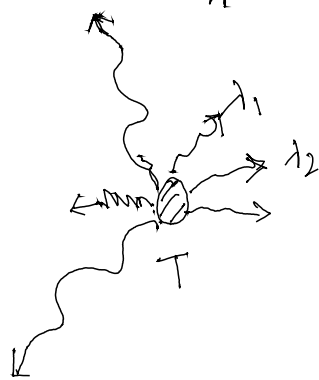
$T_{w2} > T_{w1}$

boiling point of water = $(T_{w})_2$
 " " solution = T_{s2} ($T_{s2} > T_{s1}$)

Thermal Radiation

$$T > 0 \text{ K}$$

$$\lambda \Rightarrow 0.1 \text{ to } 100 \text{ } \mu\text{m}$$



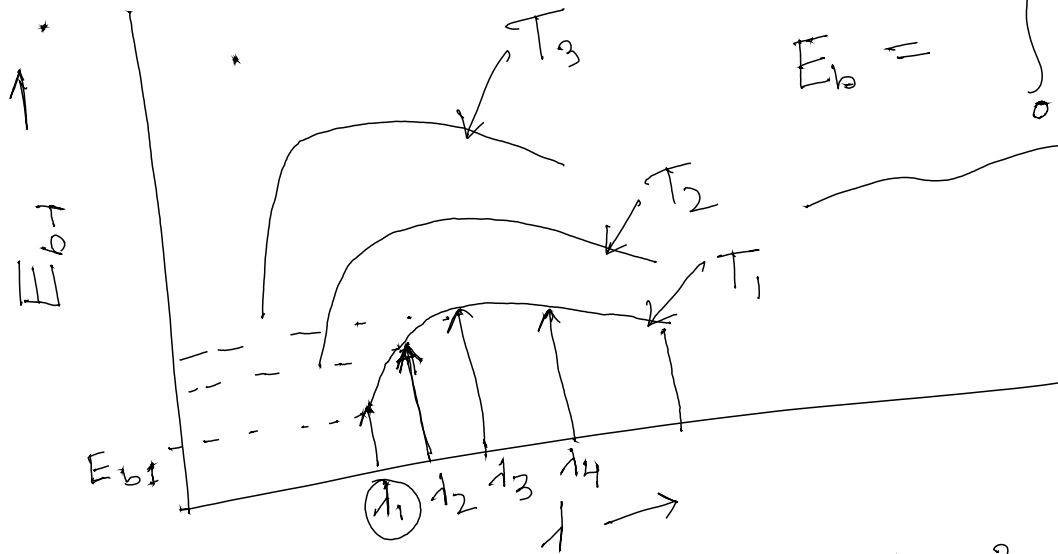
Electromagnetic wave $\rightarrow c \Rightarrow$ speed of light
 $\lambda = \frac{c}{\nu}$
 $\rightarrow$ frequency

Particle like photon

$$E = h \nu$$

$\rightarrow$ Planck's constant
 $= 6.625 \times 10^{-34} \text{ J/s}$

Speed of light in vacuum
 $c = 3 \times 10^8 \text{ m/s}$



$$E_b = \int_0^{\infty} E_{b\lambda}(T) d\lambda$$

The $\lambda \Rightarrow 0.1 \text{ } 0.2 \text{ } 0.3 \text{ } 0.5 \text{ } 1 \text{ } 2 \text{ } 5 \text{ } 10 \text{ } 50 \text{ } 100$
 Energy radiation $\Rightarrow$ Emissive power (total)

$$E_b \Rightarrow \text{W/m}^2$$

$$E_{b\lambda} \Rightarrow \text{W/m}^2 \cdot \text{m}$$

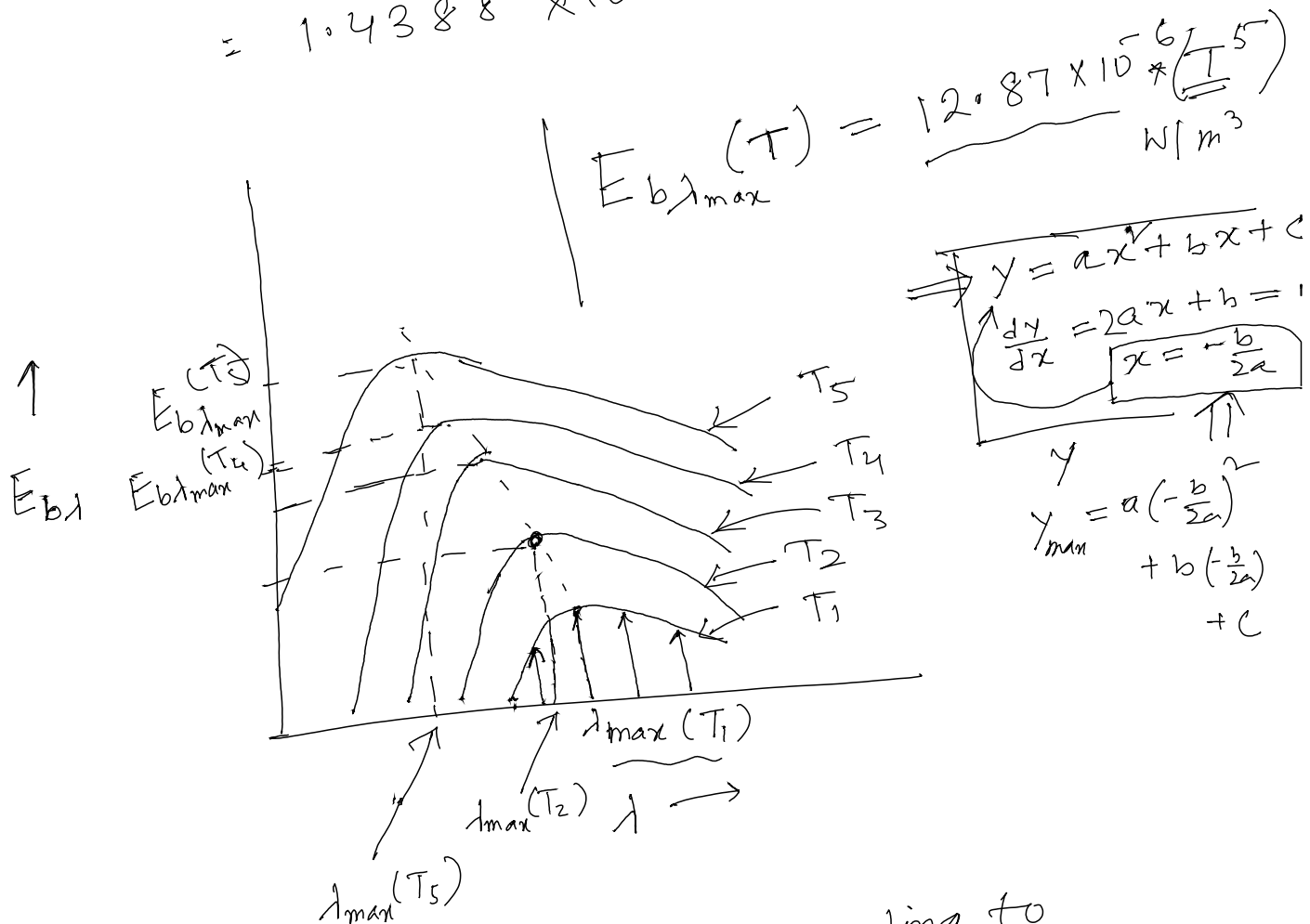
Monochromatic emissive power.

$$E_{b\lambda}(T) = \frac{C_1}{\lambda^5 [e^{C_2/\lambda T} - 1]} \Rightarrow \frac{dE_{b\lambda}(T)}{d\lambda} = 0$$

λ_{max}

$C_1 = \text{First radiation constant.}$
 $= 3.7415 \times 10^{-16} \text{ W m}^2$

$C_2 = \text{Second radiation const.}$
 $= 1.4388 \times 10^{-2} \text{ mK}$



Maximum wavelength corresponding to maximum $E_{b\lambda}$ is smaller for higher temp.

$\lambda_{max} T = \text{const.} = 2.898 \times 10^{-3} \text{ mK}$

Wien's Displacement Law.

$\lambda_{max} \downarrow T \uparrow$

$\lambda_{max} \text{ for } T = 500 \text{ K}$

$$E_b(T) = \int_0^{\infty} E_{b\lambda} d\lambda = \int_0^{\infty} \frac{C_1}{\lambda^5 [e^{\frac{C_2}{\lambda T}} - 1]} d\lambda$$

Total emissive power

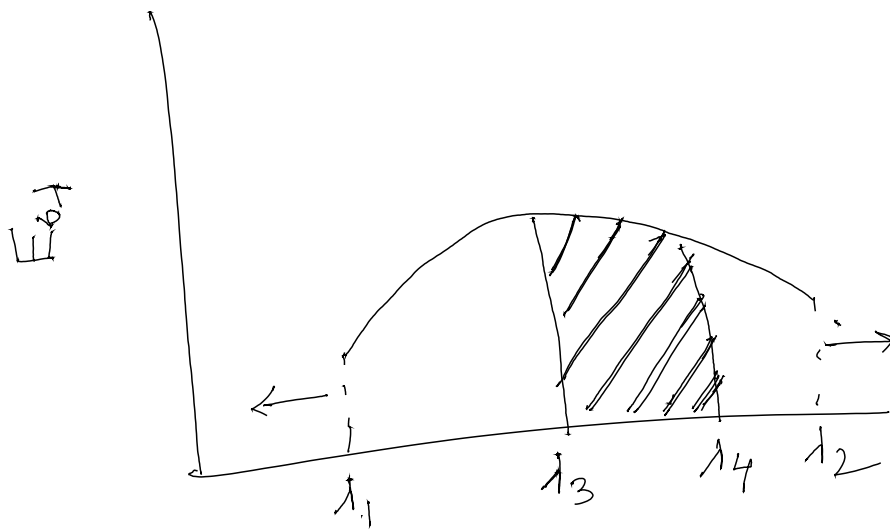
$$E_b(T) = \sigma T^4$$

Stefan Boltzmann const.

Black body : Absorb 100% radiation
emit maximum radiation.

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}$$

10-50



$$E_b(\lambda_3 \rightarrow \lambda_4) = \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda$$

$E_{b\lambda}$

$$\Rightarrow \frac{E_b(\lambda_3 \rightarrow \lambda_4)}{(\sigma T^4)} = \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{\sigma T^4} = \text{fraction of emissive power radiates in wave length range } \lambda_3 \rightarrow \lambda_4$$

Real Surface

⇒ Real body radiate less energy compared to black body at the same temperature.

⇒ The surface do not have regular spectral distribution (discrete / not continuous)

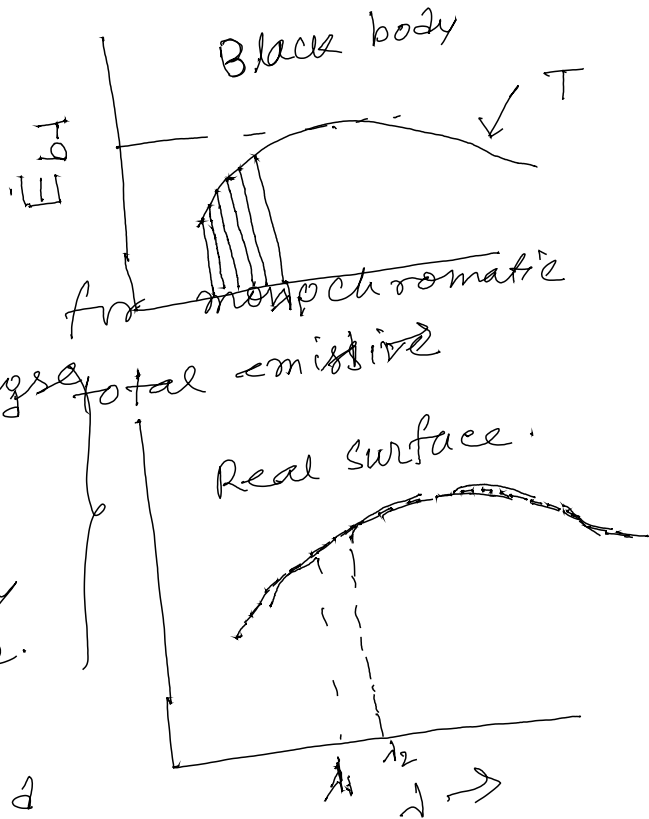
A gray surface is one whose monochromatic emissive power at all wavelength is same fraction of the black body monochromatic emissive power.

Real surface is approximated by a gray surface

The ratio $\frac{E_{\lambda}}{E_{b\lambda}} = \frac{E}{E_b} = \epsilon$ is known as emissivity.

$\epsilon(T) \Rightarrow$ is a function of temp.

but for simplicity we take average emissivity for a surface for all temp.



black λ_1	$\rightarrow$	$\frac{100}{70}$	T
Gray λ_1	$\rightarrow$	70	
black λ_2	$\rightarrow$	50	}
Gray λ_2	$\rightarrow$	35	

black λ_3	$\rightarrow$	10	}
gray λ_3	$\rightarrow$	7	

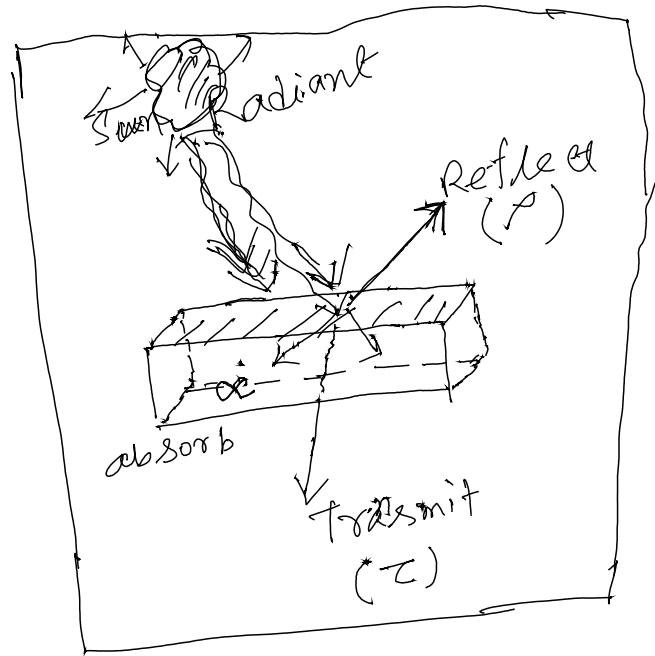
Radiation properties
 $\alpha + \tau + \rho = 1$

black body

No Reflection $\Rightarrow \rho = 0$

No transmission $\Rightarrow \tau = 0$

$$\alpha = 1$$

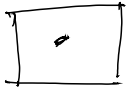


opaque surface

$$\tau = 0$$

$$\alpha + \rho = 1$$

α for monochromatic and total emissive power are same.



Kirchhoff's law

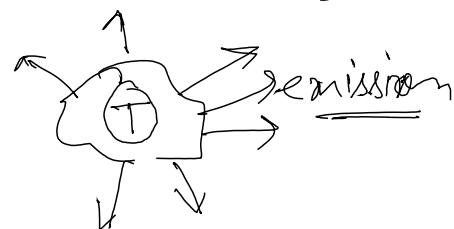
$$\alpha = \epsilon \text{ for } (1, T)$$

The absorptivity of a surface at a temp. and wave length is equal to the emissivity of the surface at the same temp. and Wave length.

*** Emissivity (power) depends on the surface temp. and on source temp. and absorptivity (power) depends on source temp.

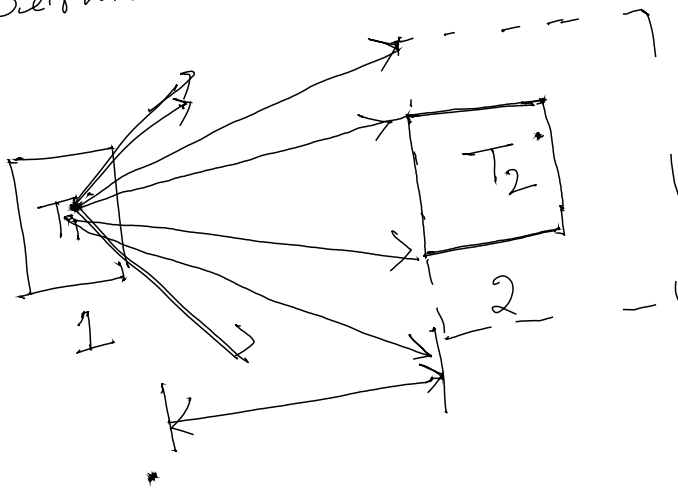
$$E_b = \alpha T^4$$

$$\left. \begin{array}{l} \text{Emissive power} \end{array} \right\} E = \epsilon \alpha T^4$$



Heat exchange by radiation

Heat transfer occurs between two body when a temperature diff. exist.



$$E_{b1} = \sigma T_1^4 \quad \frac{W}{m^2}$$

$$Q_{b1} = A_1 \sigma T_1^4 \quad W$$

$$Q_{1 \rightarrow 2} = F_{1-2} A_1 \sigma T_1^4$$

0

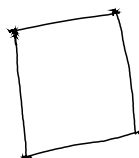
Radiation from a body travels in all direction and a fraction of the radiation reaches to another body



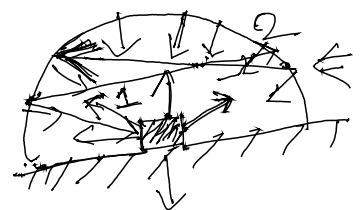
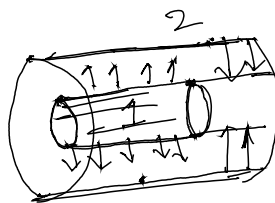
Shape factor

Shape factor of surface 1 with respect to surface 2 is defined as fraction of radiant energy emission from surface 1 reaching the surface 2.

F_{1-2}



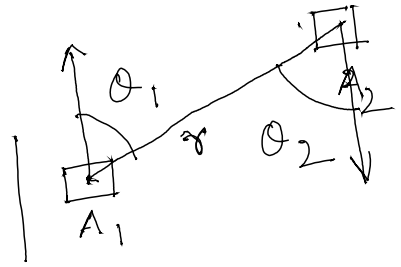
$F_{1-1} = 0$



$F_{1-2} = 1$

$F_{2-1} = F_{2-2} \neq 0$

$$F_{1-2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2 dA_1 dA_2}{\pi r^2}$$



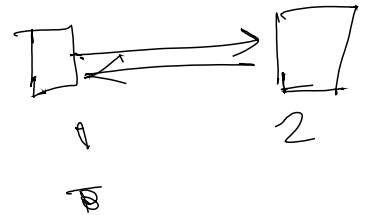
Reciprocity Relation

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$T_1 > T_2$$

$$Q = Q_{1-2} - Q_{2-1}$$

$$= A_1 F_{1-2} E_{b1} - A_2 F_{2-1} E_{b2}$$



At equilibrium $Q = 0$ &

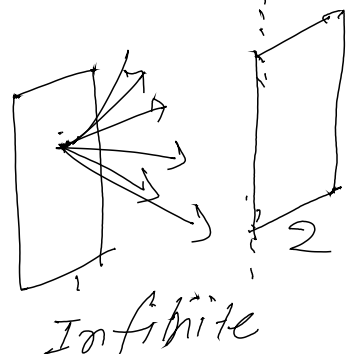
$$\underline{\underline{E_{b1} = E_{b2}}}$$

$$\Rightarrow A_1 F_{12} E_{b1} = A_2 F_{21} E_{b2}$$

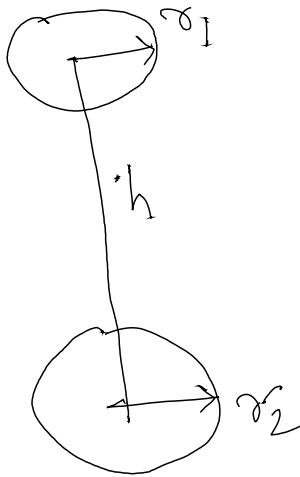
$$\Rightarrow \boxed{A_1 F_{12} = A_2 F_{21}} \checkmark \checkmark$$

$\Rightarrow$ Shape factor is equal to 1 for any surface completely enclosed by another surface

$\Rightarrow$ Shape factor is 1 for infinite parallel planes.



$\Rightarrow$



$$F_{1-2} = \frac{1}{2} \left[X - \sqrt{X^2 - 4 \left(\frac{R_2}{R_1} \right)^2} \right]$$

$$X = 1 + \frac{1 + R_2^2}{R_1^2}$$

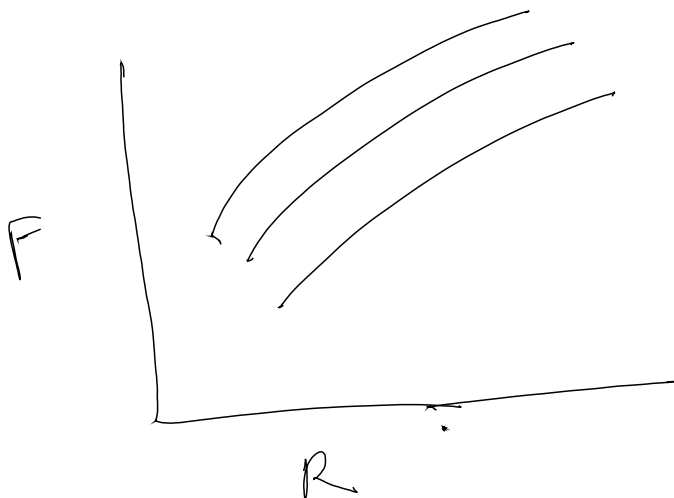
$$R_1 = \frac{r_1}{h}, \quad R_2 = \frac{r_2}{h}$$

$$0.67 = Z$$

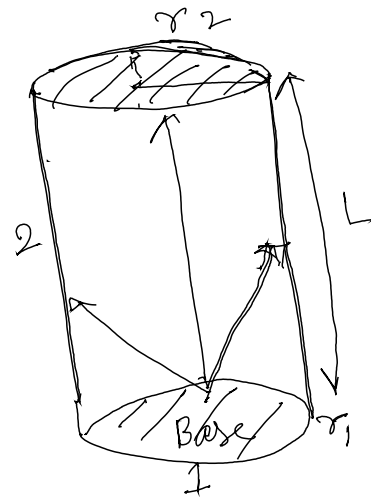
$$1.33 = Y$$

$$F_{1-2} = 0.14$$

$$F_{12} + F_{13} = 1$$



$$R = \frac{r_2}{r_1} = 1$$



$$(2\pi r L) \text{ (2)}$$

$$\pi r^2 \text{ (1)}$$

$$F_{1-2} \checkmark$$

$$F_{2-1} = \checkmark$$

$$F_{2-2} \neq 0 \checkmark$$

$$F_{1-1} = 0$$

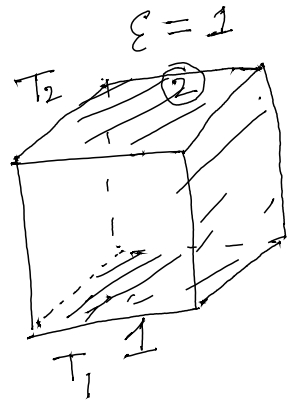
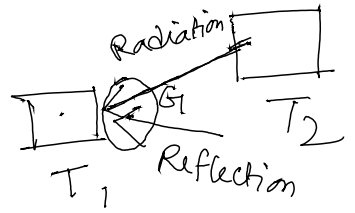
Heat exchange by radiation between surfaces

Black

$$Q = A_1 F_{12} (E_{b1} - E_{b2})$$

$$Q = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

Black body $\Rightarrow \rho = 0$
Reflectivity



Gray Surface

$$Q = \frac{E_{b1} - E_{b2}}{\frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}}$$

$$Q = \frac{A_1 (E_{b1} - E_{b2})}{\frac{1 - \epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1 - \epsilon_2}{\epsilon_2}}$$

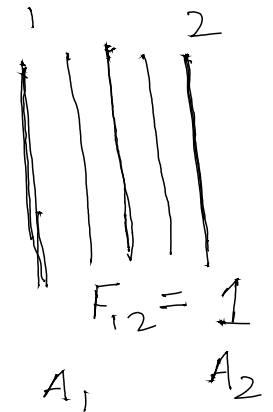
$$= \frac{A_1 (E_{b1} - E_{b2})}{\left(\frac{1}{\epsilon_1} - 1\right) + 1 + \left(\frac{1}{\epsilon_2} - 1\right)}$$

$$Q = \frac{A_1 (E_{b1} - E_{b2})}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}$$

$$Q = 1/A_1 (E_{b1} - E_{b2}) \quad \text{black body}$$

$$\frac{T_1 > T_2}{A_1 F_{12} F_{1-2}}$$

$$\epsilon E_b + \rho G$$

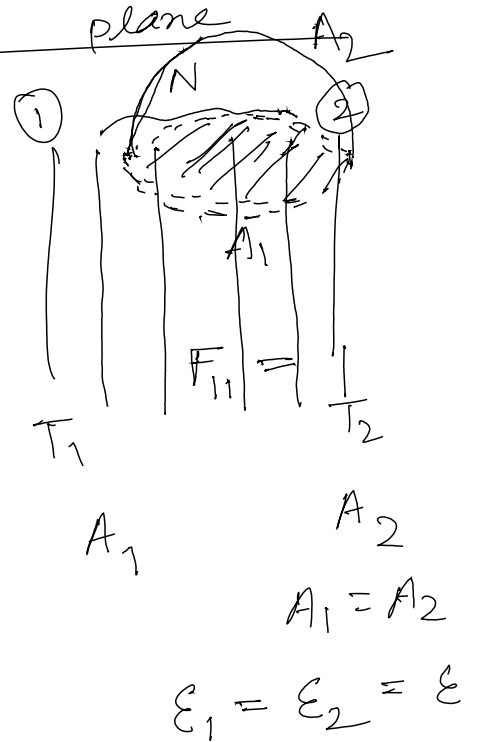


$$A_1 = A_2$$

$\epsilon_1 = 1$
 $\epsilon_2 = 1$
black body

Heat transfer? between two infinite parallel planes

$F_{12} = 1$ (For infinite parallel plane)



$$Q = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \left(\frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{2}{\epsilon} - 1} \right)$$

$$Q = \left(\frac{1}{N+1} \right)$$

$$Q = \left(\frac{1}{N+1} \right) \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{2}{\epsilon} - 1} \quad \text{Radiation shield}$$

N of shield between plane 1 & 2

⇒ Two parallel gray plane which are very large have emissivities $\epsilon_1 = 0.8$ & $\epsilon_2 = 0.7$. Surface having $T_1 = 866.5 \text{ K}$ & surface

$$T_2 = 588.8 \text{ K}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$$

$$\text{Find } \left(\frac{Q_{12}}{A_1} \right) = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = 15010 \text{ W/m}^2$$

$$\frac{Q_{12}}{A_1} \text{ for black surface, } \Rightarrow \sigma (T_1^4 - T_2^4) = 25710 \text{ W/m}^2$$

Deter $F_{22} = ?$ ✓✓

$$F_{11} + F_{12} = 1 \quad \checkmark$$

$$F_{12} = 1$$

$$A_1 F_{12} = A_2 F_{21}$$

$$A_1 = \pi r^2$$

$$A_2 = 2\pi r^2$$

$$\Rightarrow (\pi r^2) F_{12} = 2(\pi r^2) F_{21}$$

$$\Rightarrow F_{21} = \frac{F_{12}}{2} = \frac{1}{2}$$

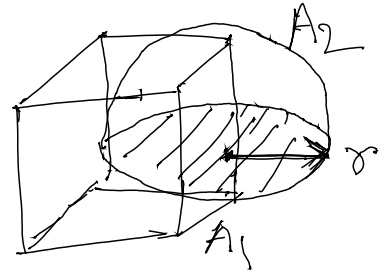
$$F_{21} + F_{22} = 1$$

$$\Rightarrow F_{22} = 1 - F_{21} = 1 - \frac{1}{2} = \frac{1}{2} = 0.5$$



$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{31} + F_{32} + F_{33} = 1$$



$$F_{11} = 0$$

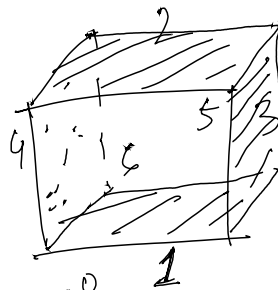
$$F_{12} = 1$$



$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

$$F_{21} + F_{22} + F_{23} + F_{24} = 1$$

$$F_{31} + F_{32} + F_{33} + F_{34} = 1$$



$$F_{11} + F_{12} + F_{13} + F_{14} + F_{15} + F_{16} = 1$$

$$F_{21} + F_{22} + F_{23} + F_{24} + F_{25} + F_{26} = 1$$

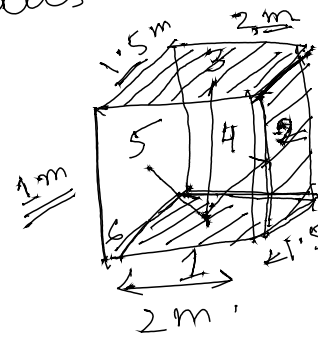
mine rate of heat transfer from floor to roof and floor to side walls if the

1) Surfaces are considered as black surface

2) Surfaces are considered as gray surface.

⇒ Emissivity of floor is 0.8

⇒ " of walls & roof 0.5



$$T_1 = 1000K$$

$$T_2 = T_3 = T_4 = T_5 = T_6 = 600K$$

2 & 5 identical

4 & 6 identical.

$$\left. \begin{aligned} F_{12} = F_{15} &= 0.14 \\ F_{14} = F_{16} &= 0.18 \\ F_{13} &= 0.36 \end{aligned} \right\} \begin{aligned} F_1 + F_{12} + F_{13} + F_{14} + F_{15} + F_{16} &= 1 \\ 0.14 + 0.18 + 0.36 + 0.14 + 0.14 &= 1 \\ 0.14 + 0.36 + 0.18 + 0.14 &= 0.82 \end{aligned}$$

$$Q_{1-2} = Q_{1-5} = A_1 F_{12} (T_1^4 - T_2^4) = 6909$$

$$A_1 = (2 \times 1.5) = 3$$

$$14805$$

$$Q_{13} =$$

$$Q_{14} = Q_{16} =$$

$$A_3 = (2 \times 1.5)$$

$$A_2 = (1.5 \times 1) \Rightarrow$$

$$A_4 = (2 \times 1)$$

$$Q_{12} = Q_{15} = A_1 F_{12} (T_1^4 - T_2^4)$$

$$= \frac{1 - \epsilon_1}{A_1 \epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}$$

$$= \frac{1 - 0.8}{3 \times 0.8} + \frac{1}{3 \times 0.14} + \frac{1 - 0.5}{(1.5 \times 1) \times 0.5}$$

$$= 8324$$

$$8820$$

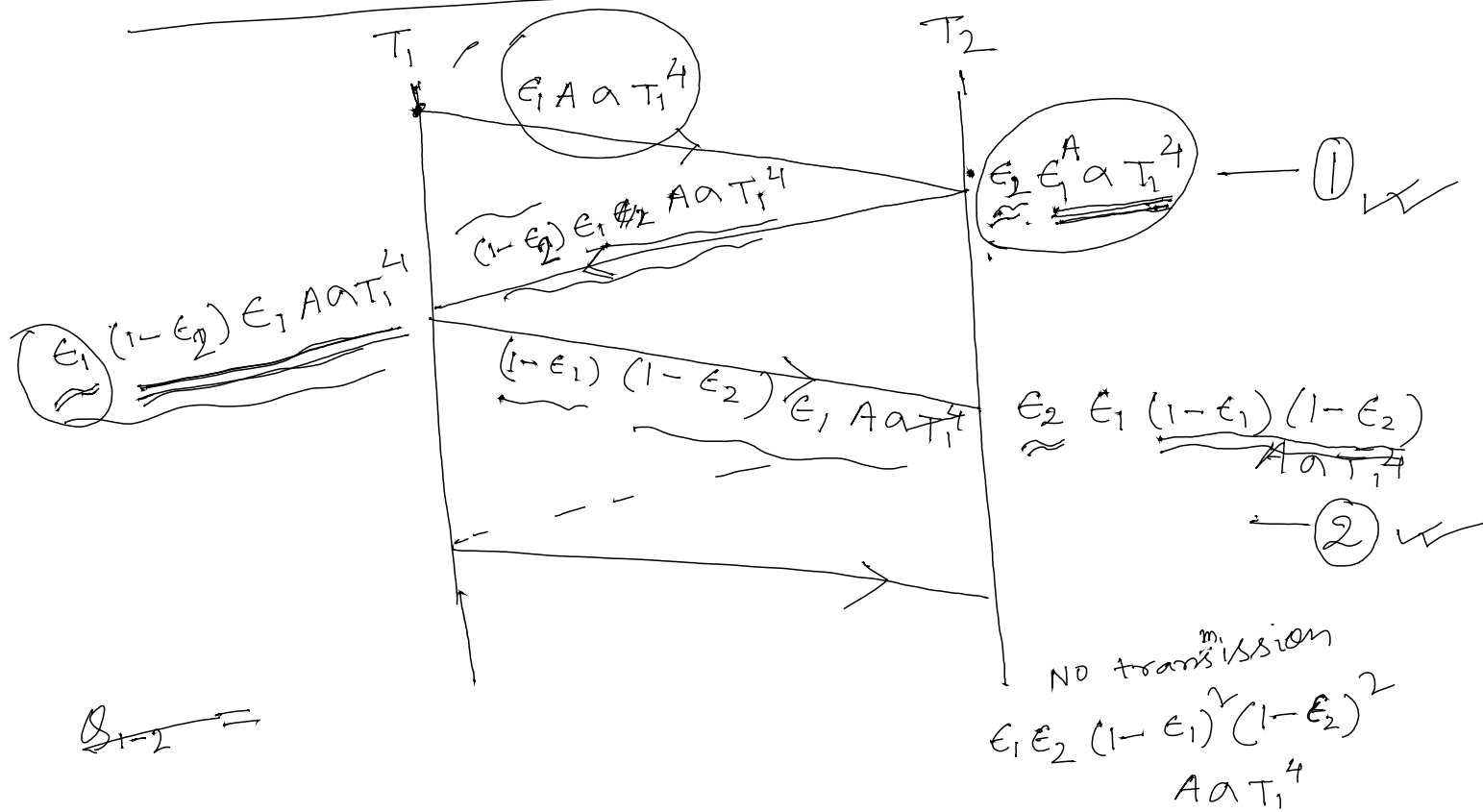
$$(2 \times 1.5 \times 1)$$

$$\epsilon_1 = 0.8$$

$$A_1 = (1.5 \times 2) = A_3$$

$$\epsilon_2 = 0.5$$

Heat transfer between two infinite parallel plane



$$Q_{1-2} = \epsilon_1 \epsilon_2 A \sigma T_1^4 \left[1 + \frac{(1-\epsilon_1)(1-\epsilon_2)}{(1-\epsilon_1)^3 (1-\epsilon_2)^3} + \dots \right]$$

$$(1-\epsilon_1)(1-\epsilon_2) = x \quad \underline{x < 1}$$

$$Q_{1-2} = \epsilon_1 \epsilon_2 A \sigma T_1^4 \left[1 + x + x^2 + x^3 + \dots \right]$$

$$= \epsilon_1 \epsilon_2 A \sigma T_1^4 \left[\frac{1}{1-x} \right] \checkmark$$

$$= \epsilon_1 \epsilon_2 A \sigma T_1^4 \left[\frac{1}{1 - (1-\epsilon_1)(1-\epsilon_2)} \right]$$

$$= \epsilon_1 \epsilon_2 A \sigma T_1^4 \left[\frac{1}{1 - [x + \epsilon_1 \epsilon_2 - \epsilon_1 - \epsilon_2]} \right] \checkmark$$

$$Q_{1-2} = A_1 \alpha T_1^4 \left[\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \right]$$

$$Q_{1-2} = A_1 \alpha T_1^4 \left[\frac{1}{\frac{1}{\epsilon_2} + \frac{1}{\epsilon_1} - 1} \right]$$

$$Q_{2-1} = A \alpha T_2^4 \left[\frac{1}{\frac{1}{\epsilon_2} + \frac{1}{\epsilon_1} - 1} \right]$$

$$Q = Q_{1-2} - Q_{2-1} = \frac{A \alpha (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}$$

N Radiation Shield

$$Q = \frac{1}{(N+1)} \frac{A \alpha (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}$$

~